2007 Information, Decision and Control

Analysis of Flip Ambiguities in Distributed Network Localization

Anushiya A Kannan*†, Barış Fidan†‡, Guoqiang Mao*† and Brian D.O.Anderson†‡
*School of Electrical and Information Engineering, University of Sydney, NSW 2006, Australia.
†National ICT Australia Limited[1],Locked Bag 9013, Alexandria, NSW 1435, Australia.
‡Research School of Information Sciences & Engineering, Australian National University, ACT, Australia.
Email: {Anushiya.Kannan,Baris.Fidan,Guoqiang.Mao,Brian.Anderson}@nicta.com.au

Abstract

Accurate self-localization capability is highly desirable in wireless sensor networks. A major problem in trilateration based wireless sensor network localization that may introduce large errors in the location estimates is flip ambiguity. Recently the notion of robust quadrilaterals has been introduced as a way to identify possible flip ambiguities that otherwise corrupt localization computations. In this paper, we provide a formal geometric analysis of flip ambiguity problems using similar notions. The two main objectives of the paper are filling some technical gaps in the recent relevant works and developing a generic formal method for quantifying the likelihood of suffering from flip ambiguities for arbitrary sensor neighborhood geometries.

1. Introduction

A typical sensor network consists of a large number of sensor nodes densely deployed at positions which may not be predetermined due to constraints on the implementation environment and the cost of deployment of sensors. In most sensor network applications, the information gathered by these micro-sensors will be meaningless unless the location from where the information is obtained is known. This makes localization capabilities highly desirable in sensor networks.

Sensor network localization algorithms [1]–[4] estimate the locations of sensors with unknown location information with respect to a few sensors with known location information (known as anchors) by using inter-sensor measurements such as distance and bearing measurements. In this paper, we focus on distance measurement based localization algorithms.

Based on the approach of processing the individual inter-sensor distance data, distance-based localization algorithms can be considered in two main classes [4]: centralized algorithms and distributed algorithms. Centralized algorithms [4]–[7] use a single central processor to collect all the individual inter-sensor distance data and produce a map of the entire sensor network, while distributed algorithms [4], [8]–[10] rely on self-localization of each node in the sensor network using the local distance and position information it collects from its neighbors. One particular technique used in the later context is *trilateration* [4], [11], [12], where at each (local) step the location of a node is estimated using the information about its neighbor nodes whose positions or position estimates are available, together with the distance measurements from these neighbor nodes.

A fundamental problem in distance-based sensor network localization is whether a given sensor network is uniquely localizable or not, i.e. whether the sensor network geometry corresponding to a given set of inter-sensor distance measurements is unique or not. A particular framework that is useful for analyzing and solving this problem, especially in the trilateration context, is rigid graph theory [11]–[14]. In this framework, the sensor network to be localized is modelled by a graph, where vertices of the graph represent sensor nodes and edges of the graph connect the neighbor node pairs with known inter-sensor distance measurements.

The underlying graph is termed rigid if the sensor network has the following property: Any continuous replacement of sensors from their initial positions which maintains known inter-sensor distances actually preserves all inter-sensor distances. The graph is termed globally rigid if the known inter-sensor distances determine the geometry of the network up to congruence, i.e. determine all the inter-sensor distances uniquely, so that the network is localizable to within arbitrary translation, rotation, and reflection. Rigid graphs can be non-globally rigid just when one or both of two ambiguities exist: flip ambiguities and continuous flex ambiguities. For a sensor network with a globally rigid graph, having at least three noncollinear anchor nodes suffices to pin down the residual uncertainty, ensuring localizability. These ambiguities are further described in Section 2. In this paper we focus on flip ambiguities, which are more likely to happen (as compared to discontinuous flex ambiguities) in a typical localization algorithm and which may introduce significant errors in the location estimates.

Flip ambiguities typically arise when a node's neighbors are placed/estimated collinearly or nearly collinearly due to erroneous distance measurements; this includes the case where a node has only two neighbors (which are automatically collinear). Recently the notion of *robust quadrilaterals* has been introduced as a way to identify possible flip ambiguities that otherwise corrupt computations [12] in trilateration based localization procedures. Here, a *quadrilateral* is defined as

[1]National ICT Australia is funded by the Australian Government's *Backing Australia's Ability* initiative, in part through the Australian Research Council.

a complete graph with four vertices each pair of which is connected by an edge. In the context of sensor networks, a *quadrilateral* is a quadraple of nodes all of which are neighbors of each other, i.e. the inter-agent distance of any node pair within a *quadrilateral* is measurable. A quadrilateral is labelled as *robust* (with respect to distance measurement errors) if any of its four nodes (call node A) is uniquely localizable with a predefined accuracy level $\delta_S > 0$ (i.e. where the localization error is at most δ_S) once the positions of the other three nodes are precisely known and the measurements of the distances of node A from these three nodes are available where the corresponding absolute measurement errors are less than a predefined error bound $\bar{\epsilon} > 0$.

