Applying Distributed Constraint Optimization Approach to the User Association Problem in Heterogeneous Networks

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***Abstract*—User association has emerged as a distributed resource allocation problem in the heterogeneous networks (HetNets). Although an approximate solution is obtainable using the approaches like combinatorial optimization and game theory based schemes, these techniques can be easily trapped in local optima. Furthermore, the lack of exploring the relation between the quality of the solution and the parameters in the HetNet (e.g. the number of users and BSs), at what levels, impairs the practicability of deploying these approaches in a real world environment. To address these issues, this paper investigates how to model the problem as a distributed constraint optimization problem (DCOP) from the point of the view of the multi-agent system. More specifically, we develop two models named Each Connection As Variable (ECAV) and Each BS and User As Variable (EBUAV). Hereinafter, we propose a DCOP solver which not only sets up the model in a distributed way but also enables us to efficiently obtain the solution by means of a complete DCOP algorithm based on distributed message-passing. Naturally, both theoretical analysis and simulation show that different qualitative solutions can be obtained in terms of an introduced parameter η which has a close relation with the parameters in the HetNet. It is also apparent that there is 6% improvement on the throughput by the DCOP solver comparing with other counterparts when $\eta = 3$. Particularly, it demonstrates up to 18% increase in the ability to make BSs service more users when the number of users is above 200 while the available RBs are limited. In addition, it appears that the distribution of RBs allocated to users by BSs is better with the variation of the volume of RBs at the macro BS.**

***Index Terms*—HetNets, user association, DCOP, solution quality.**

I. INTRODUCTION

A heterogeneous network (HetNet) is composed of several tiers including macrocells, picocells, and femtocells. Different cells supply service to a variety of zones ranging from the outdoor to indoor environment. A challenging problem in the HetNet is integrating resources (spectrum, power, sub-channel) to optimize the system performance (throughput, energy efficiency). Thus, a series of resource allocation problems, such as user/cell association, inter-cell interference management, have attracted considerable attention [1–5]. In this paper, we focus on the user association problem in the downlink of the HetNet which aims to assign mobile users to different BSs in different tiers while satisfying the QoS constraint on the rate required by each user.

The prevalent solution schemes for the user association problem are broadly divided into three categories [6, 7] including 1) stochastic geometry based scheme, 2) game theory based scheme, and 3) combinatorial optimization based scheme. The methods from the first category mainly include Max-SINR (maximum signal-to-interference-plus-noise ratio), Max-RSS (maximum received signal strength) methods with the intention of setting up the connection between a user and a BS with large SINR (signal-to-interference-plus-noise ratio). Thus, it is difficult to guarantee the load balance between macrocell BSs and small cell BSs [8] since the SINR or the signal between the users and macrocell BSs is always larger. Although a bias is added to the users' power received from the small cell BSs, the way to determine the bias is also a difficult problem.

The schemes from the other two categories, by contrast, are able to avoid such unfairness. More precisely, game theory based scheme aims at modeling the users or BSs as players and then investigating the interaction between these players [6]. Particularly, Nash bargaining and the matching theory are the specific methods which have been widely used for solving user association problem [9, 10]. The methods in combinatorial optimization based scheme formulates the problem as a non-convex mixed integer programming which is transformed to a convex one by relaxing the discrete domain of variables into the continuous one. Then the solution is obtained by means of a numerical technique such as Lagrange dual decomposition (LDD) [4, 11, 12]. However, In the game theory based schemes, the players (BSs or users) can not act in a rational manner all the time due to the fact that different players (e.g. BSs) always have different optimization objectives [6]. On the other hand, the relaxation in combinatorial optimization based scheme leads to a duality gap between the primal and dual

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problems. Further, the lack of exploring the relation between the quality of the solution and the parameters in the HetNet (e.g. the number of users, BSs) impairs the practicability of deploying these approaches in a real world environment.

On the other hand, interest in applying the multi-agent system for addressing resource allocation problem in the HetNet has been on the rise [13, 14]. This is attributable to the following reasons. First, with the aid of intelligent agents, the control and responsibilities within the multi-agent system are sufficiently shared among agents. In this way, the system can tolerate failures of one or more agents. Second, the operation of adding agents to a multi-agent system is more efficiently than adding new capabilities to a monolithic system. Third, the consensus problem in the field of multi-agent system, aiming at designing an appropriate control input to make a group of agents converge to a consistent quantity of interest, has been further researched [15, 16]. In recent years, DCOP has emerged as a credible framework of multi-agent system to tackle a series of distributed resource allocation problems such as energy-efficient smart environment configuration [17], target location in the sensor networks [18] and the management of water resources systems [19]. Therefore, it motivates us to use DCOP technique to revisit the user association problem from the point of the view of the multi-agent system.

In this paper, we develop a DCOP solver including the building of ECAV/EBUAV-η model and the execuation of a complete DCOP algorithm. The solver enables us to obtain the optimal or suboptimal solution. Naturally, both theoretical analysis and simulation show that different qualitative solutions can be guaranteed in terms of different assignments of η. In brief, the contribution is as follows:

- To the best of our knowledge, this is the first attempt to bridge multi-agent system and the user association problem in the HetNet by bringing about the technique of DCOP. Also, we introduce a parameter η which have benefit to make a trade-off between the performance and complexity of the DCOP solver.
- Based on the complete DCOP algorithm, the lower bound of the solution can be guaranteed through a theoretical analysis of the parameter η, allowing for deploying the DCOP solver in a real world environment.
- The simulation shows that the resource allocation strategy provided by the DCOP solver is better than the ones obtained by the Max-SINR and LDD based schemes. Particularly, it has a better robustness when the number of users increases while the available resources are limited.

The rest of this paper is organized as follows. In Section II, we briefly introduce related works of user association problem and DCOP applications. In Section III, we describe the system model of k-tier HetNet. The DCOP models of user association problem are presented in Section IV. After that, we propose a DCOP solver along with the theoretical analysis of the solution obtained by the solver in Section V. Then, we explore the performance of our proposed method by comparing with the Max-SINR and LDD based schemes in Section VI. Finally, Section VII draws the conclusion.

II. Related works

In this section, we firstly introduce some novel methods for solving user association problem. Then, we present the definition of DCOP along with DCOP applications in real life.

A. User Association Problem

User association problem can be formulated as a distributed resource allocation problem in which a unit resource refers to a resource block (RB) that covers a certain frequency range and time duration [20] according to the LTE (Long Term Evaluation) technology. In past decades, noticeable research effort has been dedicated to the development of distributed methods, especially game theory based and combinatorial optimization based schemes [4, 5, 21, 22], partly due to the fact that less feedback overhead is needed between users and BSs.

Game theory based scheme models the users, BSs or both as players and explores the solution through the interaction between the players. For instance, [5] and [22] modeled the users as players. When the Nash equilibrium is achieved, the solution (pure strategy) is a stable profile of actions with which no player can obtain a personal gain by changing to another strategy [23]. The difference between these two research methods relied on the various objective functions formulated in the system model. Ha and Le in [5] solved a joint user association and power control problem. An iterative algorithm was proposed with a minimization of effective interference as the metric of interest. Zhen et al. in [22] solved a multi-dimensional resource optimization problem with the objective of suppressing the interference and improving the overall system throughput while ensuring the QoS of femtocell users. Liu et al. [10] formulated the user association as a bargaining problem by modeling different BSs as competing players who bargained with each other for the sake of attaining mutual advantages. Pantisano et al. [9] exploited a matching game by regarding small cell base stations and users as players. To solve this game, they proposed a distributed algorithm which enabled the players to self-organize into a stable matching that guaranteed the required applications' QoS.

As for the combinatorial optimization based scheme, the formulation of the system model falls into the scope of non-convex mixed integer programming [21, 24–27], which is always NP-hard. The way to obtain the solution has been very challenging and most attempts to address the problem have yielded encouraging results. Fooladivanda and Rosenber [21] transformed the non-convex problem to a convex one by relaxing the discrete domain of the variable in the association constraint into the continuous one. After that, the numerical technique for solving convex problems could be applied to obtain the suboptimal solution, which was always an upper bound of the optimal solution. Similarly, after relaxing the constraints of both resources and energy in the system model of RES powered HetNets, Han et al. [27] applied an optimal offline algorithm using discretization and dual decomposition. In [4], Hamidreza and Bhargava put forward an LDD based iterative algorithm with which the users and BSs made their respective decisions using local SINR information. A global

QoS, expressed in terms of minimum achievable long-term rate or maximum outage probability, was achieved. Ye et al. [26] developed a unified framework, where resource allocation was cast as a network utility maximization problem. this particular problem was overcome by a dual subgradient based algorithm which converged towards the solution.

