Connectivity-based Distance Estimation in Wireless Sensor Networks

Baoqi Huang, Changbin Yu, Brian D.O. Anderson, Guoqiang Mao

***Abstract*—Distance estimation is of great importance for localization and a variety of applications in wireless sensor networks. In this paper, we develop a simple and efficient method for estimating distances between any pairs of neighboring nodes in wireless sensor networks based on their local connectivity information, namely the numbers of their common one-hop neighbors and non-common one-hop neighbors. The proposed method involves two steps: estimating an intermediate parameter through a Maximum-Likelihood Estimator (MLE) and then mapping this estimate to the associated distance estimate. In the first instance, we present the method by assuming that signal transmission satisfies the ideal unit disk model but then we expand it to the more realistic log-normal shadowing model. Finally, simulation results show that localization algorithms using the distance estimates produced by this method can deliver superior performances in most cases in comparison with the corresponding connectivity-based localization algorithms.**

I. Introduction

Wireless sensor networks, comprised of hundreds or thousands of small and inexpensive nodes with constrained computing power, limited memory and short battery lifetime, can be used to monitor and collect data in a region of interest. Accurate and low-cost sensor (the word "sensor" connotes a node of unspecified location) localization is a critical requirement for a wide variety of applications in wireless sensor networks, and great efforts have been invested in developing localization algorithms including both distance-based algorithms and connectivity-based algorithms.

In reality, exact distance measurement is usually unavailable and has to be estimated from information such as received signal strength (RSS), time of arrival (TOA), or time difference of arrival (TDOA) [1]. In large-scale sensor networks, it is, however, impractical to localize all sensors by using additional hardware such as GPS receivers and measuring devices due to cost constraints. On the other hand, although classes of connectivity-based localization algorithms without using any additional measuring devices have been proposed [2], [3], achieving a high localization accuracy usually demands a comparatively large number of anchor nodes, hereafter termed simply anchors, whose positions are known a priori.

In a static wireless sensor network, two nodes are termed one-hop neighbors or simply neighbors as long as they can communicate with each other. An intuitive observation shows that two geographically close neighboring nodes often share more common one-hop neighbors than two distant nodes. In this paper, we quantify and exploit this observation to develop a method for estimating the distance between any pair of neighboring nodes based on their local connectivity information, i.e. the numbers of their common one-hop neighbors and non-common one-hop neighbors. Our method involves two steps: first, an intermediate parameter relating the distance and the numbers of different types of neighbors is estimated based on a Maximum Likelihood Estimator (MLE); second, through a mapping function we can obtain the distance estimate from the estimate in the first step. After presenting this method for the unit disk model, we expand it to the more realistic log-normal shadowing model. Such distance estimates can be directly used by distance-based localization algorithms, and in comparison with traditional connectivity-based localization algorithms, a significant improvement on localization accuracies is reported through simulations.

The advantages of the proposed method are: independent of additional hardware; totally distributed; energy efficient due to its simple mechanism and computations. Prior to our work, [4], [5] came up with the method of estimating distances based on the same idea as ours; their treatments rest on empirical observations rather than theoretical foundations. In comparison to their work, our paper: (1) formalizes and gives mathematical proofs of the correctness of the method; (2) bases the method on the MLE in both the unit disk model and the more realistic log-normal shadowing model; (3) reports the performance improvement on localization after using the distance estimates produced by the method through simulations.

The remainder of the paper is organized as follows: Section II introduces the method of estimating distances in the unit disk model and Section III expands it to the log-normal shadowing model. Section IV investigates the performance improvement on localization by using the proposed method through simulations. Finally, Section V concludes the paper.

II. Estimating Distances in the Unit Disk Model

We first introduce the network model used here and then present the method in the unit disk model.

A. Network Model

Considering a static wireless sensor network where nodes are randomly and uniformly distributed in 2-dimensional region, a homogeneous Poisson process provides an accurate model for the distribution of nodes as the network size approaches infinity [6]. Let λ denote the node density (the number of nodes per unit square) and the probability mass function of the number of nodes N in an area D is given by

$$Pr(N=n)=\frac{(\lambda D)^n}{n!}e^{-\lambda D} \tag{1}$$

B. Huang and B.D.O. Anderson are with the Australian National University and National ICT Australia Ltd., Canberra. Email:{Baoqi.Huang,Brian.Anderson}@anu.edu.au.

