Optimization of Subcarrier Allocation in Highly Dynamic Cellular Relay Networks

Xue Han[1], Guoqiang Mao[2], Zichen Liu[1], Liang Huang[1], Manli Qian[1], Jinglin Shi[1]
1. Institute of Computing Technology, Chinese Academy of Sciences, Beijing, China
2. School of Electrical and Information Engineering, the University of Sydney
Email:{hanxue, liuzichen, huangliang, qianmanli, sjl}@ict.ac.cn; guoqiang.mao@sydney.edu.au

***Abstract*—Cellular relay networks prove to be a cost-effective approach that offers significant performance benefits in coverage extension, cell-edge throughput enhancement and increased spectral efficiency, compared with traditional cellular networks. Radio resource management for cellular relay networks, where the network topology is highly dynamic, is particularly challenging and interesting. The highly dynamic topology may be caused by an ever increasing number of users accessing Internet and other multimedia services using mobile devices carried by pedestrians, in a train or in vehicles. Traditional radio resource management techniques developed for static networks may not be applicable in highly dynamic networks. This paper studies the subcarrier allocation problem in highly dynamic cellular relay networks with an objective to maximize the overall throughput. This optimization problem is formulated and solved as an expectation maximization problem. Statistical multiplexing gain is explicitly explored to further improve the channel utilization and spectral efficiency. Numerical evaluations are performed which demonstrate that the proposed scheme offers higher system throughput and improved radio resource utilization compared with existing schemes.**

I. Introduction

Recently, the use of relay stations (RSs) in cellular networks has been shown to be a cost-effective approach for improving system performance in a number of aspects, such as coverage extension, cell-edge throughput enhancement and increased spectral efficiency [1]. While these RSs provide opportunities in cellular relay networks on performance improvement, some interesting challenges also arise, caused by increased network dynamics, increased degrees of freedom and more complicated resource allocation schemes, particularly in networks with a large numbers of dynamic users. Radio resource management for cellular relay networks that efficiently utilizes spectrum, where the network topology is highly dynamic, is particularly challenging and interesting. The highly dynamic topology may be caused by an ever increasing number of users accessing Internet and other multimedia services using mobile devices carried by pedestrians, in a train or in vehicles.

Optimum radio resource allocation that maximizes the overall network throughput in OFDMA cellular relay networks has been investigated from the perspectives of optimum channel/subcarrier[1] allocation, optimum power allocation and optimum relay selection [2]. In [3], [4], the authors focused on the optimization of downlink throughput. Particularly, in [3], channel allocation was optimized to maximize the downlink throughput in a half-duplex cellular relay network with little or no user mobility. However, RSs in [3] simply amplify and forward the received signals using the same channel. This arrangement may not deliver the optimum throughput. A better performance can often be achieved if RSs use different channels to forward the received signals, called *subcarrier-paring*. Considering the use of subcarrier-pairing, in [4], the authors investigated optimal power allocation over subcarriers to maximize the instantaneous rate of the RS links. It was shown that the subcarrier pairing can improve the data rates of both the amplify-and-forward (AF) and decode-and-forward (DF) relay links.

In [5], [6], the optimization of uplink throughput in OFDMA relay networks were considered. Particularly in [5], Li et al. considered an OFDMA relay network with multiple sources, multiple RSs, and a single destination. Optimal radio resource allocation was investigated that balances the load of RSs while maximizing the total throughput. In [5], it was however assumed that perfect Channel State Information (CSI) is available at a central place, which is very difficult to realize due to the huge overhead involved in collecting the CSI in real time. In [6], Cui et al. considered distributive channel, power and rate allocation for an OFDMA cellular system with fixed RSs. They considered the impact on packet transmission errors due to imperfect CSI by using the system goodput instead of the traditional sum-ergodic capacity as the performance measure. Further, system fairness was also taken into account by considering weighted total goodput as the optimization objective.

While the above studies lead to new insight into the design of improved radio resource allocation scheme for cellular relay networks, the proposed solutions are mainly for static networks or networks with relatively stable topologies. Further, the underlying characteristics of traffic sources are not considered. The burstiness in traffic, particularly in video traffic, which is becoming the dominating traffic in cellular networks, may result in a lower utilization of the spectrum than expected.

Motivated by the above observations, the subcarrier allocation optimization in highly dynamic cellular relay networks is considered in this paper. The statistical multiplexing effect, which arises when multiple bursty sources transmitting together, is exploited to further improve the efficient utilization of radio resources. The objective is to maximize the overall throughput and achieves an efficient radio resource utilization.