Although there is no sufficient evidence to infer that one method is better than the others, the simulation results in [4] and [22] show that the performance of the methods in game theory based and combinatorial optimization based schemes are better than some methods in the stochastic geometry based schemes, e.g. Max- SINR. However, both of these two schemes are confined to the following two aspects: 1) the HetNet is a dynamic system in which there is frequent variation of the number of users, the configuration of the resource at BSs; 2) there is little work on exploring the relation between the quality of the solution and the parameters in the HetNet, thus impairing the practicability of deploying these approaches in a real world environment. Consequently, it inspires us to model the user association problem as a multi-agent system.

B. Distributed Constraint Optimization Problem

DCOP is a set of cooperation techniques for multi-agent system, effective for optimizing the global objective function which is an aggregation of distributed cost functions. It is generally presented as a four tuples model $<\mathcal{A},\mathcal{V},\mathcal{D},\mathcal{C}>$ where $\mathcal{A}=\{a_1,a_2,...,a_{|\mathcal{A}|}\}$ [1] is a set of agents, $\mathcal{V}=\{v_1,v_2,...,v_n\}$ is a set of variables, $\mathcal{D}=\{d_1,d_2,...,d_n\}$ consists of all the domains from different variables and $\mathcal{C}=\{c_1,c_2,...,c_{|\mathcal{C}|}\}$ is the set of constraints between variables. Each variable $v_i \in \mathcal{V}$ belongs to a unique agent $a_i \in \mathcal{A}$, and each constraint $c \in \mathcal{C}$ is defined as a mapping from the assignments of m variables to a positive real value

$$R(c): d_{i_1} \times d_{i_2} \times \cdots d_{i_m} \rightarrow \mathbb{R}^+, \tag{1}$$

The objective of a DCOP is to find a set of assignments of all the variables, denoted as $\mathcal{X}^*$, which maximize the utility, namely the sum of all constraint rewards:

$$\underset{\mathcal{X}^*}{\operatorname{argmax}} \sum_{\mathcal{C}} R(c). \tag{2}$$

To obtain the optimal solution, a large literature exists on the solution methods which fall into two categories including the ADOPT [28] based algorithms which rely on the message propagation, and the DPOP [29] based algorithms which are depended on the inference strategy.

Recently, DCOP technique has played an increasingly essential role when we model practical problems as multi-agent systems [30–32]. An agent, characterized by the autonomy and distributivity, is capable of making decision independently without a centralized controller. Katsuya, Kayo and Yasuki [30] employed the DCOP technique to model a rescue system so that a real-time evacuation guidance was provided for the victims. Enembreck and Barthes modeled the distributed meetings scheduling problem as a DCOP where the time slots were modeled as variables, and the set of meetings potentially scheduled within a time slot was modeled as the domain of a variable.

[1] $|X|$ denotes the cardinality of X

According to the survey made by [33], the aforementioned problems, at what levels, could be regarded as a kind of distributed resource allocation problems. Thus, DCOP techniques have the potential of enabling the design of the user association problem in the HetNet. Besides, considering the dynamic environment (e.g. the mobility of users or the plug-and-play property of BSs in small cells [34]), the variant of DCOP modeling framework, named StochDCOP [35], is an effective tool to capture these variations in the dynamic environment by modeling the sources of uncertainty as random, uncontrollable variables.

In order to apply advanced DCOP techniques to the user association problem in the HetNet, the first pahse in the corresponding research is to model the problem as a multi-agent system using DCOP framework. Moreover, one of the challenges is the construction of the constraints on account of the rate QoS affected by the distinctions of the configuration at different BSs, e.g. transmit power or resource. Another challenge is the large amount of the users, resource, which may potentially reduce the performance of solution scheme. These challenges led to the modeling methods in the existing DCOP applications are not available for the user association problem.

III. System Model

A tier in the HetNet indicates a macrocell, picocell, or femtocell where each tier contains a set of BSs with the same configurations, e.g., the transmit power and resource. According to the OFDMA (Orthogonal Frequency Division Multiple Access) technique, the resource configured at a BS refers to a set of resource blocks (RBs) where each RB consists of a certain time duration and certain bandwidth [36]. A two-tier HetNet is given in Fig.1 using a multilayer graph where each tier consists of a BS, respectively denoted as $\mathcal{B}_1$ and $\mathcal{B}_2$. Four users are deployed in the HetNet, denoted as $\mathcal{U}=\{\mathcal{U}_1,\mathcal{U}_2,\mathcal{U}_3,\mathcal{U}_4\}$. $\mathcal{B}_1$ is capable of providing service to $\{\mathcal{U}_1,\mathcal{U}_2,\mathcal{U}_3\}$, while $\mathcal{B}_2$ is able to provide service to $\{\mathcal{U}_1,\mathcal{U}_2,\mathcal{U}_4\}$.

Given a k-tier HetNet including $\mathcal{NB}$ BSs and $\mathcal{NU}$ users respectively denoted by $\mathcal{B}=\{\mathcal{B}_1,\mathcal{B}_2,...,\mathcal{B}_{\mathcal{NB}}\}$ and $\mathcal{U}=\{\mathcal{U}_1,\mathcal{U}_2,...,\mathcal{U}_{\mathcal{NU}}\}$. Assuming the channel state information is available at the BSs, also, the BSs from different tiers share the total bandwidth such that there are both intra- and inter-tier interference when the BSs allocate RBs to the users instantaneously. Therefore, the SINR experienced by user $\mathcal{U}_j$ served by $\mathcal{B}_i$ in the kth tier is given by

$$\mathrm{SINR}_{ij} = \frac{P_k g_{ij}}{\sum_{\mathcal{B}_l \in \mathcal{B}/\{\mathcal{B}_i\}} P_k g_{lj} + BN_0}, \tag{3}$$

where P_k is the transmit power for the BSs in the kth tier, g_{ij} is the channel power gain between $\mathcal{U}_j$ and $\mathcal{B}_i$, $\mathcal{B}/\{\mathcal{B}_i\}$ represents all the BSs in $\mathcal{B}$ except $\mathcal{B}_i$, B is the bandwidth and N_0 is noise power spectral density. The channel power gain includes the effect of both path loss and fading. Path loss is

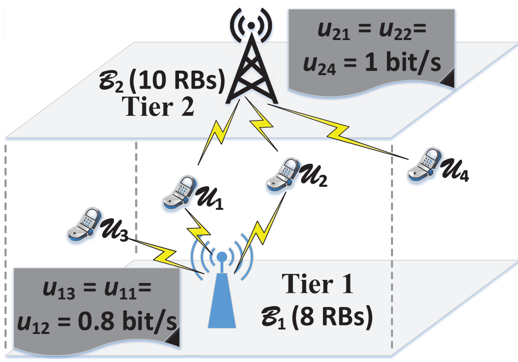


Fig. 1: A two-tiers HetNet

assumed to be static and its effect is captured in the average value of the channel power gain, while the fading is assumed to follow the exponential distribution. Then the efficiency of user $\mathcal{U}_j \in \mathcal{U}$ powered by BS $\mathcal{B}_i \in \mathcal{B}$, denoted as e_{ij} is

$$e_{ij} = log_2(1 + \mathrm{SINR}_{ij}). \tag{4}$$

Consider the bandwidth B, time duration T and the scheduling interval Γ configured at each RB, we attain the unit rate at $\mathcal{U}_j$ upon one RB as

$$u_{ij} = \frac{BTe_{ij}}{\Gamma}. \tag{5}$$

On the basis of formula (5), the rate received at $\mathcal{U}_j$ with n_{ij} RBs provided by $\mathcal{B}_i$ in the kth tier is

$$r_{ij} = n_{ij}u_{ij}. \tag{6}$$

The quality-of-service (QoS) constraint of each user is expressed as the minimum total rate the user should receive. Denoting the rate requiremnt from all the users as γ, the minimum number of RBs required to satisfy γ is estimated by:

$$n_{min}^{ij} = \lceil \frac{\gamma}{u_{ij}} \rceil \tag{7}$$

in which $\lceil \cdot \rceil$ is a ceiling function.

IV. Problem Formulation with DCOP

The primary step for solving the user association problem is to satisfy the basic QoS rate requirements from the users. To this end, the following sections develop the ECAV and EBUAV model which play an important role for designing a DCOP solver. Finally, a parameter η is introduced to restrict the scale of these two models.