C. Yu is with the Australian National University, Canberra. Email:Brad.Yu@anu.edu.au. Correspondence: RSISE Building 115, The Australian National University, Canberra ACT 0200, Australia, Tel: +61 2 61258670, Fax: +61 2 61258660.

Guoqiang Mao is with the University of Sydney and National ICT Australia Ltd., Sydney. Email:Guoqiang@ee.usyd.edu.au.

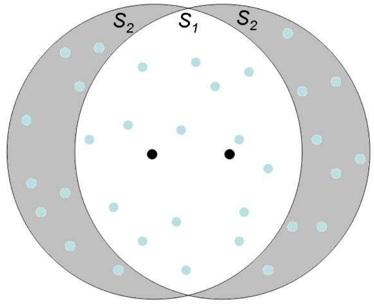


Fig. 1. Communication coverage of two neighboring nodes in the unit disk model.

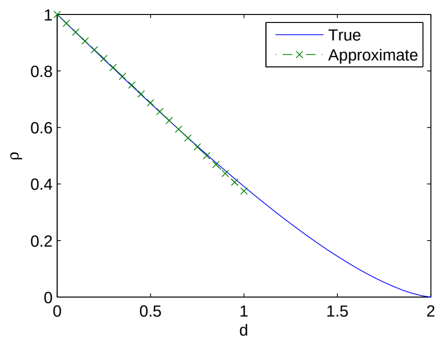


Fig. 2. The true and approximate functions from d to ρ ($r = 1$).

where $Pr(\cdot)$ denotes the probability of a statistical event. Obviously, N is a Poisson random variable with mean λD.

Moreover, throughout this paper we assume that:

Assumption 1: Nodes are uniformly and randomly deployed with density λ in an infinite plane.

Assumption 2: Each node knows the neighborhood information (i.e. the list of one-hop neighbors) of all its one-hop neighbors.

Assumption 1 avoids the boundary effect and Assumption 2 ensures that local connectivity information is available for each node to carry out the distance estimation method.

B. Estimating Distances

Given a static wireless sensor network conforming Assumption 1, the node transmission range is identically r as we are discussing with the unit disk model.

As shown in Fig. 1, provided two neighboring nodes A and B separated by distance d ($d \leq r$), two circles with radii r and centered at A, B represent their individual communication coverage, and intersect and create three disjoint regions. The nodes residing in the middle one are common one-hop neighbors of A and B and the nodes residing in the left (right) one are non-common one-hop neighbors of A (B). Define S_1 to be the area of the middle one and both the areas of the left and the right ones are $\pi r^2 - S_1$, denoted S_2. Moreover, we let three random variables M, P, Q denote the numbers of the three kinds of neighbors and define a key parameter ρ:

$$\rho = \frac{E(2M)}{E(2M + P + Q)} \tag{2}$$

where $E(\cdot)$ denotes the expected value of a random variable.

As pointed out in [7], M, P, Q are mutually independent Poisson distributed random variables with the means $\lambda S_1, \lambda S_2, \lambda S_2$. Equivalently, ρ can also be expressed as

$$\rho = \frac{S_1}{S_1 + S_2} = \frac{S_1}{\pi r^2} \tag{3}$$

According to the geometries among d, r and S_1, we have

$$S_1 = 2r^2 \arccos(\frac{d}{2r}) - d\sqrt{r^2 - \frac{d^2}{4}} \tag{4}$$

In effect, S_1 is a monotonically decreasing function of d, and so is ρ. Hence, as long as ρ is available, we can compute d by using its inverse function, termed a mapping function.

The actual values of M, P and Q can be easily obtained in the light of Assumption 2 and can be furthered employed to estimate ρ according to Theorem 1.