In this paper, we provide a formal geometric analysis of flip ambiguity problems using similar notions. The two main objectives of the paper are filling some technical gaps in the recent relevant works and developing a generic formal method for quantifying the likelihood of suffering of an arbitrary quadrilateral from flip ambiguities for arbitrary sensor neighborhood geometries. One particular aim of the analysis and quantification is to establish a criterion to determine whether a given quadrilateral should be used in localization or not (in order to avoid the errors to be introduced by this quadrilateral).

2. Ambiguities in Sensor Network Localization

Consider a sensor network of m anchor nodes with known locations and $n - m$ sensor nodes with unknown locations. For simplicity, let the nodes lie on a plane such that node i has location (x_i, y_i). Let us define a graph $G = (V, E)$, where V is the set of all nodes in the sensor network and E is the set of all links between one-hop neighbors. Assume that G is connected. As mentioned in Section 1, if G is rigid but not globally rigid, there are two types of ambiguities that may prevent sensor network localizability up to congruence [12], [15]: Flip (Fig. 1) and discontinuous flex (Fig. 2) ambiguities.

In flip ambiguities, a vertex (sensor node) has a set of neighbors which leads to the possibility of the neighbors forming a mirror through which the vertex (sensor node) can be reflected. Fig. 1 depicts an example of flip ambiguity.

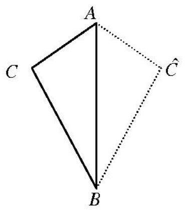


Fig. 1: Flip ambiguity: vertex C can be reflected across the line connecting the collinear neighbors A, B to the new location $\hat{C}$ without violating the distance constraints.

In discontinuous flex ambiguities, the removal of an edge allows the remaining part of the graph to be flexed to a different realization (which cannot be obtained from the original realization by translation, rotation or reflection) and the removed edge can be reinserted with the same length. Fig. 2 depicts an example. Use of graph rigidity and global rigidity

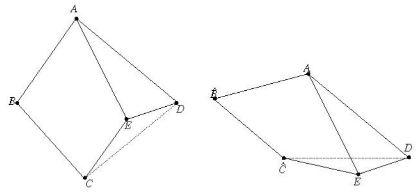


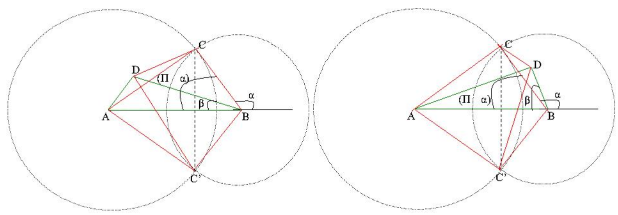
Fig. 2: Discontinuous flex ambiguity: removing the edge CD, flexing the edges AB, BC and CE, and reinserting the edge CD we obtain a new realization corresponding to the distance constraints.

notions in sensor network localization is well described and their importance is well demonstrated from both the algorithmic and the analytic aspects in the recent literature [4], [11], [13].

In analyzing the ambiguity problems in sensor network localization, a particular issue requiring attention is the effect of the noise in distance measurements. For example, it is common knowledge that in the presence of noisy distance measurements, a node is likely to have a flip ambiguity problem if its neighbors are nearly collinear. However there is little work in quantifying this relationship. A recent work focusing on robust distributed localization of sensor networks with certain distance measurement errors and ambiguities caused by these errors is presented in [12]. In this paper, certain criteria are provided in the selection of the subgraphs of the representative graph of a network to be used in a localization algorithm robustly against such errors. The analysis in [12], however, is not complete and there may be other criteria that may better characterize robustness of a given sub-network against distance measurement errors. We elaborate more on this in the next section.

3. Substantial Flip Ambiguities

In this section, we consider localization for quadrilaterals in a sensor network. We focus on the particular problem of estimating the location of node C using the given fixed positions of neighbor nodes A, B and D (see Fig.3) and inter-sensor measurements $\hat{d}_{AC}$, $\hat{d}_{BC}$ and $\hat{d}_{DC}$ between node C and each of its three neighbor nodes in an arbitrary quadrilateral $ABCD$.