[1]The terms channel and subcarrier are used interchangeably in this paper.

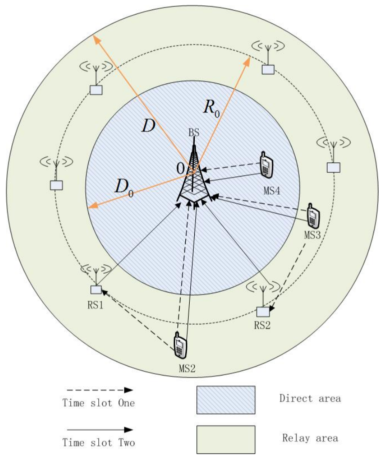

Figure 1. An Illustration of the System Model.

The main contributions of this paper are:

- A radio resource allocation scheme is proposed that optimizes subcarrier allocation in highly dynamic cellular relay networks with an objective to maximize the overall throughput. This optimization problem is formulated and solved as an expectation maximization problem.
- Compared with traditional schemes designed for static networks that rely on the availability of accurate user information, the scheme only uses statistical information, e.g. user density instead of accurate number of users, when performing optimization.
- Statistical multiplexing effect is explicitly explored to improve the utilization of radio resources at the expense of a controlled (small) link access collision probability.
- Simulations are performed which demonstrate that the proposed scheme can achieve a higher system throughput and a higher channel utilization than those in the literature.

The rest of this paper is organized as follows. In Section II, the system model of cellular relay network is described. The optimal subcarriers allocation problem is formulated in Section III, as well as the design of the optimization algorithm. SectionIV presents the simulation results. Finally, Section V concludes this paper.

II. System Model

In this paper, a cellular network composed of a single base station (BS) and a fixed number of relay stations (RSs) is considered. Denote the number of RSs by M. Further, mobile stations (MS) are distributed in the cellular network area, denoted by V, following a Poisson distribution with a known node density $\rho\left(v\right)$, $v \in V$. In dynamic networks, it is often difficult to obtain the accurate number of users due to both mobility and overhead involved in collecting the user information in real time. Therefore in this paper, it is assumed that only limited statistical knowledge about user information is available [7], which can often be obtained through empirical estimation or online measurements. Further, the situation that $\rho\left(v\right)$ is constant throughout the network area is considered for simplicity, i.e. $\rho\left(v\right) = \rho$. The analysis can be readily extended to accommodate the situation that $\rho\left(v\right)$ may not be constant throughout the region V. This system is illustrated in Fig.1.

Without loss of generality, it is considered that the BS is at the origin O. The coverage radius of the BS is known to be D. Following the same model as that used in [8], RSs are assumed to be placed uniformly on a circle centered at O and with a known radius $R_0 < D$. The coverage area of the BS is divided into two regions: the direct area and the relay area. The direct area is a disk centered at the BS and with a radius D_0. The relay area is an annulus centered at the origin and with an inner radius D_0 and an outer radius D. MSs in the direct area will communicate with the BS directly. MSs in the relay area will generally suffer greater path loss and therefore communicate with the BS via their respective closest RSs. A more intricate study involves considering the set of RSs and the BS partitioning the network area into Voronoi cells and each MS only communicate with the RS (BS) within the same cell. The performance improvement is likely to be marginal using the latter partitioning scheme but the complexity will be significantly increased. Therefore in this paper, only the earlier simpler partitioning scheme is considered.

The entire spectrum is divided into a set of orthogonal subcarriers N with flat channel responses. It is further assumed that the wireless channel between each pair of transmitting and receiving nodes is frequency selective. Noise in the channel is assumed to be additive white Gaussian noises (AWGN). RSs operate in a DF mode and only use half-duplex operation [5], [6], i.e., RSs cannot transmit and receive at the same time. Due to the half-duplex operation, a transmission frame can be divided into two time slots of equal length. Considering uplink, in time slot one, only MSs (both in the direct area and in the relay area) transmit while RSs and the BS receive. In time slot two, the MSs in the direct area and the RSs, which have successfully decoded data from their associated MSs in time slot one, will transmit at the same time while the BS receives. It is assumed that the channel remains unchanged for at least one time slot. To reduce the access collision probability and improve the system throughput, the entire frequency band is divided into two parts for the use of the MSs and BS communication via RSs and the direct communication between MSs and BS separately.