Theorem 1: Given three independent Poisson distributed random variables M, P and Q which define $\rho = \frac{E(2M)}{E(2M+P+Q)}$, the MLE for ρ, termed $\hat{\rho}$, is

$$\hat{\rho} = \begin{cases} 1, & \text{if M=P=Q=0} \quad (5) \\ \dfrac{2M}{2M + P + Q}, & \text{otherwise} \quad (6) \end{cases}$$

Proof: Establish a statistical model: measured data are observations of M, P and Q, denoted $\phi = [\ m \quad p \quad q\ ]$ where m, p, q are non-negative integers; the unknown parameters are $\theta = [\ \rho \quad \lambda\ ]$. The likelihood function of this model is

$$\mathcal{L}(\theta, \phi) = Pr(M = m) \times Pr(P = p) \times Pr(Q = q) \tag{7}$$

The MLE is the solution to the following equation set

$$\frac{\partial \ln \mathcal{L}(\theta, \phi)}{\partial \theta} = 0 \tag{8}$$

which yields

$$\begin{cases} \dfrac{m}{\rho} - \dfrac{p+q}{1-\rho} + \lambda \pi r^2 = 0 & (9) \\ \dfrac{m+p+q}{\lambda \pi r^2} - (2 - \rho) = 0 & (10) \end{cases}$$

By eliminating λ, we can obtain

$$2m = (2m + p + q)\rho \tag{11}$$

If $2m + p + q > 0$, i.e. m, p and q are simultaneously 0, the solution for ρ is $\frac{2m}{2m+p+q}$; otherwise, the solution for ρ is not well-defined. But because $\rho = 1$ maximizes the likelihood when $2m + p + q = 0$, we obtain the MLE for ρ, termed $\hat{\rho}$, as

$$\hat{\rho} = \begin{cases} 1, & \text{if M=P=Q=0} \\ \dfrac{2M}{2M + P + Q}, & \text{otherwise} \end{cases}$$

■

By substituting (4) into (3) and applying the first order Taylor series expansions on $d = 0$, we can obtain

$$\rho \approx 1 - \frac{2d}{\pi r} \tag{12}$$

As depicted in Fig. 2, (12) displays a good approximation to the true function from d to ρ when $0 \leq d \leq r$. As such, the mapping function is approximately

$$\hat{d} \approx \frac{\pi r}{2}(1 - \hat{\rho}) \tag{13}$$

which enables us to obtain the estimate of d, i.e. $\hat{d}$, from $\hat{\rho}$.

III. Estimating Distances in the Log-normal Shadowing Model

Before expanding the method to the more realistic log-normal shadowing model, we make the following assumptions (as is commonly the case in the literature):

Assumption 3: The attenuations of the transmitting powers between any pairs of nodes are independent and identically distributed (i.i.d.) [1];

Assumption 4: Communication links are symmetric, namely that node v can directly receive packets from node u as long as node u can directly receive packets from node v;

Assumption 5: All nodes transmit at a fixed power level.

Although these assumptions may not fully reflect a real network environment, they still enable us to obtain some results as estimates for more realistic situations.

A. Log-normal Shadowing Model

The *log-normal shadowing model* predicts the received signal power by a receiver with distance d from a transmitter, denoted $P_R(d)$, to be log-normally distributed around the ensemble average received power, denoted $\overline{P_R(d)}$; this model is based on a wide variety of measurement results [8] as well as analytical evidence [9], and is typically modeled as [1]

$$P_R(d)(dBm) = \overline{P_R}(d_0)(dBm) - 10\alpha \log \frac{d}{d_0} + Z \tag{14}$$

where Z is a random variable representing the shadowing effect, normally distributed with mean zero and variance σ^2; $\overline{P_R}(d_0)$ (dBm) is the ensemble average received signal power in dBm at a short reference distance d_0; α is the path-loss exponent. Both σ and α are a priori known constants; typically, σ is as low as 4 and as high as 12, and α varies between 2 in free space to 6 in heavily built urban areas [8].