(a) $\beta < \pi - \alpha$ **(b)** $\beta \geq \pi - \alpha$

Fig. 3: Localization of the node D using the positions of nodes A,B and C and inter sensor measurements $\hat{d}_{AC}$, $\hat{d}_{BC}$ and $\hat{d}_{DC}$ between node C and each of these nodes.

We focus on the following generic localization method, which has also been focused in [12]:

Step 1. Using $\hat{d}_{AC}$ and $\hat{d}_{BC}$ only, find the two possible locations $\hat{C}$ and $\hat{C}'$ for node C as intersection points of the circles $\mathcal{C}(A, \hat{d}_{AC})$ with center A and radius $\hat{d}_{AC}$ and $\mathcal{C}(B, \hat{d}_{BC})$ with center B and radius $\hat{d}_{BC}$.

Step 2. Using $\hat{d}_{DC}$, decide on which of $\hat{C}$ and $\hat{C}'$ to choose as the location estimate of C.

Remark 1: The above processing order of the inter-sensor measurements $\hat{d}_{AC}$,$\hat{d}_{BC}$,$\hat{d}_{DC}$ is without loss of generality, and the analysis below applies to any other order as well with appropriate index modifications.

For analysis purposes based on the specified inter-sensor measurement order, the position of D can be grouped into two cases with respect to the nodes A,B and C: $\beta < \pi - \alpha$ and $\beta \geq \pi - \alpha$ where α and β are defined in Fig.3. When the distances $\left|D\hat{C}\right|$,$\left|D\hat{C}'\right|$ between node D and the two possible location estimates $\hat{C}$,$\hat{C}'$ are very close, the measured distance $\hat{d}_{DC}$ is not sufficient to select one of $\hat{C}$ and $\hat{C}'$ as the location estimate of C. In these situations, there is a possibility for a flip ambiguity to occur in step 2.

The generic task of sensor localization is estimating the location of each sensor such that the magnitude of the corresponding estimation error is less than or equal to some given bound $\delta_S > 0$. Hence an error caused by a flip ambiguity where $\left|\hat{C}\hat{C}'\right| < \delta_S$ is not substantial in terms of the localization task. To take this observation into consideration, we define an *substantial flip ambiguity* as a flip ambiguity where the distance between the two possible positions $\hat{C}$ and $\hat{C}'$ is at least δ_S. A flip ambiguity which is not a substantial flip ambiguity is called a *negligible flip ambiguity*. In the following, we consider the identification of the substantial flip ambiguities only.

Now assuming that the measurements used in Step 1 are accurate and the error in measurement of distance $|DC|$ is less than some known small constant $\bar{\epsilon} > 0$, i.e., $\hat{d}_{DC} \in [|DC| - \bar{\epsilon}, |DC| + \bar{\epsilon}]$ or equivalently $|DC| \in [\hat{d}_{DC} - \bar{\epsilon}, \hat{d}_{DC} + \bar{\epsilon}]$, the flip ambiguity problem is likely to occur if $\left|\left|D\hat{C}\right| - \left|D\hat{C}'\right|\right| < 2\bar{\epsilon}$. Here, $\bar{\epsilon}$ could be represented by σ, 2σ or 3σ when the noise in measurement $\hat{d}_{DC}$ is represented with a zero-mean, σ standard deviation Gaussian noise. Note that this noise is taken to be zero mean Gaussian and $\bar{\epsilon}$ is chosen as $\bar{\epsilon} = 3\sigma$ in [12].

4. Effects of Neighborhood Geometry on Flip Ambiguities

In this section, we focus on the flip ambiguity likelihood indicator $\left|\left|D\hat{C}\right| - \left|D\hat{C}'\right|\right|$ and analyze the effect of the geometry of $\triangle ABD$ and measurements $\hat{d}_{AC}$ and $\hat{d}_{BC}$ on this quantity. To do that, we consider the following problem.

Problem 1: Given fixed positions of A and B, for which positions of D and measurements $\hat{d}_{AC}$, $\hat{d}_{BC}$ is $\left|\left|D\hat{C}\right| - \left|D\hat{C}'\right|\right|$ less than the threshold $\bar{\epsilon}_1 \triangleq 2\bar{\epsilon}$ for $\left|\hat{C}\hat{C}'\right| \geq \delta_S$, where $\hat{C}$ and $\hat{C}'$ denote the positions of the two possible points having distance $\hat{d}_{AC}$ and $\hat{d}_{BC}$ from A and B respectively?