It is assumed that traffic being transmitted from MSs to the BS is bursty and the burstiness of the traffic sources is measured by a known parameter p. That is, in a randomly chosen time slot, with probability p, a traffic source is active; and with probability $1-p$, the traffic source is silent. Our results can be readily extended to handle the situation that traffic is heterogeneous. It is a common knowledge that most realtime traffic is bursty. For typical voice traffic, p is around 0.4 and for typical video traffic, p may be even less, e.g. $0.1-0.2$ [9].

III. Optimization of Subcarrier Allocation

Given the above system model, the subcarrier allocation shall be optimized to maximize the expected total system throughput. It is assumed that the power used by each subcarrier is fixed at P. During the optimization, the statistical multiplexing effect will be exploited.

Denote by K_r the (random) number of MSs in the relay area and we have $E(K_r) = \pi(D^2 - D_0^2)\rho$. Denote by K_b the number of MSs in the direct area and we have $E(K_b) = \pi D_0^2 \rho$. Notice that $K = K_r + K_b$ where K is the total number of MSs. Let N_r be the set of subcarriers used in MSs and BS communication via RSs; let N_b be the set of subcarriers used in the direct communication between MSs and BS. The number of subcarriers in N_r and N_b is denoted by n_r and n_b separately. Then the total number of subcarriers $N = n_r + n_b$.

Assume that each MS accesses its allocated subcarriers randomly with probability p (consistent with the traffic model introduced in SectionII). Let K_r^a be the (random) number of active MSs in the relay area and K_b^a be the (random) number of active MSs in the direct area. Collision happens when either $K_b^a > n_b$ or $K_r^a > n_r$, i.e. the number of active MSs is larger than the number of allocated channels. Let Pr_c^{BS} and Pr_c^{RS} be the probabilities that collision occurs in the direct area and in the relay area respectively. It is important that the collision probability is controlled below an acceptable threshold, denoted by ε.

Next, an analytical expression for Pr_c^{BS} will be obtained. The expression for Pr_c^{RS} can be obtained using a similar technique and hence is omitted. Given the earlier definition of collision, it easily follows that: $Pr_c^{BS} = Pr(K_b^a > n_b) = \sum\limits_{i=n_b+1}^{\infty} Pr(K_b^a = i)$. Let k_b be an arbitrary positive integer. Conditioned on $K_b = k_b$, the conditional distribution of K_b^a is given by: $Pr(K_b^a = i | K_b = k_b) = \left(\begin{array}{c} k_b \\ i \end{array}\right) p^i (1-p)^{k_b - i}$. Given the above two equations on Pr_c^{BS} and on $Pr(K_b^a = i | K_b = k_b)$, the unconditional distribution of K_b^a can be obtained by first using the Baye's formula and then using the total probability theorem:

$$\begin{aligned} & Pr(K_b^a = i) \\ & = \sum_{k_b=i}^{\infty} \left(\begin{array}{c} k_b \\ i \end{array}\right) p^i (1-p)^{k_b - i} \frac{(E(K_b))^{k_b}}{k_b!} e^{-E(K_b)} \end{aligned} \tag{1}$$

Finally, Pr_c^{BS} can be obtained:

$$Pr_c^{BS} = \sum_{i=n_b+1}^{\infty} Pr(K_b^a = i) \tag{2}$$

On the basis of the above equation on Pr_c^{BS} (and Pr_c^{RS}), which allows us to exploit the statistical multiplexing effect while keeping the collision probability below an acceptable threshold ε, we shall next study the subcarrier allocation optimization problem.

Define a set of indicator variables $I_{k,m,n} \in \{0,1\}$ for time-slot one, where $0 \le m \le M$, $1 \le k \le K$ and $1 \le n \le N$. Let $I_{k,m,n} = 1$ if the n^{th} subcarrier is allocated for communication between the k^{th} MS and its associated m^{th} RS; $I_{k,m,n} = 0$ otherwise. To simplify the notation, the 0^{th} RS is used to represent the BS. Therefore $I_{k,0,n} = 1$ implies that the n^{th} subcarrier is allocated for direct communication between the BS and the k^{th} MS. The signal-to-interference-plus-noise ratio (SINR) from the k^{th} MS to the m^{th} RS using the subcarrier n is denoted as $\alpha_{k,m,n} = \frac{P l_{km} |h_{km}^n|^2}{\Gamma W N_0}$ where l_{sd} denotes the path loss of the channel between source node s and destination d, h_{sd}^n represents the small-scale fading on the link between s and d over subcarrier n, Γ is the SNR gap related to a target bit error rate (BER), $\Gamma = -\log(5BER)/1.5$ and N_0 is assumed to be AWGN. Accordingly link rate can be calculated as $R_{k,m,n}^{(1)} = W \log_2(1 + \alpha_{k,m,n})$ where W represents the bandwidth of each subcarrier. Define another set of indicator variables $J_{m,n} \in \{0,1\}$ for time slot two, where $1 \le m \le M + K$ and $1 \le n \le N$. Let $J_{m,n} = 1$ if the n^{th} subcarrier is allocated for communication between the $m^{th}(1 \le m \le M)$ RS and the BS or for direct communication between the m^{th} MS (for notational convenience, the MSs are numbered from $M+1$ to $M+K$ in the second time slot. Therefore $M+1 \le m \le M+K$) and the BS; $J_{m,n} = 0$ otherwise. The SINR from MS or RS m to BS on subcarrier n in time slot two is $\beta_{m,n} = \frac{P l_{mb} |h_{mb}^n|^2}{\Gamma W N_0}$. Accordingly link rate in time-slot two can be calculated as $R_{m,n}^{(2)} = W \log_2(1 + \beta_{m,n})$.