Given a transmitter A and a receiver B, if $P_R(d)$ is no less than some specified value P_c, a directional communication link exists from A to B; equivalently, a directional communication link also exists from B to A due to Assumption 4. In particular, if the shadowing effect Z vanishes, i.e. $\sigma = 0$, the log-normal shadowing model is equivalent to the unit disk model with the communication range of

$$r = d_0 (\frac{\overline{P_R}(d_0)}{P_c})^{\frac{1}{\alpha}} \tag{15}$$

which is a constant given α. Otherwise, the probability that two nodes with distance d can communicate is

$$g(d) = \int_{k_1 \ln \frac{d}{r}}^{\infty} \frac{e^{-\frac{z^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma} dz \tag{16}$$

where $k_1 = \frac{10\alpha}{\ln 10}$.

B. Distributions of M, P, Q

We still use M, P, Q to denote the numbers of common and non-common one-hop neighbors associated with A and B. The following theorem and corollary provide their distributions in the log-normal shadowing model.

Theorem 2: Suppose a sensor network where nodes are randomly and uniformly distributed with density λ in a disk of radius R; given two nodes A and B, let M be the number of their common one-hop neighbors and P and Q be the numbers of their non-common one-hop neighbors. M, P and Q are Poisson random variables in the limiting case of $R \to \infty$.

Proof: Let (x_1, y_1) and (x_2, y_2) be the positions of A and B. Given an arbitrary node C, there exist four cases with regard to communications between A, B and C:

1) C can directly communicate with both A and B;
2) C can directly communicate with A but not B;
3) C can directly communicate with B but not A;
4) C cannot directly communicate with either A or B.

Apparently, M is the number of nodes satisfying the Case 1; P (or Q) is the number of nodes satisfying the Case 2 (or 3). Supposing the disk area is D, the probabilities that C satisfies the i-th ($i = 1, 2, 3, 4$) case, termed p_i, are

$$p_1 = \frac{1}{D} \int\int_D g(d_1) g(d_2) dx dy \tag{17}$$

$$p_2 = \frac{1}{D} \int\int_D g(d_1)(1 - g(d_2)) dx dy \tag{18}$$

$$p_3 = \frac{1}{D} \int\int_D (1 - g(d_1)) g(d_2) dx dy \tag{19}$$

$$p_4 = \frac{1}{D} \int\int_D (1 - g(d_1))(1 - g(d_2)) dx dy \tag{20}$$

where $d_1 = \sqrt{(x - x_1)^2 + (y - y_1)^2}$ and $d_2 = \sqrt{(x - x_2)^2 + (y - y_2)^2}$.

To obtain the number of one-hop neighbors of A, i.e. $M+P$, we conduct a test for each node in the network except for A and B to decide whether it satisfies the Case 1 or 2, and then the total number of successful tests is $M + P$. Due to Assumption 3, all of the tests are independent of each other and hence, the test process is a *Bernoulli process*. Then, $M+P$ follows a Binomial distribution with the total number of tests $n = \lambda D - 2$ and the success probability $p = p_1 + p_2$. Moreover, if the limiting case of $\lim_{n\to\infty} np$ converges, the distribution of $M + P$ is Poisson with expected value $\lim_{n\to\infty} np$.

[1]Even though field measurements in real applications seem to indicate that the attenuations between two links with a common node are correlated [10], this i.i.d assumption is generally considered appropriate for far field transmission and is widely used in the literature [10]–[12].

Because of

$$\lim_{n\to\infty} np = \lim_{D\to\infty} [\lambda \int\int_D g(d_1)dxdy] \quad (21)$$

, we let (x_1, y_1) be the origin and $d_1 = R$, and transform (21) into the Polar coordinate system

$$\begin{aligned} \lim_{n\to\infty} np &= \lim_{R\to\infty} [\lambda \int_0^R \int_0^{2\pi} g(x)xdxd\theta] \\ &= 2\pi\lambda \int_0^\infty x \int_{\frac{k_1}{\sigma}\ln\frac{x}{r}}^\infty \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}} dzdx \quad (22) \end{aligned}$$