We approach Problem 1 in two steps: In the first step assuming that the position of D is also given, we find the measurements $\hat{d}_{AC}$ and $\hat{d}_{BC}$ minimizing $\left|\left|D\hat{C}\right| - \left|D\hat{C}'\right|\right|$ as described in Problem 2 below. In the second step, based on the solution of Problem 2, we find the set of all acceptable positions for D.

Problem 2: Given fixed positions of neighbor nodes A,B and D (in Fig.3); for which measurement couple $(\hat{d}_{AC}, \hat{d}_{BC})$ is the value of $\left|\left|D\hat{C}\right| - \left|D\hat{C}'\right|\right|$ minimum given that $\left|\hat{C}\hat{C}'\right| \geq \delta_S$, where $\hat{C}$ and $\hat{C}'$ denote the positions of the two possible points having distance $\hat{d}_{AC}$ and $\hat{d}_{BC}$ from A and B respectively?

As a partial answer to Problem 2, it is claimed in [12] that $\left|\left|D\hat{C}\right| - \left|D\hat{C}'\right|\right|$ is minimized when the quadrilateral $AB\hat{C}D$ is symmetric about the perpendicular bisector of $[AB]$. However this claim can be falsified with a number of counter-examples. One such counter-example is depicted in Fig.4, where we fix a certain position for $\hat{C}$ and consider the possible locations for D such that $D\hat{C} \perp \hat{C}\hat{C}'$ denoting the angle $\angle \hat{C}D\hat{C}'$ by ψ.

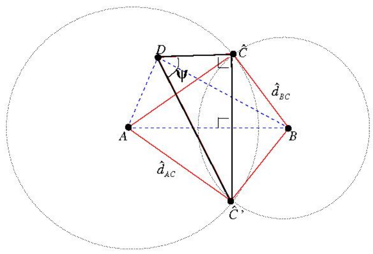


Fig. 4: A counter-example for the argument in [12] claiming that $D\hat{C} - D\hat{C}'$ is minimum at left-right symmetric $AB\hat{C}D$.

We have $\left|D\hat{C}'\right| = \frac{\left|\hat{C}\hat{C}'\right|}{\sin\psi}$ and $\left|D\hat{C}\right| = \frac{\left|\hat{C}\hat{C}'\right|\cos\psi}{\sin\psi}$, which implies that $\left|\left|D\hat{C}\right| - \left|D\hat{C}'\right|\right| = \left|\hat{C}\hat{C}'\right| \frac{1-\cos\psi}{\sin\psi} = \left|\hat{C}\hat{C}'\right| \tan\frac{\psi}{2}$. Now consider the affect of D moving along the line perpendicular to $\hat{C}\hat{C}'$ and passing through $\hat{C}'$. As D moves, ψ will change. For larger values of $\left|D\hat{C}\right|$, $\left|\left|D\hat{C}\right| - \left|D\hat{C}'\right|\right|$ gets smaller, which contradicts the claim in [12] saying that left-right symmetry in $AB\hat{C}D$ results in the minimum possible value of $\left|\left|D\hat{C}\right| - \left|D\hat{C}'\right|\right|$.

In our analysis we first solve the following problem which can be shown to be equivalent to Problem 2.

Problem 3: Given fixed positions of neighbor nodes A,B and D (in Fig.3); for which measurement couple $(\alpha, \hat{d}_{BC})$ is the value of $\left|\left|D\hat{C}\right| - \left|D\hat{C}'\right|\right|$ minimum given that $\left|\hat{C}\hat{C}'\right| \geq \delta_S$, where $\hat{C}$ and $\hat{C}'$ denote the positions of the two possible points having distance $\hat{d}_{BC}$ from B and absolute bearing angle α with respect to $\overrightarrow{AB}$?

Note that the relation between $\hat{d}_{AC}$ and α can be obtained using trigonometric analysis of $\triangle ABC$ of Fig.3, as follows.

$$\left|A\hat{C}\right|^2 = \left|B\hat{C}\right|^2 + |AB|^2 + 2\left|B\hat{C}\right||AB|\cos\alpha$$
$$\alpha = cos^{-1}\left(\frac{\left|A\hat{C}\right|^2 - \left|B\hat{C}\right|^2 - |AB|^2}{2\left|B\hat{C}\right||AB|}\right) \qquad (1)$$

The detailed formal analysis of Problem 3 can be found in [16]. Here, we present the essentials of this analysis. From Fig.3, the measurement error square d^2_{error} can be derived as