Let R_b represents the total throughput of the direct communication between MSs and BS. Let R_r be the total throughput of MSs and BS communication via RSs. Conditioned on $K_b = k_b$, then $R_b = \frac{1}{2} \sum\limits_{n=1}^{n_b} (\sum\limits_{k=1}^{k_b} I_{k,0,n} R_{k,0,n}^{(1)} + \sum\limits_{m=M+1}^{M+k_b} J_{m,n} R_{m,n}^{(2)})$. Conditioned on $K_r = k_r$, $R_r = \frac{1}{2} \sum\limits_{n=1}^{n_r} \sum\limits_{m=1}^{M} \sum\limits_{k=1}^{k_r} \min\left\{R_{k,m,n}^{(1)}, R_{m,n}^{(2)}\right\}$.

Considering the randomness in K_b and K_r, it is more meaningful to use $E(R_b)$ and $E(R_r)$ in the optimization:

$$\begin{aligned} E(R_b) \ = \ & \frac{1}{2} \ \sum_{n=1}^{n_b} \sum_{k_b=1}^{\infty} (\sum_{k=1}^{k_b} I_{k,0,n} R_{k,0,n}^{(1)} \\ & + \sum_{m=M+1}^{M+k_b} J_{m,n} R_{m,n}^{(2)}) \Pr(K_b = k_b) \end{aligned} \tag{3}$$

For R_r, noting that $\min\left\{R_{k,m,n}^{(1)}, R_{m,n}^{(2)}\right\}$ can be transformed as $R_r^* = W I_{m,k,n} J_{m,n} \log_2(1 + 2\frac{\alpha_{k,m,n}\beta_{m,n}}{\alpha_{k,m,n} + \beta_{m,n}})$ under the condition that I and J are relaxed to be $I_{m,k,n} \ge 0$ and $J_{m,n} \ge 0$ respectively [3]. Accordingly, it follows that

$$\begin{aligned} E(R_r) \ & = \ \mathrm{E}(\frac{1}{2} \sum_{n=1}^{n_r} \sum_{m=1}^{M} \sum_{k=1}^{K_r} I_{k,m,n} J_{m,n} R_r^*) \\ & = \ \frac{1}{2} \ \sum_{n=1}^{n_r} \sum_{m=1}^{M} \sum_{k_r=1}^{\infty} (\sum_{k=1}^{k_r} I_{k,m,n} J_{m,n} R_r^*) \Pr(K_r = k_r) \end{aligned} \tag{4}$$

where $\Pr(K_r = k_r) = \frac{(E(K_r))^{k_r}}{k_r!} e^{-E(K_r)}$ due to the Poisson distribution of MSs.

Based on the above analysis, the throughput maximization problem, denoted by P1, can be formulated as follows:

$$
\begin{aligned}
P1: \quad & \max_{I,J,n_b} E(R_b) + E(R_r) \qquad (5)\\
s.t.\ C1.1: \quad & I_{k,m,n} \geq 0, \forall m,k,n;\ C1.2:\ J_{m,n} \geq 0, \forall m,n\\
C1.3: \quad & \sum_{k=1}^{k_b} I_{k,0,n} \leq \left\lceil \frac{k_b}{n_b} \right\rceil, \sum_{m=M+1}^{M+k_b} J_{m,n} \leq \left\lceil \frac{k_b}{n_b} \right\rceil,\\
& \forall n \in N_b, \forall k_b \in [1,\infty)\\
C1.4: \quad & \sum_{m=1}^{M} \sum_{k=1}^{k_r} I_{k,m,n} J_{m,n} \leq \left\lceil \frac{k_r}{n_r} \right\rceil,\\
& \forall n \in N_r, \forall k_r \in [1,\infty)\\
C1.5: \quad & Pr_c^{BS} < \varepsilon, C1.6:\ Pr_c^{RS} < \varepsilon
\end{aligned}
$$