A. ECAV

The key step towards modeling a user association problem as a DCOP is mapping the entities in the HetNet, e.g. the users, BSs and resource to the four tuples in the DCOP model. Motivated by the modeling method of the distributed meeting schedule problem in [31], we firstly introduce the definition of candidate BS according to the protentional connection between the BSs and users:

Definition 1. Candidate BS: we declare $\mathcal{B}_i \in \mathcal{B}$ is a candidate BS of $\mathcal{U}_j \in \mathcal{U}$ if $\mathcal{B}_i$ is able to satisfy the rate QoS requirement of $\mathcal{U}_j$ without overload of resource.

In other words, Definition 1 indicates that $r_{ij} \geq \gamma$ and $n_{min}^{ij} \leq \mathcal{R}_i$ where $\mathcal{R}_i$ is the aggregate RBs configured at $\mathcal{B}_i$. On the basis of Definition 1, we define each BS $\mathcal{B}_i$ as a an agent, denoted as a_i, and each connection between a user $\mathcal{U}_j$ and one of its corresponding candidate BS $\mathcal{B}_i$ as a variable, denoted as v_i^j. As a result, $\mathcal{A} = \{a_1, a_2, ..., a_{\mathcal{NB}}\}$ and $\mathcal{V} = \{v_i^j | j \in \mathcal{NU}, i \in |\mathcal{CB}_j|\}$. Each variable has a binary domain $d_i^j = \{0, n_{min}^{ij}\}$ where v_i^j is 0 if $\mathcal{B}_i$ does not allocate RB to $\mathcal{U}_j$. Otherwise, v_i^j has the value of n_{min}^{ij}. Thus $\mathcal{D} = \{d_i^j | d_i^j = \{0, n_{min}^{ij}\}\}$. An intra-constraint connecting n variables in a_i is formulated as an n-ary constraint which represents the number of RBs consumed at $\mathcal{B}_i$. Note that a user may have more than one candidate BS. Therefore, there is also an inter-constraint between the variables in different agents. We use $\mathcal{C} = \mathcal{C}_{inter} \cup \mathcal{C}_{intra}$ to denote the set of intra- and inter-constraints in the ECAV model, then the reward $R(c)$ of each constraint is given as follows. For $\forall c \in \mathcal{C}_{inter}$

$$R(c) = \begin{cases} 0, & \text{for } \sum_{v_i^j \in \psi(c)} \frac{Val(v_i^j)}{n_{min}^{ij}} \leq 1 \quad \text{(8a)} \\ -\infty, & \text{otherwise} \quad \text{(8b)} \end{cases}$$

For $\forall c \in \mathcal{C}_{intra}$

$$R(c) = \begin{cases} -\infty, & \text{for } \sum_{v_i^j \in \psi_c} v_i^j > \mathcal{R}_i \quad \text{(9a)} \\ \sum\limits_{v_i^j \in \psi(c)} r_{ij}, & \text{otherwise} \quad \text{(9b)} \end{cases}$$

In constraint (8a), $\psi(c) \subseteqq \mathcal{V}$ is the set of variables connected by constraint c. $Val(v_i^j)$ represents the assignment of v_i^j. A small reward (we use $-\infty$ in this paper) is assigned if any constraint is violated (e.g. formulation (8b) and (9a)). Otherwise, the reward of an inter-constraint is 0 if the unique connection between users and BSs can be gauranteed (formulation (8a). Also, the reward of an intra-constraint is the rate achieved at the user if there is no overloading of resource occurred at the BS (formulation 9b)). To sum up all the rewards of the constraints, we get the following equation

$$\sum_{c \in \mathcal{C}} R(c) = \sum_{\mathcal{B}_i \in \mathcal{B}} \sum_{\mathcal{U}_j \in \mathcal{U}} r_{ij} \tag{10}$$

where $\sum_{\mathcal{B}_i \in \mathcal{B}} \sum_{\mathcal{U}_j \in \mathcal{U}} r_{ij}$ is the sum of the rates achieved at all the users, which can be regarded as a metric of the throughput in the HetNet. Thus, the larger $\sum_{c \in \mathcal{C}} R(c)$ is, the better the throughput should be.

Example 2. Recall the two-tier HetNet in Fig.1. We assume the candidate BSs of $\mathcal{U}_1$ and $\mathcal{U}_2$ are both $\{\mathcal{B}_1, \mathcal{B}_2\}$, and the candidate BSs of $\mathcal{U}_3$ and $\mathcal{U}_4$ are respectively $\{\mathcal{B}_1\}$ and $\{\mathcal{B}_2\}$. Moreover, we assume the total RBs configured at $\mathcal{B}_1$ and $\mathcal{B}_2$ is 8 and 10. For simplicity, the unit rate is 0.8 bit/s between a user and $\mathcal{B}_1$, while 1 bit/s between a user and $\mathcal{B}_2$. Fig.2a displays the ECAV model of this instance using a constraint graph where two BSs are modeled as agents a_1 and a_2. The variables in a_1 are v_1^1, v_1^2 and v_1^3, and the variables in a_2 are v_2^1, v_2^2 and v_2^4. Given the threshold rate 3 bit/s, the number of RBs needed by the users connecting with $\mathcal{B}_1$ is at least $\lceil \frac{3}{0.8} \rceil = 4$, hence the domain of each variable in a_1 is $\{0, 4\}$. Similarly, the domain of each variable in a_2 is $\{0, 3\}$. The black lines connecting the variables in each agent consist of a 3-nry intra-constraint, and the red line connecting the variables in two agents is an intra-constraint. Take the intra-constraint in a_1 as an example, the reward is $-\infty$ when the value of each variable is 4 since the total number of RBs consumed by these users is 12 which is more than 8 RBs configured at $\mathcal{B}_1$. Otherwise, the reward is 0.8 × 4 × 2 = 6.4 (bit/s) if the assignments of v_1^1, v_1^2 and v_1^3 are 4, 4 and 0. Considering the inter-constraint between v_2^1 and v_1^1, the reward is $-\infty$ if the value of v_2^1 is 3 and the value of v_1^1 is 4. In this case, it violates the assumption of unique connection since $\mathcal{U}_2$ will be served by $\mathcal{B}_1$ and $\mathcal{B}_2$ at the same time.

B. EBUAV

As an n-ary constraint in the ECAV model produces $\binom{n}{2}$ edges in the corresponding constraint graph, a message propagation based DCOP algorithm [28, 37, 38] consumes more computational resource and requires more running time when there is an increase in terms of users and BSs. To perform the DCOP algorithm in an efficient manner, we propose the EBUAV model motivated by the dual coding technology [39] which has been widely used in the DCOP framework aiming at transforming an n-ary constraint into binary one. More precisely, we construct the set of agents and variables by $\mathcal{A} = \{a_1, a_2, ..., a_{\mathcal{NB}+\mathcal{NU}}\}$ and $\mathcal{V} = \{v_1, v_2, ..., v_{\mathcal{NB}+\mathcal{NU}}\}$ where the agents (variables) are divided into two categories including "user agents (variables)" and "BS agents (variables)". Denoting the candidate BSs of user $\mathcal{U}_j \in \mathcal{U}$ by $\mathcal{CB}_j$, then the set of domains of user variables is formulated as $\mathcal{D}_\mathcal{U} = \{d_1, d_2, ..., d_{\mathcal{NU}}\}$, in which $d_j = \{0\} \bigcup \{n_{min}^{ij} | \mathcal{B}_i \in \mathcal{CB}_j\}, d_j \in \mathcal{D}_\mathcal{U}$. Besides, the domains of BS variables are formulated by $\mathcal{D}_\mathcal{B} = \{d_1, d_2, ..., d_{|\mathcal{NB}|}\}$, where each element $d_i \in \mathcal{D}_\mathcal{B}$ indicates all possible combinations of RBs assigned to m users by $\mathcal{B}_i \in \mathcal{B}$, denoted as

$$d_i \triangleq \{(x_1, ..., x_m) | x_j = \{0, n_{min}^{ij}\}, j = 1, ..., m\} \tag{11}$$

It is worth noting that the data structure used for storing $d_i \in \mathcal{D}_\mathcal{B}$ is a binary tree which has benefits to effectively deploy a search strategy on the domain space. Futher, we condense the storage space of the binary tree leveraging on the binary decision diagram (BDD) which can be denoted as an acyclic graph including a root and directed edges. It consists of the decision and terminal nodes in which terminal nodes are classified into 0-terminal and 1-terminal nodes. Particularly, a decision node, in this paper, is represented as a boolean variable $v_\mathcal{U}$. It has two child nodes called low child and high child. The edge from node $v_\mathcal{U}$ to a low (or high) child represents that $v_\mathcal{U}$ has an assignment of 0 (resp. the number of RBs allocated to the user).