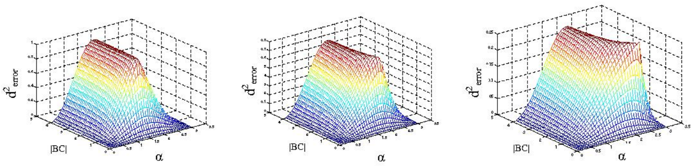
$$\begin{aligned} & d^2_{error} \\ = & \left(\frac{\left|D\hat{C}\right| - \left|D\hat{C}'\right|}{2}\right)^2 \\ = & \Bigg(\frac{1}{2}\Big(\sqrt{|DB|^2 + \left|B\hat{C}\right|^2 + 2|DB|\left|B\hat{C}\right|\cos(\alpha+\beta)} - \\ & \sqrt{|DB|^2 + \left|B\hat{C}\right|^2 + 2|DB|\left|B\hat{C}\right|\cos(\alpha-\beta)}\Big)\Bigg)^2 \end{aligned} \qquad (2)$$

If we analyze the effect of variations in α on d^2_{error} for an arbitrary fixed $\left|B\hat{C}\right|$, after some manipulation, we have

$$\begin{aligned} & \frac{\partial(d^2_{error})}{\partial\alpha} \\ = & \frac{1}{2}\left(\left|D\hat{C}\right| - \left|D\hat{C}'\right|\right)^2 \frac{|DB||B\hat{C}|}{|D\hat{C}||D\hat{C}'|}(\sin\alpha\cos\beta) \\ & \left(1 + \frac{\tan\beta(\left|D\hat{C}'\right| + \left|D\hat{C}\right|)}{\tan\alpha(\left|D\hat{C}'\right| - \left|D\hat{C}\right|)}\right) \end{aligned} \qquad (3)$$

In (3), $|DB|$, $|BC|$, $|D\hat{C}|$ and $|D\hat{C}'|$ are all distances between two nodes hence they are greater than zero and since $0 \leq \alpha \leq \pi$, $\sin\alpha \geq 0$. So by analyzing $\cos\beta\left[1+\frac{\tan\beta}{\tan\alpha}\left(\frac{|D\hat{C}'|+|D\hat{C}|}{|D\hat{C}'|-|D\hat{C}|}\right)\right]$, we see that $\frac{\partial(d^2_{error})}{\partial\alpha}$ decreases from a positive value to a negative value continuously as α moves from 0 to π. Hence d^2_{error} has only one extremum and it is a maximum. So d^2_{error} has its minimum at the boundary $\alpha = 0$ or $\alpha = \pi$.

The variation of d^2_{error} with respect to $\hat{d}_{BC}$ and α is plotted in Fig.5. We deduce from the above analysis, the constrain $|\hat{C}\hat{C}'| \geq 2\delta$ and Fig.5 that the minimizing $(\alpha, \hat{d}_{BC})$ pair lies on the boundary curve $\hat{d}_{BC}\sin\alpha = \delta$ where $\delta \triangleq \delta_S/2$, which is plotted in Fig.6.



(a) $\beta = \frac{\pi}{2}$ **(b)** $\beta = \frac{\pi}{3}$ **(c)** $\beta = \frac{\pi}{6}$

Fig. 5: The plots of d^2_{error} for three different β with $|DB| = \frac{R}{2}$.

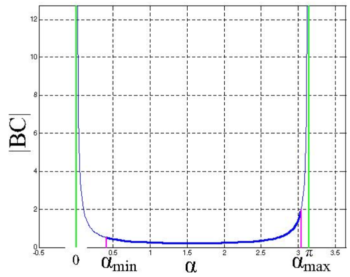


Fig. 6: Plot of $|BC|$ vs α at the boundary $|BC|\sin\alpha = \delta = \delta_S/2$.

Furthermore, by the definition of *substantial flip ambiguity* and the fact that neighbors of a node in a sensor network should lie within the transmission range R as depicted in Fig. 7, we see that $\left|\left|D\hat{C}\right| - \left|D\hat{C}'\right|\right|$ is minimized on the portion of the boundary curve $\hat{d}_{BC}\sin\alpha = \delta$ where

$$\delta \leq \left|B\hat{C}\right| \leq R \text{ and } \alpha_{min} \leq \alpha \leq \alpha_{max} \qquad (4)$$
$$\alpha_{min} = \sin^{-1}\left(\frac{\delta}{\sqrt{\left(\sqrt{R^2-\delta^2} - |AB|\right)^2 + \delta^2}}\right)$$
$$\alpha_{max} = \pi - sin^{-1}\left(\frac{\delta}{R}\right)$$

This portion is highlighted in Fig.6.