When x is no less than some value, say a, $\frac{k_1}{\sigma}\ln\frac{x}{r} \geq 1$ and

$$\begin{aligned} \lim_{n\to\infty} np &= 2\pi\lambda[\int_0^a x \int_{\frac{k_1}{\sigma}\ln\frac{x}{r}}^\infty \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}} dzdx \\ &\quad + \int_a^\infty x \int_{\frac{k_1}{\sigma}\ln\frac{x}{r}}^\infty \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}} dzdx] \\ &\leq 2\pi\lambda[\int_0^a x \int_{\frac{k_1}{\sigma}\ln\frac{x}{r}}^\infty \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}} dzdx \\ &\quad + \int_a^\infty x \int_{\frac{k_1}{\sigma}\ln\frac{x}{r}}^\infty \frac{ze^{-\frac{z^2}{2}}}{\sqrt{2\pi}} dzdx] \quad (23) \end{aligned}$$

It is easy to judge that the two integrals in the right hand side of the inequality definitely converge and then $\lim_{n\to\infty} np < \infty$. Therefore, $M+P$ follows a Poisson distribution. We can obtain the same result for $M+Q$. Regarding M, P, Q, because their success probabilities p_1, p_2 and p_3 in the corresponding Bernoulli processes are less than that of $M+P$, $\lim_{n\to\infty} np < \infty$ when $p = p_1, p_2, p_3$ and thus it is straightforward that M, P and Q follow Poisson distributions. ■

Corollary 1: M, P and Q are mutually independent in the limiting case of $R \to \infty$.

Proof: Define two events $M \leq m$ and $Q \leq q$. Then,

$$\begin{aligned} & Pr(M \leq m \cap Q \leq q) \\ =& \sum_{j=0}^{q} Pr(M \leq m \cap Q = j) \\ =& \sum_{j=0}^{q} [Pr(M \leq m | Q = j) Pr(Q = j)] \\ =& \sum_{j=0}^{q} \sum_{i=0}^{m} [Pr(M = i | Q = j) Pr(Q = j)] \quad (24) \end{aligned}$$

Consider the limiting case of $R \to \infty$ (equivalently $n \to \infty$)

$$\begin{aligned} & \lim_{n\to\infty} Pr(M \leq m \cap Q \leq q) \\ =& \lim_{n\to\infty} \sum_{j=0}^{q} \sum_{i=0}^{m} [Pr(M = i | Q = j) Pr(Q = j)] \\ =& \lim_{n\to\infty} \sum_{j=0}^{q} \sum_{i=0}^{m} [\frac{((n-j)\frac{p_1}{1-p_3})^i e^{-(n-j)\frac{p_1}{1-p_3}}}{i!} \frac{(np_3)^j e^{-np_3}}{j!}] \end{aligned} \quad (25)$$

According to the proof of Theorem 2, when $p = p_1, p_2, p_3$, $\lim_{n\to\infty} np < \infty$, and then $\lim_{n\to\infty} p = 0$. Hence,

$$\begin{aligned} & \lim_{n\to\infty} Pr(M \leq m \cap Q \leq q) \\ =& \lim_{n\to\infty} [\sum_{j=0}^{p} \sum_{i=0}^{m} [\frac{(np_1)^i e^{-np_1}}{i!} \times \frac{(np_3)^j e^{-np_3}}{j!}]] \\ =& \lim_{n\to\infty} [\sum_{i=0}^{m} [\frac{(np_1)^i e^{-np_1}}{i!}] \times \sum_{j=0}^{p} [\frac{(np_3)^j e^{-np_3}}{j!}]] \\ =& \lim_{n\to\infty} [\sum_{i=0}^{m} Pr(M = i) \times \sum_{j=0}^{q} Pr(Q = j)] \\ =& \lim_{n\to\infty} [Pr(M \leq m) \times Pr(Q \leq q)] \quad (26) \end{aligned}$$

Therefore, M and Q are independent as $R \to \infty$. Similarly, we can obtain that M, P and Q are mutually independent through the similar approach. ■

In the log-normal shadowing model, Theorem 2 and Corollary 1 assure us to apply Theorem 1 to arrive at the estimate of ρ. According to [12], the number of one-hop neighbor of a node in the log-normal shadowing model equals $\lambda\pi r^2 e^{\frac{2\sigma^2}{k_1^2}}$, which is in fact $E(M+P)$ and $E(M+Q)$. Furthermore, based on the proof of Theorem 2, we have

$$E(M) = \lambda \int_0^\infty \int_0^{2\pi} g(x) g(\sqrt{x^2 + d^2 - 2xd\cos\theta}) xdxd\theta \quad (27)$$