The definition of the reward of the constraints are depended on the concept of **Consistency**:

Definition 3. Consistency: Given a user variable v_j with assignment $Val(v_j) \in d_j \in \mathcal{D}_\mathcal{U}$ and a BS variable v_i with assignment $Val(v_i) \in d_i \in \mathcal{D}_B$, we say the assignments of a BS variable and a user variable are consistent if $Val(v_j) = x_j, x_j \in Val(v_i)$.

For simplicity, we use $Consis(Val(v_i), Val(v_j))$ to represent the consistency between a BS variable and a user variable. According to Definition 3, the user $\mathcal{U}_j$ prefers (or not) to connect with the BS $\mathcal{B}_i$ if the constraint between them satisfies (cannot satisfy) the consistency condition. Then, we have the reward upon each constraint between a user and BS variable as follows

$$R(c) = \begin{cases} r_{ij}, & Consis(Val(v_i), Val(v_j)) \quad \text{(12a)} \\ -\infty, & \text{otherwise} \quad \text{(12b)} \end{cases}$$

Obviously, The sum of all the rewards is also the throughput in the HetNet.

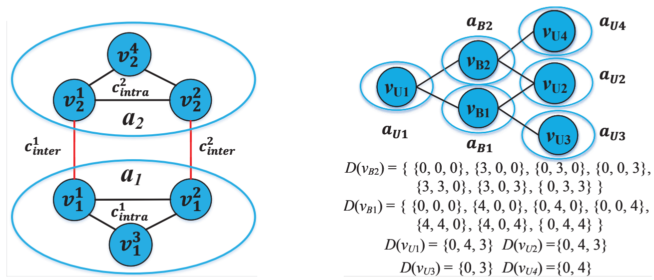
Example 4. Fig.2b is the constraint graph of the EBUAV model of the instance in Fig.1. The variables are $\{v_{\mathcal{B}_1}, v_{\mathcal{B}_2}, v_{\mathcal{U}_1}, ..., v_{\mathcal{U}_4}\}$ controlled by the agents $\{a_{\mathcal{B}_1}, a_{\mathcal{B}_2}, a_{\mathcal{U}_1}, ..., a_{\mathcal{U}_4}\}$. Take $v_{\mathcal{U}_1}$ and $v_{\mathcal{B}_1}$ as examples, the domain of $v_{\mathcal{U}_1}$ is $\{0, 4, 3\}$, where 0 means $\mathcal{U}_1$ does not connect with any BS, while 4 or 3 means $\mathcal{U}_1$ respectively connects with $\mathcal{B}_1$ or $\mathcal{B}_2$. The domain of $v_{\mathcal{B}_1}$ is denoted as $\mathcal{D}(v_{\mathcal{B}_1})$. Each value in the domain is in the format of a three-tuple (x_1, x_2, x_3), where x_1, x_2, x_3 respectively represents the number of RBs allocated to $\mathcal{U}_1, \mathcal{U}_2$ and $\mathcal{U}_3$ by $\mathcal{B}_1$. Particularly, $\{0, 0, 0\}$ refers that $\mathcal{B}_1$ does not allocate any RBs to $\mathcal{U}_1, \mathcal{U}_2$ and $\mathcal{U}_3$. There is no value $(4, 4, 4)$ in $\mathcal{D}(v_{\mathcal{B}_1})$ because the total number of RBs consumed by these three users is 12 which is more than 8 RBs configured at $\mathcal{B}_1$. $\mathcal{D}_{v_{\mathcal{B}_1}}$ is represented as a binary tree in Fig.3a, and formulated as a BDD which has fewer nodes and edges in Fig.3b. With the definition of consistency, the reward of the constraint between $v_{\mathcal{U}_1}$ and $v_{\mathcal{B}_1}$ is 3.2 bit/s when the assignment of $v_{\mathcal{U}_1}$ is 4, while the assignment of $v_{\mathcal{B}_1}$ is $(4, 0, 0)$, $(4, 4, 0)$ or $(4, 0, 4)$. Otherwise, the utility is $-\infty$

The difference between EBUAV and ECAV model is as follows:

- The number of agents ($\mathcal{NB}+\mathcal{NU}$) in the EBUAV model is more than the number of agents ($\mathcal{NB}$) in the ECAV model, but shows a linear growth with the increment of users and BSs. The extra agents have a light influence on the performance of DCOP algorithms.
- The constraint in the EBUAV model is 2-nry rather than the n-ry $(n \geq 2)$ in terms of the constraint in the ECAV model. As a result, there is a decline in the amount of the messages delivered between agents.

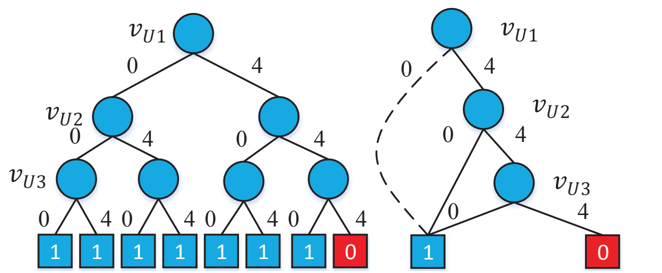
C. ECAV/EBUAV-η

The size of the domain for a BS variable is at worst $O(2^m)$ if there are m users potentially connecting with the same BS. The critical factor to give rise to the exponential growth of memory is a large amount of candidate BSs of some users. However, some candidate BSs of users can be ignored because they are far from the user. Although these BSs are able to



(a) ECAV model (b) EBUAV model

Fig. 2: The ECAV/EBUAVmodel of the instance in Fig.1



(a) Binary tree (b) BDD

Fig. 3: Store the domain of variable $v_{\mathcal{B}_1}$ with binary tree and BDD

satisfy the rate QoS requirement of a user, they have to expend a lot of RBs. Such situation is ordinarily discouraged and seldom occurs in an optimal solution.

Assuming the candidate BSs of a user is $\mathcal{CB} = \{\mathcal{B}_1, \mathcal{B}_2, ..., \mathcal{B}_{|\mathcal{CB}|}\}$ with $\mathcal{B}_1 \preceq \mathcal{B}_2 \preceq ... \preceq \mathcal{B}_{|\mathcal{CB}|}$ where $\mathcal{B}_p \preceq \mathcal{B}_q$ means the number of RBs consumed at $\mathcal{B}_p$ is smaller than that consumed at $\mathcal{B}_q$. We then come up with a parameter η with which we make each user select top η candidate BSs from $\mathcal{CB}$, denoted as $\hat{\mathcal{CB}}$. For the sake of simplicity, we use the term of candidate users, abbreviated as $\hat{\mathcal{CU}}$ to denote the set of users potentially served by a BS. As $|\hat{\mathcal{CU}}| < m$, it is sufficient for us to believe that the complexity of the ECAV/EBUAV-η becomes smaller when we select a feasible value for η.

V. DCOP Solver and Solution

The technical challenges for desgining a DCOP solver lie in setting up the ECAV/EBUAV-η model in a distributed way and ensuring the resulting DCOP's solution is acceptable and feasible. Thus not do we need to develop the algorithm for modeling and addressing the user association problem, but we need to make an analysis on the quality of obtained solution.