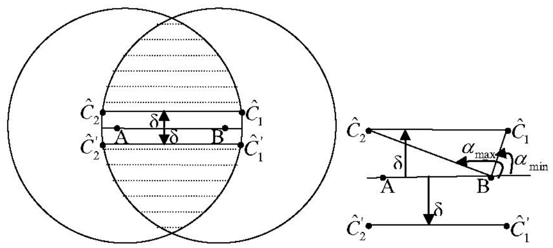
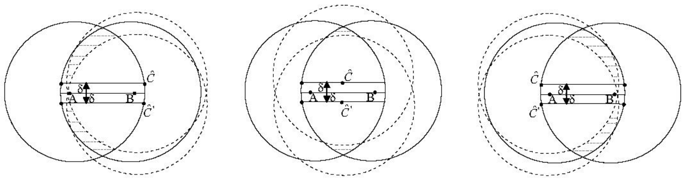


Fig. 7: Shaded region is the set of all the possible locations of $\hat{C}$ and $\hat{C}'$ within the neighborhood of A and B to cause a substantial flip. Here $|A\hat{C}_1| = |A\hat{C}'_1| = R$ and $|B\hat{C}_2| = |B\hat{C}'_2| = R$.

The above discussion completes the first step of our approach to solving Problem 1. Next, we consider the second step: We find the possible locations for D in which $\left|\left|D\hat{C}\right| - \left|D\hat{C}'\right|\right| \geq 2\bar{\epsilon}$ when $\left|\left|D\hat{C}\right| - \left|D\hat{C}'\right|\right|$ is at its minimum. The collection of these locations will give us the region for D such that estimation of C is not likely to suffer from substantial flip ambiguity.

Note first that D, as a neighbor of both A and B, should be located in the intersection of the disks with the bordering circles $\mathcal{C}(A, R)$ with center A and radius R, and $\mathcal{C}(B, R)$ with center B and radius R. To locate node C using the location of node D, node D should be located either in the circle $\mathcal{C}(\hat{C}, R)$ with center $\hat{C}$ and radius R or in the circle $\mathcal{C}(\hat{C}', R)$ with center $\hat{C}'$ and radius R or in both. If D is in the circle $\mathcal{C}(\hat{C}, R)$ and not in the circle $\mathcal{C}(\hat{C}', R)$, then D is not within the transmission range of $\hat{C}'$ and node C could be located uniquely at $\hat{C}$ without any ambiguity. Likewise if

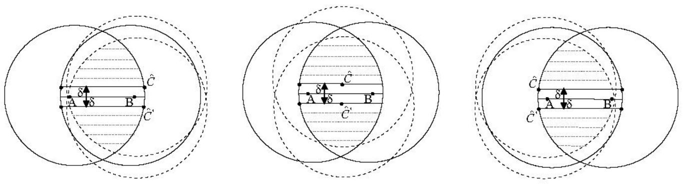
D is in the circle $\mathcal{C}(\hat{C}', R)$ and not in the circle $\mathcal{C}(\hat{C}, R)$, then node C could be located uniquely at $\hat{C}'$ without any ambiguity. The shaded area of Fig. 8 shows where D could be located such that it will be in the transmission range of only one estimated location of C for three different scenarios, $\alpha = \alpha_{min}$, $\alpha_{min} < \alpha < \alpha_{max}$ and $\alpha < \alpha_{max}$.



(a) $\alpha = \alpha_{min}$ **(b)** $\alpha_{min} < \alpha < \alpha_{max}$ **(c)** $\alpha = \alpha_{max}$

Fig. 8: Possible locations of D to be a neighbor of only one of $\hat{C}$,$\hat{C}'$ for given A, B.

If D is in the intersection of the disks with the bordering circles $\mathcal{C}(\hat{C}, R)$ and $\mathcal{C}(\hat{C}', R)$, i.e. if D is located in the area shaded in Fig. 9, there is a possibility for a substantial flip. In order to prevent substantial flip ambiguity, we could restrict the position of node D in the shaded area enclosed by the hyperbola $\mathcal{H}(\hat{C}, \hat{C}', \bar{\epsilon}) \triangleq \left\{ P | \left| \left|P\hat{C}\right| - \left|P\hat{C}'\right| \right| = 2\bar{\epsilon} \right\}$ with foci at $\hat{C}$ and $\hat{C}'$(which will give $\left| \left|D\hat{C}\right| - \left|D\hat{C}'\right| \right| \geq 2\bar{\epsilon}$) in Fig. 10 which can be expressed as follows:



(a) $\alpha = \alpha_{min}$ **(b)** $\alpha_{min} < \alpha < \alpha_{max}$ **(c)** $\alpha = \alpha_{max}$

Fig. 9: Possible locations of D to be a neighbor of both $\hat{C}$ and $\hat{C}'$ for given A, B.

$$\frac{(x - x_C)^2}{\bar{\epsilon}^2} - \frac{y^2}{\sqrt{\delta^2 - \bar{\epsilon}^2}} \geq 1 \tag{5}$$

Here (x, y) is the coordinate of D and without loss of generality, we take AB as x-axis and midpoint of AB as origin; also $(x_C, 0)$ is the coordinate of the midpoint of $\hat{C}\hat{C}'$. The $y-$coordinates of $\hat{C}$ and $\hat{C}'$ have the same magnitude and different signs and are denoted by y_C and $-y_C$ respectively.

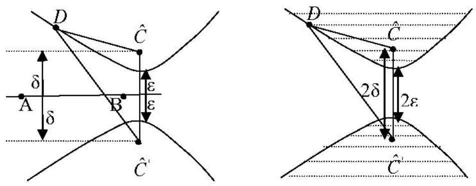


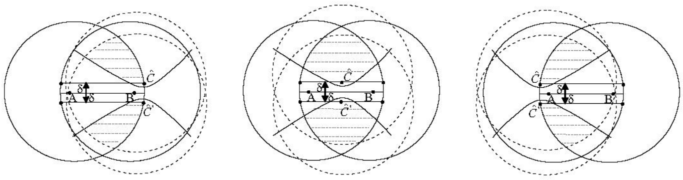
Fig. 10: Possible locations of D for a given $\hat{C}$, $\hat{C}'$ and $d_{error} \geq \bar{\epsilon}$

.

From the possible region of node D shown in Fig. 9, if we eliminate the region for a possible substantial flip ambiguity, that is where $\left| \left|D\hat{C}\right| - \left|D\hat{C}'\right| \right| < 2\bar{\epsilon}$, D should be located in the intersection of the regions enclosed by the circles $\mathcal{C}(A, R)$, $\mathcal{C}(B, R)$, $\mathcal{C}(\hat{C}, R)$, $\mathcal{C}(\hat{C}', R)$ and region enclosed by the hyperbola $\mathcal{H}(\hat{C}, \hat{C}', \bar{\epsilon})$ as shown in Fig. 11. This can be stated as follows:

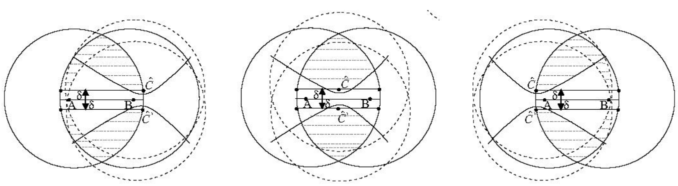
$$\begin{aligned} &(x - x_C)^2 + (y - y_C)^2 \leq R^2 \\ &\text{and } (x - x_C)^2 + (y + y_C)^2 \leq R^2 \\ &\text{and } (x - x_A)^2 + y^2 \leq R^2 \\ &\text{and } (x - x_B)^2 + y^2 \leq R^2 \\ &\text{and } \frac{(x - x_C)^2}{\bar{\epsilon}^2} - \frac{y^2}{\sqrt{\delta^2 - \bar{\epsilon}^2}} \geq 1 \end{aligned} \tag{6}$$

Here (x_C, y_C), $(x_C, -y_C)$,$(x_A, 0)$ and $(x_B, 0)$ are the coordinates of $\hat{C}$, $\hat{C}'$, A and B respectively. From the above analysis, the union of the shaded regions of Fig. 8 and Fig. 11 which is shown in Fig. 12, gives the set of possible locations of D for which substantial flip ambiguity is guaranteed to be avoided.



(a) $\alpha = \alpha_{min}$ **(b)** $\alpha_{min} < \alpha < \alpha_{max}$ **(c)** $\alpha = \alpha_{max}$

Fig. 11: Possible locations of D for a given A, B, $\hat{d}_{AC}$, $\hat{d}_{BC}$, $d_{error} \geq \bar{\epsilon}$ and D is in the transmission range of $\hat{C}$ and $\hat{C}'$.