where $\lceil x \rceil$ represents the smallest integer greater than or equal to x. To exploit the statistical multiplexing effect, a single subcarrier can be allocated to multiple MSs so long as the probability that these MSs transmit at the same time is controlled below a given threshold. When no statistical multiplexing is considered, $\sum_k I_{k,0,n} = 1$. In P1, C1.3 and C1.4 mean that the number of MSs that a subcarrier is allocated to is not greater than the ratio of the total number of users to the total number of subcarrier. These ensure a subcarrier is fairly allocated (i.e. not allocated to an excessive number of MSs). Constraint C1.5 and C1.6 make sure that the collision probability is below a pre-specified level ε. In the following sections, we shall explain how the optimization problem is solved.

A. Determination of n_b

The optimization problem P1 is solved in two stages. In the first stage, we determine the number of subcarriers that are allocated for direct communication between MSs and the BS, i.e. determine the value of n_b. Note n_r can be determined in the same way easily and is omitted here. In the second stage, we determine the allocation of subcarriers to each MS and RS.

The objective in the first stage is to find the minimum n_b such that the collision probability is below ε. First note that Pr_c^{BS} in Eq. (2) can be rewritten as

$$
\begin{aligned}
& Pr_c^{BS}\\
&= \sum_{i=n_b+1}^{\infty} \frac{p^i (E(K_b))^i}{i!} [\sum_{k_b=i}^{\infty} \frac{(E(K_b))^{k_b-i} (1-p)^{k_b-i} e^{-E(K_b)}}{(k_b-i)!}]\\
&= C \sum_{i=n_b+1}^{\infty} \frac{p^i (E(K_b))^i}{i!} \qquad (6)
\end{aligned}
$$

where $C = \sum_{k_b=i}^{\infty} \frac{(E(K_b))^{k_b-i}(1-p)^{k_b-i} e^{-E(K_b)}}{(k_b-i)!} = \sum_{k_b=0}^{\infty} \frac{(E(K_b))^{k_b}(1-p)^{k_b} e^{-E(K_b)}}{k_b!}$ is a constant independent of n_b.

According to [10], for a Poisson random variable X with mean λ, $\Pr(X \geq x) \leq \frac{e^{-\lambda}(e\lambda)^x}{x^x}$ when $x > \lambda$. Using the above inequality and Eq.(6),

$$
\begin{aligned}
Pr_c^{BS} &= C \sum_{i=n_b+1}^{\infty} \frac{p^i (E(K_b))^i}{i!} \qquad (7)\\
&\leq C e^{pE(K_b)} \frac{e^{-pE(K_b)} (epE(K_b))^{n_b+1}}{(n_b+1)^{n_b+1}}
\end{aligned}
$$

Let $g(n_b) = \frac{(epE(K_b))^{n_b+1}}{(n_b+1)^{n_b+1}}$, C1.5 can be transformed as follows:

$$
Pr_c^{BS} \leq C g(n_b) \leq \varepsilon \qquad (8)
$$

The minimum n_b can be determined from Eq.(8). Denote that minimum value by n_b^*.

B. Optimization of $I_{m,k,n}$ and $J_{m,n}$

Given $\hat{n_b} \geq n_b^*$, the set of subcarriers allocated for the direct communication between MSs and the BS denoted by $\hat{N}_b$ can be fixed. Then, P1 can be transformed to P2 as follows:

$$
\begin{aligned}
& P2: \max_{I,J} E(R_b) + E(R_r),\ s.t. C1.1 - 1.2 \qquad (9)\\
& C2.3: \sum_{k=1}^{k_b} I_{k,0,n} \leq \left\lceil \frac{k_b}{\hat{n_b}} \right\rceil, \sum_{m=M+1}^{M+k_b} J_{m,n} \leq \left\lceil \frac{k_b}{\hat{n_b}} \right\rceil\\
& \qquad \forall n \in \hat{N}_b, \forall k_b \in [1,\infty)\\
& C2.4: \sum_{m=1}^{M} \sum_{k=1}^{k_r} I_{k,m,n} J_{m,n} \leq \left\lceil \frac{k_r}{\hat{n_r}} \right\rceil\\
& \qquad \forall n \in \hat{N}_r,\ k_r \in [1,\infty)\\
& C2.5: \hat{n_b} \geq n_b^*, C2.6:\ \hat{n_r} \geq n_r^*
\end{aligned}
$$