A. DCOP Solver for User Association

Algorithm 1 is the pseudo code for determining $\hat{\mathcal{CB}}$ and $\hat{\mathcal{CU}}$. It takes time of $O(\mathcal{NB})$ for a user to determine $\mathcal{CB}$ from line 1 to 5. Hereinafter, $\hat{\mathcal{CB}}$ is estimated through line 6 to 15. The time complexity mainly rests on the "$\preceq$" operation by means of a sorting algorithm whose time complexity is $\mathcal{NB}^2$, like bubble sort. To sum up, the time expended for the determination of $\hat{\mathcal{CB}}$ is $O(\mathcal{NB}^2 + 2\mathcal{NB})$. A user $\mathcal{U}_j$ informs its candidate BS $\mathcal{B}_p \in \hat{\mathcal{CB}}_j$ by sending a message named

Algorithm 1 $\hat{\mathcal{CB}}_j$ of user $\mathcal{U}_j \in \mathcal{U}$ and $\hat{\mathcal{CU}}_i$ of user $\mathcal{B}_j \in \mathcal{B}$

Each User:
1: **for** $\mathcal{B}_i \in \mathcal{B}$ **do**
2:     **if** $r_{ij} \geq \gamma$ **then**
3:         $\mathcal{CB}_j \bigcup \{\mathcal{B}_i\}$
4:     **end if**
5: **end for**
6: "$\preceq$" operation on $\mathcal{CB}_j$ using a sorting algorithm
7: **if** $|\mathcal{CB}_j| > \eta$ **then**
8:     **for** p from 1 to η **do**
9:         $\hat{\mathcal{CB}}_j \bigcup \{\mathcal{B}_p \in \mathcal{CB}_j\}$
10:         send **CB_Msg** to $\mathcal{B}_p$ $\triangleright$ a user sends message to the candidate BS
11:     **end for**
12: **else**
13:     $\hat{\mathcal{CB}}_j \leftarrow \mathcal{CB}_j$
14:     send **CB_Msg** to all $\mathcal{B}_p \in \hat{\mathcal{CB}}_j$
15: **end if**
Each BS:
16: **if** message = **CB_Msg** from $\mathcal{U}_j$ **then**
17:     $\hat{\mathcal{CU}}_i \bigcup \{\mathcal{U}_j\}$
18: **end if**

CB_Msg. After that, $\mathcal{B}_p$ in turn sets up $\hat{\mathcal{CU}}_p$ as soon as $\mathcal{B}_p$ recieves **CB_Msg** (line 16).

Algorithm 2 presents the pseudo code for the construction of the ECAV/EBUAV-η model. In the case of the ECAV-η model, each BS models itself as an agent in line 21. The variables along with the domains are set up by each user (line 3 and 4). After that, the rewards depended on intra- and inter-constraints are respectively determined by the users and BSs in line 5 and 22. As for the EBUAV-η model, a BS models itself as an agent/variable and sets up the domain, constraint and rewards from line 25 to 28. Note that the building of a BS agent/variable relies on the construction of the binary tree and BDD (line 28) and procedure BuildBinaryTree(Root, N_{RB})). The time complexity is, at worst, $O(2^{\frac{|\hat{\mathcal{CU}}_j|}{2}})$.

The terminative signal of the model building is depended on the pseudo code in line 30 where $\mathcal{B}_i$ sends message **CU_Msg** to the users not belonged to the current $\hat{\mathcal{CU}}_i$ in order to confirm the completeness of $\hat{\mathcal{CU}}_i$. If $\mathcal{U}_j$ has already identified the $\hat{\mathcal{CB}}_j$, it sends back **End_Msg** to the BS (line 15 and 16). After that, the BS broadcasts **Start_Algo_Msg** to inform the users to run the DCOP algorithm (from line 31 to 33). Ideally, any DCOP complete algorithm can be utilized to obtain the optimal solution based on the EBUAV-η model. However, the DPOP based algorithms have exponential growth on the storage occupation with respect to both the domain size and number of variables. Thus, in this paper, we utilize a message propagation based algorithm, namely BnB-ADOPT [41], which is not only asynchronously executed by agents but has fewer exchanging messages. After that, the BSs who have redundant resource can proceed a greedy strategy by allocating the RBs to the users with the largest rate in order to improve the throughput in the HetNet. With the vast availability of incomplete DCOP algorithms [42–44] which have benefits to supply a suboptimal solution with theoretical error bound, it is a future job to apply the incomplete algorithms to our proposed DCOP solver.

Algorithm 2 ECAV/EBUAV-η

Each User: $\mathcal{U}_j \in \mathcal{U}$
procedure SETECAV
  for $\mathcal{B}_i \in \mathcal{C}\hat{\mathcal{B}}_j$ **do**
    $\mathcal{V} \bigcup \{v_i^j\}$
    $d_i^j \leftarrow \{0, n_{min}^{ij}\}, \mathcal{D} \bigcup \{d_i^j\}$
    $\mathcal{C} \bigcup \{c_{intra}^j\}$, $R \bigcup \{R(\mathcal{C}_{intra}^i)\}$ based on (9)
  end for
end procedure
procedure SETEBUAV
  $\mathcal{U}_j$ is modeled as an agent $a_j \in \mathcal{A}$ and variable $v_j \in \mathcal{V}$
  $\mathcal{D}(v_{\mathcal{U}_j}) \leftarrow \{0, n_{min}^1, ..., n_{min}^{|\mathcal{C}\hat{\mathcal{B}}_j|}\}$,$\mathcal{D} \bigcup \{\mathcal{D}(v_{\mathcal{U}_j})\}$
  for $\mathcal{B}_p \in \mathcal{C}\hat{\mathcal{B}}_j$ **do**
    $\mathcal{C} \bigcup \{c_i^j\}, R \bigcup R(c_i^j)$ calculated based on (12a) and (12b)
  end for
end procedure
if message = **CU_msg** from $\mathcal{B}_i \in \mathcal{B}$ and $\mathcal{B}_i \notin \mathcal{C}\hat{\mathcal{B}}_j$ **then**
  Send **End_Msg** to BS
else if message = **Start_Algo_Msg then**
  Implementing DCOP algorithm
end if
Each BS: $\mathcal{B}_i \in \mathcal{B}$
procedure SETECAV
  $\mathcal{B}_i$ is modeled as an agent $a_i \in \mathcal{A}$
  $\mathcal{C} \bigcup \{\mathcal{C}_{inter}^i\}$, $R \bigcup \{R(c_{inter}^j)\}$ based on (8)
end procedure
procedure SETEBUAV
  $\mathcal{B}_i$ is modeled as an agent $a_i \in \mathcal{A}$ and $v_i \in \mathcal{V}$
  $Root \leftarrow \mathcal{C}\hat{\mathcal{U}}_i^1$, $N_{RB} \leftarrow 0$ ▷ select a root
  $\mathcal{D}(v_{\mathcal{B}_i}) \leftarrow$ BuildBinaryTree(Root, N_{RB})
  Storing the binary tree of $\mathcal{D}(v_{\mathcal{B}_i})$ by BDD with the algorithms in [40]
end procedure
Send **CU_Msg** to $\mathcal{U}/\mathcal{C}\hat{\mathcal{U}}_i$
if get **End_Msg** from all $\mathcal{U}/\mathcal{C}\hat{\mathcal{U}}_i$ **then**
  Send **Start_Algo_Msg** to the users in $\mathcal{C}\hat{\mathcal{U}}_i$
end if
Implementing DCOP algorithm
if there are RBs left at $\mathcal{B}_i$ **then** ▷ greedy method
  Allocating RBs to the users with large SINR
end if
procedure BUILDBINARYTREE(Node, N_{RB})
  if Node.children = **NULL then**
    return
  else
    N_{RB} = N_{RB} + Node.RB ▷ allocated RBs
    if $N_{RB} > N_i$ **then**
      Node.children $\leftarrow$ **NULL**
    else
      BuildBinaryTree(Node.left)
      BulidBinaryTree(Node.right)
    end if
  end if
end procedure

In the ECAV and EBUAV models, the number of the agents, variables, and constraints varies with the number of users in the HetNet, and has a significant impact on the performance of the DCOP algorithm. However, it is difficult to make a quantitative analysis on the structure of the models due to the stochastic characteristics of the distribution of the users. Aiming at selecting a feasible model according to the practical configurations in the HetNet, we propose a model selection strategy using a threshold of the number of users, denoted as

Algorithm 3 DCOP solver for the user association problem

1: **Initialize:** THRE_NUM_US
2: Algorithm 1
3: **if** $\mathcal{NU} \leqslant THRE\_NUM\_US$ **then**
4:   set ECAV-η model
5: **else**
6:   set EBUAV-η model
7: **end if**

THRE_NUM_US, which is an empirical value based on the simulated results (we will illustrate more details in Section VI). When the number of users is below the threshold, the ECAV model is set up. Otherwise, the EBUAV model is exploited (line 3 to 7 in Algorithm 3).

B. The Solution Quality

We use $\mathcal{X}^\eta$ and $\mathcal{X}^*$ to respectively denote the solutions obtained based on the ECAV/EBUAV-η model and the ECAV/EBUAV model by the complete DCOP algorithm. we then explore the relationship between these two solutions as follows:

Theorem 5. *If all the users are served by the BSs,* $\mathcal{X}^\eta = \mathcal{X}^*$.