And then we have

$$\rho = \frac{\int_0^\infty \int_0^{2\pi} g(x) g(\sqrt{x^2 + d^2 - 2xd\cos\theta}) xdxd\theta}{\pi r^2 e^{\frac{2\sigma^2}{k_1^2}}} \quad (28)$$

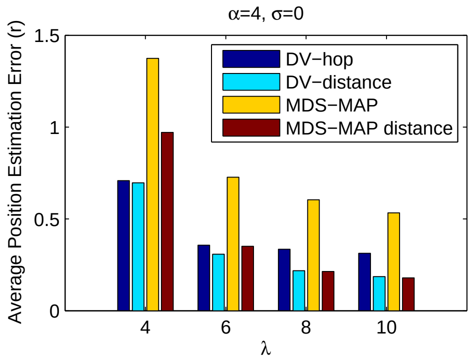
which formalizes the functional relationship between ρ and d. Though (28) is not closed-form, we can produce a piecewise linear function to approximate the inverse function of (28) as the mapping function and then estimate d from $\hat{\rho}$.

IV. Simulations

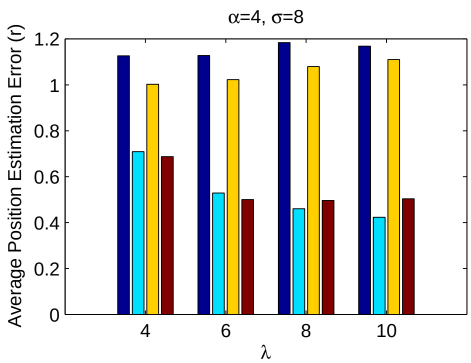
Since distance estimates are available, it is feasible to localize sensors by using a variety of distance-based localization algorithms. In order to evaluate the proposed method in a fair environment, we shall investigate two well-known connectivity-based localization algorithms which are also applicable with distance measurements, i.e. DV-hop and MDS-MAP; their distance-based versions are termed DV-distance and MDS-MAP distance respectively. Due to the length limitation, refer [2], [3] for more details about these localization algorithms.

To avoid the boundary effect, we actually generate sensor networks over a large square with size of 18×18, but only localize the nodes inside a small square with size of 6×6 and centered at the same center of the large square. However, the nodes outside of the small square are sometimes used in estimating distances between nodes within the small square. Four nodes closest to the four corners of the small square and inside of the small square are chosen as anchors.

We use a triple λ, σ and α to parameterize the configuration of a wireless sensor network. α is known to be 4 and λ takes



(a)



(b)

Fig. 3. Average position estimation errors.

a value in $4, 6, 8, 10$. Two values of σ are taken into account: 0 and 8. According to (15), r is essentially a constant given α and hence without loss of generality, we shall simply take $r = 1$ in the simulations. Consequently, d and resulting localization errors are normalized by r.

Given a certain triple, 100 independent runs are carried out and each run involves three steps: generating a wireless sensor network by a homogeneous Poisson process of density λ; estimating distances between any pair of neighboring nodes by using the proposed method; localizing sensors by DV-hop and MDS-MAP and the distance-based versions (with distances coming from the second step), and computing the average position estimation error for each localization algorithm. Finally, the average position estimation errors are averaged over the 100 independent runs corresponding to each parameter triple and each localization algorithm.

Simulation results, plotted in Fig. 3, show that DV-distance and MDS-MAP distance which use the distance estimates from the proposed method produce less average position estimation errors than the corresponding DV-hop and MDS-MAP. In summary, our distance estimation method makes good use of connectivity in wireless sensor networks and can dramatically improve localization accuracies.

V. Conclusions

In this paper, we proposed a method of estimating distances via connectivity in wireless sensor networks by dealing with the ideal unit disk model and the more realistic log-normal shadowing model. Simulation results showed that using the distance estimates produced by this method significantly improves localization accuracies in comparison to connectivity-based localization algorithms. Future work will focus on improving its practicality by relaxing some assumptions and improving its performance by utilizing more information.

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