P2 is a linear optimization problem. Although it can be solved by simplex or interior-point method, the complexity is large when the number of variables is large. To simplify the problem, MSs in the relay area are assumed to transmit using the same subcarrier in both time slots which means for a k^{th} MS located in the relay area, $I_{k,m,n} = J_{m,n}$. Further, the continuous relaxation in C1.1-1.2 from $I_{k,m,n} \in \{0,1\}$ to $I_{k,m,n} \geq 0$ and from $J_{m,n} \in \{0,1\}$ to $J_{m,n} \geq 0$ might yields a fractional-valued solution which should be rounded to 0 or 1. As an indirect approach, a low-complexity distributive solution is derived by solving the dual problem of P2. Since P2 is a linear optimization problem, the duality gap is zero. Firstly, the optimal solution for the time-slot one is solved by using Lagrange multipliers for constraint optimization, and dual decomposition. Then, subcarriers in time-slot two are determined for MSs in the direct area. The Lagrangian function of P2 is derived as follows:

$$
\begin{aligned}
& L(I,J,\varphi,\xi,\mu) = \qquad (10)\\
& \qquad \sum_{n=1}^{\hat{n_r}} \sum_{m=1}^{M} \sum_{k_r=1}^{\infty} (\sum_{k=1}^{k_r} I_{k,m,n} R_r^*) Pr(K_r = k_r)
\end{aligned}
$$

$$+\sum_{n=1}^{\hat{n_b}}\sum_{k_b=1}^{\infty}(\sum_{k=1}^{k_b} I_{k,0,n}R^{(1)}_{k,0,n}$$
$$+\sum_{m=M+1}^{M+k_b} J_{m,n}R^{(2)}_{m,n})Pr(K_b=k_b)$$
$$+\sum_{n=1}^{\hat{n_b}}\sum_{k_b=1}^{\infty}\varphi_{k_b}(\left\lceil\frac{k_b}{\hat{n_b}}\right\rceil-\sum_{k=1}^{k_b} I_{0,k,n})$$
$$+\sum_{n=1}^{\hat{n_b}}\sum_{k_b=1}^{\infty}\xi_{k_b}(\left\lceil\frac{k_b}{\hat{n_b}}\right\rceil-\sum_{m=M+1}^{M+k_b} J_{m,n})$$
$$+\sum_{n=1}^{\hat{n_r}}\sum_{k_r=1}^{\infty}\mu_{k_r}(\left\lceil\frac{k_r}{\hat{n_r}}\right\rceil-\sum_{m=1}^{M}\sum_{k=1}^{k_r} I_{k,m,n})$$

where φ , ξ and μ are the Lagrangian dual variables corresponding to constraints C2.3 and C2.4. Then, the dual objective function problem is computed as $h(\varphi,\xi,\mu) = \max_{I,J} L(I,J,\varphi,\xi,\mu)$ and the dual problem is given as

$$\min_{\varphi,\xi,\mu} h(\varphi,\xi,\mu), s.t. C1.1-1.2\, and\, C2.5-2.6. \qquad (11)$$

1) Resource Allocation for the First Hop: The dual objective function $h(\varphi,\xi,\mu)$ can be decomposed into two independent subproblems as $h(\varphi,\xi,\mu) = h_1(\varphi,\mu) + h_2(\xi)$, where

$$\begin{aligned} h_1(\varphi,\mu) &= \max_I L(I,\varphi,\mu),\ s.t.\ C1.1, C2.5-2.6 \\ &= \sum_{n=1}^{\hat{n_r}}\sum_{m=1}^{M}\sum_{k_r=1}^{\infty}(\sum_{k=1}^{k_r} I_{k,m,n}R_r^*)Pr(K_r=k_r) \\ &+\sum_{n=1}^{\hat{n_b}}\sum_{k_b=1}^{\infty}(\sum_{k=1}^{k_b} I_{k,0,n}R^{(1)}_{k,0,n})Pr(K_b=k_b) \\ &+\sum_{n=1}^{\hat{n_b}}\sum_{k_b=1}^{\infty}\varphi_{k_b}(\left\lceil\frac{k_b}{\hat{n_b}}\right\rceil-\sum_{k=1}^{k_b} I_{k,0,n}) \\ &+\sum_{n=1}^{\hat{n_r}}\sum_{k_r=1}^{\infty}\mu_{k_r}(\left\lceil\frac{k_r}{\hat{n_r}}\right\rceil-\sum_{m=1}^{M}\sum_{k=1}^{k_r} I_{k,m,n}) \end{aligned}$$