Proof: Denoting $\bar{\mathcal{X}^\eta}$ as the set of solutions where $\bar{\mathcal{X}_s^\eta} \neq \mathcal{X}^\eta, \forall \bar{\mathcal{X}_s^\eta} \in \bar{\mathcal{X}^\eta}$. According to formulation (10), we have

$$\sum_{c\in\mathcal{C}} R(c) \geqslant \sum_{\mathcal{B}_i\in\mathcal{B}} \sum_{\mathcal{U}_j\in\pi_{\bar{\mathcal{X}_s^\eta}}(\mathcal{B}_i)} r_{ij} \tag{13}$$

where $\pi_{\mathcal{X}}(\mathcal{B}_i)$ is the set of users connecting with $\mathcal{B}_i$ in the solution $\mathcal{X}$. Assuming $\mathcal{X}^\eta \neq \mathcal{X}^*$, there is at least one user who connects with another BS instead of current one in the solution $\mathcal{X}^\eta$. In this case, $\bar{\mathcal{X}_s^\eta} = \mathcal{X}^* \subseteq \bar{\mathcal{X}^\eta}$ and

$$\sum_{\mathcal{B}_i\in\mathcal{B}} \sum_{\mathcal{U}_j\in\pi_{\bar{\mathcal{X}_s^\eta}}(\mathcal{B}_i)} r_{ij} > \sum_{c\in\mathcal{C}} R(c). \tag{14}$$

A conflict exists between (13) and (14). Therefore, Theorem 5 is proved. ■

Theorem 6. *If not all the users are served by the BSs,* $U_{\mathcal{X}^\eta} \geqslant \frac{1}{2} U_{\mathcal{X}^*}$ *where* $U_{\mathcal{X}}$ *represents the global utility with solution* $\mathcal{X}$.

Proof: We define a set of users $\pi_{\mathcal{X}^*-\mathcal{X}^\eta}(\mathcal{B}_i) = \{\mathcal{U}_j | \mathcal{U}_j \in \pi_{\mathcal{X}^*}(\mathcal{B}_i) \bigcup \mathcal{U}_j \notin \pi_{\mathcal{X}^\eta}(\mathcal{B}_i), \forall \mathcal{B}_i \in \mathcal{B}\}$. Similarly, $\pi_{\mathcal{X}^*\cap\mathcal{X}^\eta}(\mathcal{B}_i) = \{\mathcal{U}_j | \mathcal{U}_j \in \pi_{\mathcal{X}^*}(\mathcal{B}_i) \bigcup \mathcal{U}_j \in \pi_{\mathcal{X}^\eta}(\mathcal{B}_i), \forall \mathcal{B}_i \in \mathcal{B}\}$. Then we can calculate $U_{\mathcal{X}^*}$ and $U_{\mathcal{X}^\eta}$ by

$$U_{\mathcal{X}^*} = \sum_{\mathcal{B}_i\in\mathcal{B}} \Big(\sum_{\mathcal{U}_{j_1}\in\pi_{\mathcal{X}^*\cap\mathcal{X}^\eta}(\mathcal{B}_i)} r_{ij_1} + \sum_{\mathcal{U}_{j_2}\in\pi_{\mathcal{X}^*-\mathcal{X}^\eta}(\mathcal{B}_i)} r_{ij_2} \Big), \tag{15}$$

$$U_{\mathcal{X}^\eta} = \sum_{\mathcal{B}_i\in\mathcal{B}} \Big(\sum_{\mathcal{U}_{j_1}\in\pi_{\mathcal{X}^*\cap\mathcal{X}^\eta}(\mathcal{B}_i)} r_{ij_1} + \sum_{\mathcal{U}_{j_2}\in\pi_{\mathcal{X}^\eta-\mathcal{X}^*}(\mathcal{B}_i)} r_{ij_2} \Big). \tag{16}$$

Subtract (16) from (15), we get

$$U_{\mathcal{X}^*} - U_{\mathcal{X}^\eta} = \sum_{\mathcal{B}_i \in \mathcal{B}} (\sum_{\mathcal{U}_{j_1} \in \pi_{\mathcal{X}^* - \mathcal{X}^\eta}(\mathcal{B}_i)} r_{ij_1} - \sum_{\mathcal{U}_{j_2} \in \pi_{\mathcal{X}^\eta - \mathcal{X}^*}(\mathcal{B}_i)} r_{ij_2}). \tag{17}$$

As $\sum_{\mathcal{U}_{j_1} \in \pi_{\mathcal{X}^* - \mathcal{X}^\eta}} r_{ij_1} \leqslant U_{\mathcal{X}^\eta}$, we can get the inequation as

$$U_{\mathcal{X}^*} - U_{\mathcal{X}^\eta} \geqslant U_{\mathcal{X}^\eta} \Leftrightarrow U_{\mathcal{X}^\eta} \geqslant \frac{1}{2} U_{\mathcal{X}^*}. \tag{18}$$

■

Thus, Theorem 6 is proved. Further, we discuss the relationship between $U_{\mathcal{X}^\eta}$ and η as follows:

Theorem 7. *If not all the users are served by the BSs,* $U_{\mathcal{X}^*} - U_{\mathcal{X}^\eta} \leqslant \sigma(\eta)(\Upsilon - \gamma)$ *where* $\sigma(\eta)$ *is the number of users with* $|\hat{\mathcal{CB}}| > \eta$*,* $\Upsilon = [2\gamma, max\{u_{ij}\}]^+, \forall i \in \mathcal{NB}, \forall j \in \mathcal{NU}$*.*

Proof: Based on the formulation (5) to (7), the pratical rate achieved at the user $\mathcal{U}_j \in \mathcal{U}$ served by BS $\mathcal{B}_i \in \mathcal{B}$ is $r_{ij} = u_{ij} \times \lceil \frac{\gamma}{u_{ij}} \rceil$. As $0 \leqslant \lceil \frac{\gamma}{u_{ij}} \rceil - \frac{\gamma}{u_{ij}} \leqslant 1$, we have $r_{ij} \leqslant u_{ij} \times (\frac{\gamma}{u_{ij}} + 1) \leqslant \gamma + u_{ij}$. In this case, the upper bound of r_{ij} is denoted by $\hat{r_{ij}}$ and estimated by

$$\hat{r_{ij}} = \begin{cases} 2\gamma, & \text{if } \gamma \geq u_{ij} \quad (19a) \\ max\{u_{ij}\}, & \text{otherwise} \quad (19b) \end{cases}$$

If a BS can use one RB to satisfy a user's rate QoS, then the practial rate will be u_{ij}. Otherwise, the rate will be $\gamma + u_{ij}$. As $u_{ij} < \gamma$, so $\gamma + u_{ij} < 2\gamma$. We let $\sigma(\eta)$ represent the number of users whose with $|\hat{\mathcal{CB}}| > \eta$. If the solution $\mathcal{X}^\eta$ is not a global optimal, then there are at most $\sigma(\eta)$ users who change their current connection to other BSs. After that, we can obtain the gap between $U_{\mathcal{X}^*}$ and $U_{\mathcal{X}^\eta}$ which is smaller than $\sigma(\eta)(\hat{r_{ij}} - \gamma)$. We make $\Upsilon = [2\gamma, max\{u_{ij}\}]^+, \forall i \in \mathcal{NB}, \forall j \in \mathcal{NU}$ where operator $[X, Y]^+$ indicates the maximum value of X and Y. Then Theorem 7 can be proved. ■

From Theorem 7, we find that if we set η with a larger value, $\sigma(\eta)$ will be smaller and the solution will be closer to the global optimal solution. In fact, when we set $\eta = 3$, the sub-optimal solution is almost the best one in our simulation.

VI. Experiments on Simulated Environments

In the simulation, a three-tiers HetNet is considered including macrocell, picocell, and femtocell. Specifically, the transmission powers of these BSs are set by 46, 35, and 20 dBm. We assume the scene is a $1000m \times 1000m$ square. One macro BS is fixed at the center of the square, and there are 5 pico BSs, 10 femto BSs, and 200 users randomly located in it. The path loss between the macro or pico BSs and the users is defined as $L(d) = 34 + 40log_{10}(d)$. Further, the pass loss between femto BSs and users is $L(d) = 37 + 30log_{10}(d)$, in which d represents the distance between the BSs and the users in meters. The noise power of all the users is equal to -111.45 dBm, which is the thermal noise at room temperature with a bandwidth of 180kHz. The scheduling interval of 1 second is considered. If there is no special illustration, the number of RBs held at different types of BSs is 200 for macro BS, 100 for pico BS and 50 for femto BS. ECAV/EBUAV-x represents the parameter $\eta = x$. Without special instructions, all the results are the average of 20 instances.

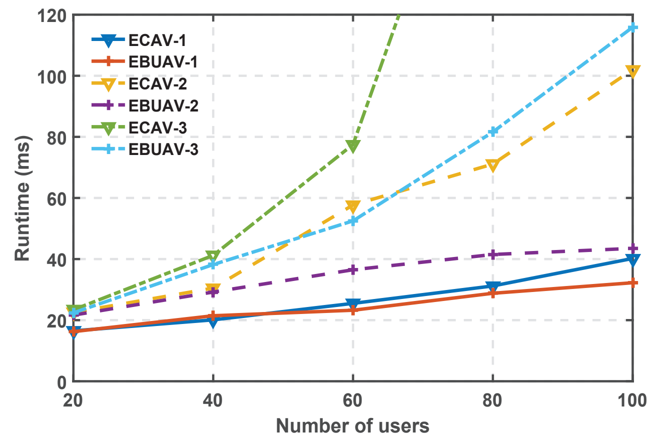


Fig. 4: The running time of ECAV and EBUAV models with $\eta = 1, 2, 3$

TABLE I: The number of constraints in the ECAV/EBUAV-η model

Num.	ECAV/EBUAV-1	ECAV/EBUAV-2	ECAV/EBUAV-3
50	0/0	30/34	56/36
100	27/33	92/46	109/42
150	61/67	140/79	143/80
200	77/102	198/118	208/123

To verify the effectiveness of the proposed DCOP solver, we firstly present the complexity of the ECAV/EBUAV-η model with respect to the variation of the number of users and η. Then, we compare the performance of the DCOP solver with Max-SINR and LDD based schemes.

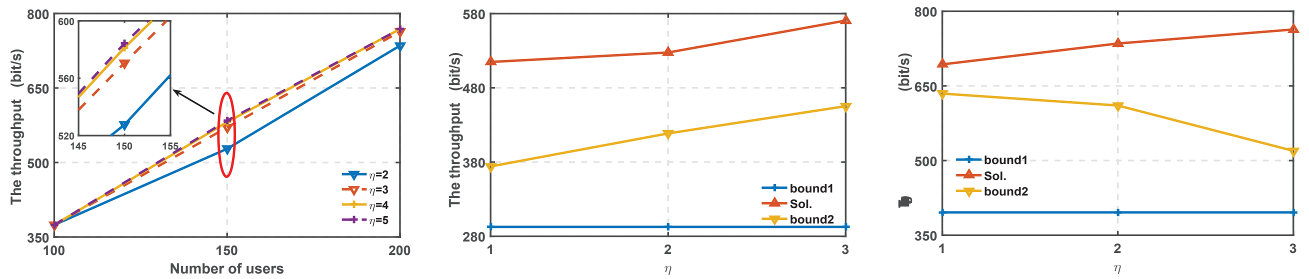
A. Complexity of ECAV/EBUAV-η Model

Table I displays the number of constraints in both models with different configurations of parameter η and the number of users in the HetNet. With the increment of the users, there are fewer constraints, but more agents in the EBUAV-η model than the ones in the ECAV-η model. Particularly, such difference is more apparent when $\eta = 2, 3$ and the number of the users is above 100. As the DCOP algorithm is implemented by each agent, the distributed behavior reduces the negative impact of the increment of users on the performance of the EBUAV-η model. This can be explained by observing Fig.4 where we compare the complexity of the ECAV/EBUAV-η model from the point of the view of the running time, by means of the same DCOP algorithm. With the same configuration of parameter η, we find that the running time based on the ECAV-η model is hardly controlled when the number of the users is increasing. On the other hand, there is a nearly linear relationship between the running time and the number of users using the EBUAV-η model. To sum up, the complexity of the EBUAV-η model is obviously less than the EVAV-η model when the number of users is more than 100. In this way, we set the threshold THRE_NUM_US by 100.

Fig.5a presents the throughput using the EBUAV-η model with $\eta = 2, 3, 4$, and 5. The difference on the throughput between $\eta = 3, 4$ and 5 is not obvious. However, there is a big gap between the throughput of $\eta = 2$ and $\eta = 3$. In Table

TABLE II: The rate (bit/s) and errors between optimal solution and the ones achieved with $\eta = 1, 2, 3$

Num.	Optimal sol.	$\eta = 1$			$\eta = 2$			$\eta = 3$		
		Sol.	$\sigma(\eta)/\Upsilon$	**Error (%)**	**Sol.**	$\sigma(\eta)/\Upsilon$	**Error (%)**	**Sol.**	$\sigma(\eta)/\Upsilon$	**Error (%)**
50	286.13	286.13	/	0	286.13	/	0	286.13	/	0
100	374.35	374.35	/	0	374.35	/	0	374.35	/	0
150	585.28	514.85	138/4.53	12.03	527.60	79/5.11	9.86	570.64	67/4.94	2.50
200	791.67	693.24	187/3.84	12.43	735.19	124/4.46	7.13	763.40	84/6.24	3.57



(a) The throughput of the EBUAV model with different configurations of η

(b) The lowver bound of the throughput with $\mathcal{NU}$ = 150

(c) The lowver bound of the throughput with $\mathcal{NU}$ = 200

Fig. 5: The performance of EBUAV-η

II, we use "Optimal Sol." to represent the optimal solution obtained by the complete DCOP algorithm using the EBUAV model, and "Sol." to represent the practical solution based on the EBUAV-η model. Sol. is the same as Optimal Sol. when $\mathcal{NU} = 50$ or 100 since all the users can be served by the BSs. Such result is consistent with Theorem 5. In contrast, the resources at the BSs are not enough to satisfy the QoS requirements from all the users when $\mathcal{NU} = 150$ or 200. By calculating $\frac{1}{2}U_{\mathcal{X}^*}$ and $U_{\mathcal{X}^*} - \sigma(\eta)(\Upsilon - \gamma)$, we estimate two lower bounds named $bound_1$ and $bound_2$ of Sol.. From Fig.5b and 5c, we can observe that both $bound_1$ and $bound_2$ is below Sol.. Besides, $bound_2$ is close to Sol.. Thus, Theorem 6 and 7 are verified. From the pecentage error calculated by $\frac{Optimal sol. - sol.}{Optimal sol.} \times 100\%$, we observe that the gap between Optimal Sol. and Sol. is $\sim 5\%$ when $\eta = 3$ and $\mathcal{NU} = 150$ or 200. Therefore, it is sufficient to use $\eta = 3$ as a feasible configuration for the EBUAV-η model.

B. The Performance of DCOP Solver

Fig.6 shows the computational complexity of different schemes from the perspective of running time. With the increment of the number of users, there has been a slow growth in the running time needed by the DCOP solver based on EBUAV-η model. Comparing with Max-SINR and LDD based schemes, DCOP solver has a better efficiency for solving the user association problem when $\eta = 3$.

In Fig.7, we compare the connected states between users and BSs in different tiers. Unsurprisingly, more users prefer to connect with the macro BS in the Max-SINR based scheme. Also, more users are out of service using Max-SINR scheme than using the other two methods. Further, we explore the variation of the connected states when there is increment of the deployed users (Fig.8). Compring with Max-SINR and LDD based schemes, the DCOP solver demonstrates up to 18% and 3% increase in the ability to make the BSs serve more users with limited resource. That is, DCOP solver can provide a novel resource allocation strategy.

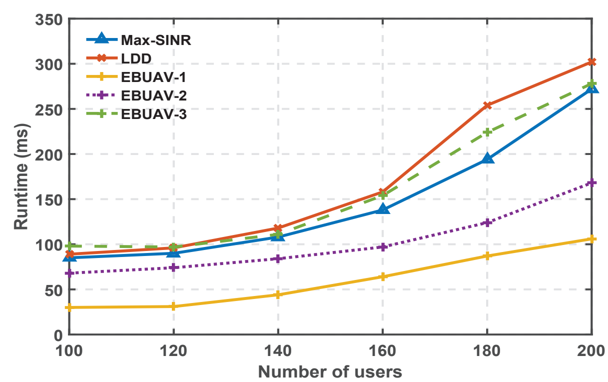


Fig. 6: The computational complexity of Max-SINR, LDD and DCOP based schemes

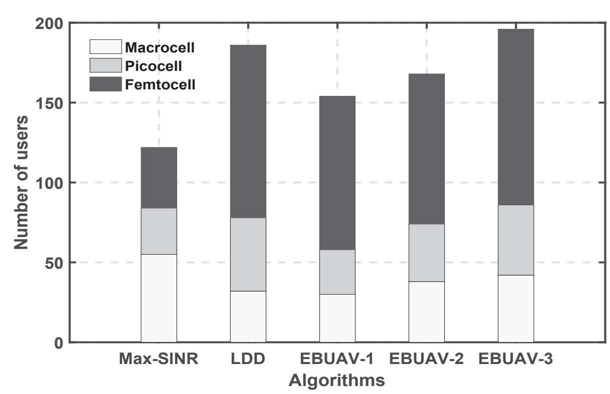


Fig. 7: The connected states between users and BSs of Max-SINR, LDD and DCOP based schemes

In Fig.9, we compare the CDF of the long-term rate calculated by the Max-SINR, LDD and DCOP based schemes.