$$\begin{aligned} h_2(\xi) &= \max_J L(J,\xi)\ s.t.\ C1.2, C2.5 \qquad (12) \\ &= \sum_{n=1}^{\hat{n_b}}\sum_{k_b=1}^{\infty}(\sum_{m=M+1}^{M+k_b} J_{m,n}R^{(2)}_{m,n})Pr(K_b=k_b) \\ &+\sum_{n=1}^{\hat{n_b}}\sum_{k_b=1}^{\infty}\xi_{k_b}(\left\lceil\frac{k_b}{\hat{n_b}}\right\rceil-\sum_{m=M+1}^{M+k_b} J_{m,n}) \end{aligned}$$

$h_1(\varphi,\mu)$ can be further decoupled into $h_1(\varphi,\mu) = h_{11}(\varphi) + h_{12}(\mu)$ as follows:

$$h_{11}(\varphi) = \sum_{n=1}^{\hat{n_b}}\sum_{k_b=1}^{\infty}\sum_{k=1}^{k_b} I_{k,0,n}R^{(1)}_{k,0,n}Pr(K_b=k_b)$$
$$+\sum_{n=1}^{\hat{n_b}}\sum_{k_b=1}^{\infty}\varphi_{k_b}((\left\lceil\frac{k_b}{\hat{n_b}}\right\rceil-\sum_{k=1}^{k_b} I_{k,0,n})$$

$$h_{12}(\mu) = \sum_{n=1}^{\hat{n_r}}\sum_{m=1}^{M}\sum_{k_r=1}^{\infty}\sum_{k=1}^{k_r} I_{k,m,n}R_r^*Pr(K_r=k_r)$$
$$+\sum_{n=1}^{\hat{n_r}}\sum_{k_r=1}^{\infty}\mu_{k_r}(\left\lceil\frac{k_r}{\hat{n_r}}\right\rceil-\sum_{m=1}^{M}\sum_{k=1}^{k_r} I_{k,m,n})$$

Given $\varphi^{(t)}_{k_b}$, as the solution of $I^{(t)}_{k,0,n}$ should be round to 0 or 1 to get an integer-valued solution, it follows that $I^{(t)}_{k,0,n} = 1$, for $k = \bar{k}^{(t)}$, $n = \bar{n}^{(t)}$. $I^{(t)}_{k,0,n} = 0$, otherwise, where $(\bar{k}^{(t)},\bar{n}^{(t)}) = \max_{k,n} R^{(1)}_{k,0,n}Pr(K_b = k_b) - \varphi^{(t)}_{k_b}$. $\varphi^{(t)}_{k_b}$ can be updated using a gradient-based method as $\varphi^{(t+1)}_{k_b} = \left[\varphi^{(t)}_{k_b} - s_1^{(t)}(\left\lceil\frac{k_b}{\hat{n_b}}\right\rceil - \sum_{k=1}^{k_b} I^{(t)}_{k,0,n})\right]^+$.

Similarly, given $\mu^{(t)}_{k_r}$, the solution is given as $I^{(t)}_{k,m,n} = 1$, for $k = \bar{k}^{(t)}$, $m = \bar{m}^{(t)}$,$n = \bar{n}^{(t)}$; $I^{(t)}_{k,m,n} = 0$ otherwise, where $(\bar{k}^{(t)},\bar{m}^{(t)},\bar{n}^{(t)}) = \underset{k,m,n}{max} R_r^*Pr(K_r = k_r) - \mu^{(t)}_{k_r}$. $\mu^{(t)}_{k_r}$ can be updated as $\mu^{(t+1)}_{k_r} = [\mu^{(t)}_{k_r} - s_2^{(t)}(\left\lceil\frac{k_r}{\hat{n_r}}\right\rceil - \sum_{m=1}^{M}\sum_{k=1}^{k_r} I^{(t)}_{k,m,n})]^+$, where s_1 and s_2 are the step size of each iteration,$[x]^+ = \max(x,0)$. A near-optimal solution for the problem can be achieved with a sufficient number of iterations.

2) Resource Allocation for the Second Hop: After the resource allocation is completed for the time slot one, each subcarrier in time slot one is assigned to one MS-BS or MS-RS pair. Then for time slot two, the subcarriers used in the RS and the BS communication are the same as the subcarriers used in the MS and RS communication in time slot one. For direction communication between MSs and the BS, the subcarrier allocation is independent of those used in time slot one. The subcarrier allocation can be optimized by solving the dual problem $h_2(\xi)$ in the same way as resolving $h_{11}(\varphi)$. Hence the details are omitted due to space limitation.