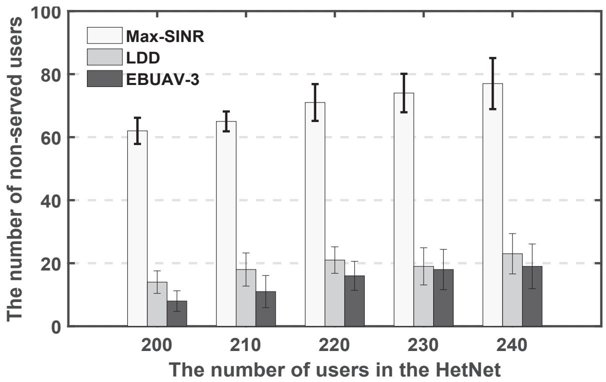


Fig. 8: Non-served users in the HetNet in the case of different configurations of $\mathcal{NU}$

We set the maximum number of iterations in the LDD based scheme as 25. From the figure, we can observe that $\mathbf{P}(r < 3)$ of DCOP solver with EBUAV-$\eta = 1, 2$ is larger than that of LDD based scheme. In the case of $\eta = 3$, no more than 5 users are out of service using the DCOP solver, while 10 14 users cannot be served using the LDD based method. This can be explained by the following reasons. Firstly, the users located at the edge of the square are hardly served by any BS; Secondly, the user $\mathcal{U}_j$ will select a BS $\mathcal{B}_i$ with the maximal QI_{ij} in each iteration of LDD based scheme [4]. In other words, the users prefer to connect with the BS which can offer better QoS even when more resources are consumed. Therefore, some BSs have to spend more RBs so that the resource at these BSs being more easily used up. For instance, considering the following scenario where both BS $\mathcal{B}_1$ and $\mathcal{B}_2$ have 10 available RBs and are able to provide service to the same user $\mathcal{U}$. The number of RBs consumed at $\mathcal{B}_1$ is 10 so that the rate at $\mathcal{U}$ is 10×0.32 = 3.2 bit/s (based on formulation 6). Also, the number of RBs consumed at $\mathcal{B}_2$ is 6 so that the rate at $\mathcal{U}$ is 6×0.5 = 3 bit/s. With LDD based scheme, $\mathcal{U}$ finally connects with $\mathcal{B}_1$ since 3.2 bit/s $>$ 3 bit/s. Clearly, as a return of more 0.2 bit/s, 4 more RBs will be consumed. However, the users which can only be served by $\mathcal{B}_1$ are out of service since none of the RBs are left at $\mathcal{B}_1$.

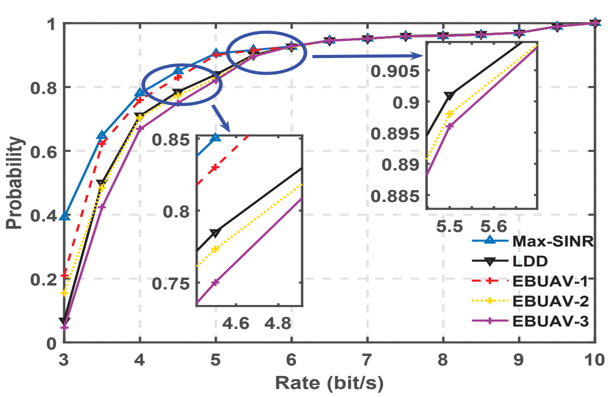


Fig. 9: The CDFs of the users' long term rate

Fig.10 shows the throughput in the HetNet with $\mathcal{NU}$ varying from 100 to 200. We can observe that both LDD and DCOP

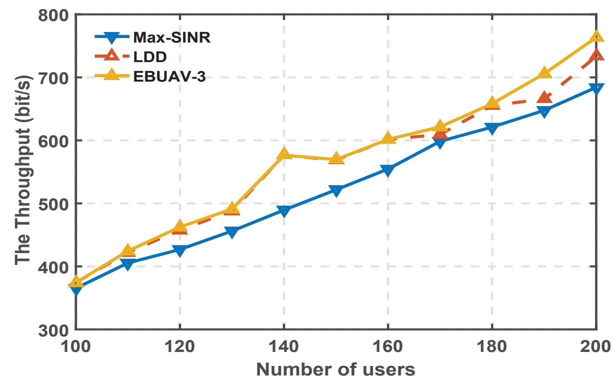


Fig. 10: The change of throughput over the variation of $\mathcal{NU}$

based schemes have a better performance than the Max-SINR scheme. When $\mathcal{NU} > 180$, it is apparent to see that there is at most 6% improvement on the throughput by the DCOP solver comparied with the LDD based scheme.

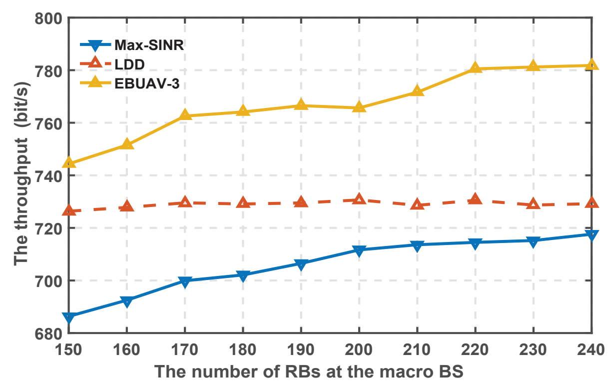


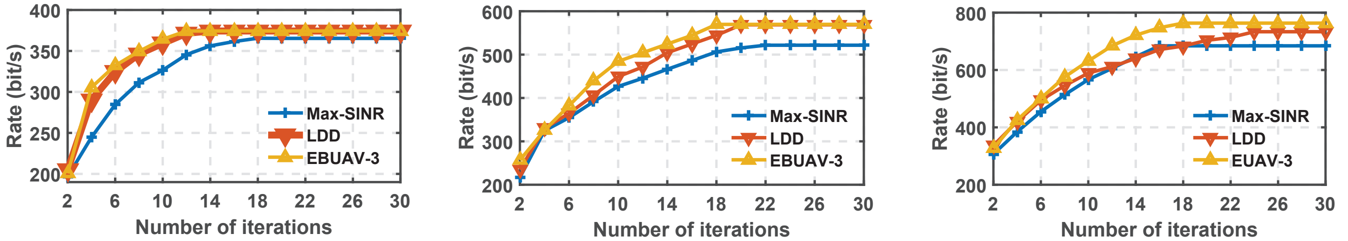
Fig. 11: The throughput against the number of RBs configured at the macro BS

We test the performance with respect to the variation of the number of RBs configured at macro BS in Fig.11. When the number of RBs falls into the range between 150 and 250, we can observe that there is little change over the throughput using LDD based scheme. In contrast, DCOP solver is capable of adjusting the resource allocation strategy with the variation of the resource configuration at macro BS.

We define an iteration in a message propagation based DCOP algorithm is a cycle that all agents (variables) finish receiving and sending messages [28]. From Fig.12, we can observe that the convergence rate of LDD and DCOP based schemes are nearly the same (after 14th iteration) when $\mathcal{NU} = 100$. However, DCOP based scheme has a better convergence rate than that of LDD based scheme when $\mathcal{NU} = 150, 200$.

VII. Conclusion and Perspectives

In this paper, we revisit the user association problem in the downlink of the multi-tier HetNet where unequal number of RBs are configured at the BSs in different tiers. We propose two models respectively named ECAV and EBUAV. To improve the performance of these two models, a parameter η is introduced to control the number of candidate BSs surrounding



(a) $\mathcal{NU} = 100$ (b) $\mathcal{NU} = 150$ (c) $\mathcal{NU} = 200$

Fig. 12: The convergence rate of Max-SINR, LDD and EBUAV-3

a user. Furthermore, the BDD techinque is carried out before implementing the DCOP algorithms in order to improve the resolution efficiency. In addition, we provide two lower bounds of practical solution in terms of the configuration of η. The simulation shows that the DCOP based scheme is able to provide a novel resource allocation strategy when $\eta = 3$. Particularly, it has a better robustness when the number of users increases but the available RBs are limited.

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