IV. Simulation

In this section, the performance of the proposed scheme is evaluated under the scenario as shown in Fig. 1. The parameter p is chosen to be 0.3 [9]. The parameters used in the simulation are drawn from [3]. The values for R_0, D and D_0 are 750m, 1000m and 500m respectively. The bandwidth W for each subcarrier is 1.25MHz. The number of RSs M is 6. For communication link between BS-MS and RS-MS, the propagation model l is $128.1+37.6\log_{10}(d)$; for BS-RS link, l is $128.1+28.8\log_{10}(d)$, where d is the distance between the communicating nodes. The AGWN N_0 is -174dBm/Hz. Γ is $-\log(5BER)/1.5$, with target BER 10^{-3}. h obeys the rayleigh distribution.

Fig.2 shows the variation of n_b^* with collision probability threshold ε according to Eq.(8). It can be seen that when ε is

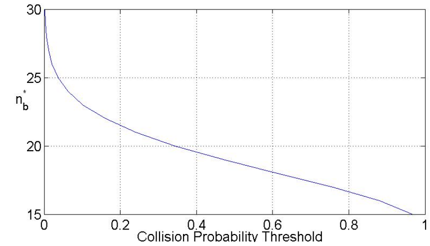

Figure 2. Variation of n_b^* with the collision probability threshold-ε.

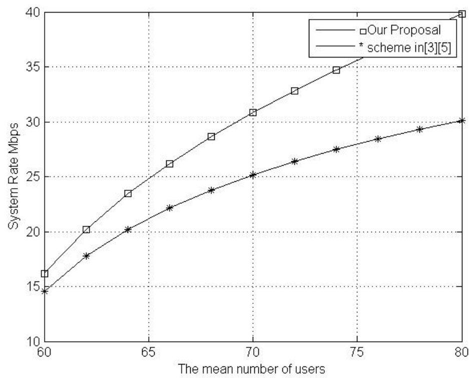

Figure 3. A comparison of the overall system throughput

less than 0.1, n_b^* is around 25. In the following performance simulation, the subcarrier available in direct area is determined as 25 which ensures the collision probability is less than 0.1.

Further simulations are performed to compare the performance of the proposed scheme with those in the literature [3], [5]. Fig.3 shows a comparison of the overall throughput between the proposed scheme and those in the literature. The simulation is repeated 10,000 times and the average value is shown in the figure. The mean number of users is varied between 60 and 80. As the mean number of users increases, the overall throughput achieved by our proposed scheme grows faster than [3], [5] which shows that the proposed scheme can achieve better spectral efficiency.

Fig.4 shows the channel utilization (the fraction of channels/subcarriers utilized in one time slot, i.e. actively transmitting traffic) as a function of the mean number of MSs for different values of p. It can be seen from Fig.4 that for different active probability p, our proposed method achieves better channel utilization than [3], [5] while the collision probability is controlled below $\varepsilon = 0.1$. It can also seen from Fig.4 that, when the value of p decreases, a more significant improvement in utilization can be achieved compared with those in [3], [5].

V. Conclusion

In this paper the optimal subcarrier allocation problem in cellular relay networks is investigated. The problem was formulated as an expectation maximization problem that maximizes the overall throughput while using some statistical information about MSs only, i.e. the density of MSs. The

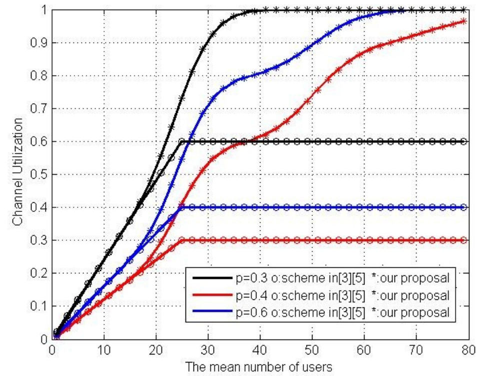

Figure 4. Channel Utilization Comparison

statistical multiplexing gain was also exploited to further improve channel utilization at the expense of some controlled collisions. A heuristic solution was proposed to solve the problem with low-complexity. Using simulations, the proposed scheme was shown to have better performance than those in the literature in both overall throughput and in the channel utilization.

In the future, we shall further investigate the fair radio resource allocation to ensure that each MS get approximately equal throughput. Spectrum reuse among RSs will also be investigated.

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