(a) $\alpha = \alpha_{min}$ **(b)** $\alpha_{min} < \alpha < \alpha_{max}$ **(c)** $\alpha = \alpha_{max}$

Fig. 12: Possible locations of D for a given A, B, $\hat{d}_{AC}$, $\hat{d}_{BC}$, $d_{error} \geq \bar{\epsilon}$ such that node C can be located uniquely.

As we can see from Fig. 12, the set of possible locations of D for which substantial flip ambiguity is guaranteed to be avoided is dependent on the estimated location of node C. We are also interested in the set of all possible locations of D for which substantial flip ambiguity is guaranteed to be avoided, independent of the estimated location of C. This region is shown as the shaded area in Fig. 13 and can be expressed as the area bounded by the hyperbolas $\mathcal{H}(\hat{C}_{\alpha_{min}}, \hat{C}'_{\alpha_{min}}, \bar{\epsilon})$, $\mathcal{H}(\hat{C}_{\alpha_{max}}, \hat{C}'_{\alpha_{max}}, \bar{\epsilon})$ and circles $\mathcal{C}(A, R)$, $\mathcal{C}(B, R)$, i.e.,

$$\begin{aligned} &\frac{(x - x_{C_{\alpha_{min}}})^2}{\bar{\epsilon}^2} - \frac{y^2}{\sqrt{\delta^2 - \bar{\epsilon}^2}} \geq 1 \\ &\text{and } \frac{(x - x_{C_{\alpha_{max}}})^2}{\bar{\epsilon}^2} - \frac{y^2}{\sqrt{\delta^2 - \bar{\epsilon}^2}} \geq 1 \\ &\text{and } (x - x_A)^2 + y^2 \leq R^2 \\ &\text{and } (x - x_B)^2 + y^2 \leq R^2 \end{aligned} \tag{7}$$

where $x_{C_{\alpha_{min}}} \triangleq x_C|_{\alpha=\alpha_{min}}$ and $x_{C_{\alpha_{max}}} \triangleq x_C|_{\alpha=\alpha_{max}}$.

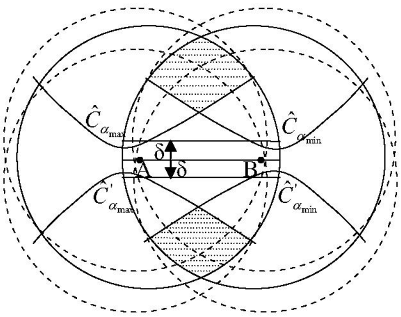


Fig. 13: Possible locations of D independent of the location of C for a given A, B, such that node C can be located uniquely.

The results of our analysis are summarized in the following propositions:

Proposition 1: Given fixed positions $(x_A, 0)$ and $(x_B, 0)$ of A and B with $x_A < x_B$ and distance measurements $\hat{d}_{AC}, \hat{d}_{BC}$ satisfying $\delta \leq \hat{d}_{AC}, \hat{d}_{BC} \leq R$, for any $D = (x, y)$ satisfying (6), i.e. lying in the shaded region of Fig. 12, the quadrilateral $ABCD$ does not suffer from substantial flip ambiguity, i.e.

$$\left|\left|D\hat{C}\right| - \left|D\hat{C}'\right|\right| \geq \bar{\epsilon}_1 = 2\bar{\epsilon}$$

for $\left|\hat{C}\hat{C}'\right| \geq \delta_S$, where $\hat{C} = (x_C, y_C)$ and $\hat{C}' = (x_C, -y_C)$ denote the positions of the two possible points having distance $\hat{d}_{AC}$ and $\hat{d}_{BC}$ from $(x_A, 0)$ and $(x_B, 0)$, respectively.

Proposition 2: Given fixed positions $(x_A, 0)$ and $(x_B, 0)$ of A and B with $x_A < x_B$, for any $D = (x, y)$ satisfying (7), i.e. lying in the shaded region of Fig. 13, and for any $\hat{d}_{AC}, \hat{d}_{BC}$ satisfying $\delta = \delta_S/2 \leq \hat{d}_{AC}, \hat{d}_{BC} \leq R$, the quadrilateral $ABCD$ does not suffer from substantial flip ambiguity, i.e.

$$\left|\left|D\hat{C}\right| - \left|D\hat{C}'\right|\right| \geq \bar{\epsilon}_1 = 2\bar{\epsilon}$$

for $\left|\hat{C}\hat{C}'\right| \geq \delta_S$, where $\hat{C} = (x_C, y_C)$ and $\hat{C}' = (x_C, -y_C)$ denote the positions of the two possible points having distance $\hat{d}_{AC}$ and $\hat{d}_{BC}$ from $(x_A, 0)$ and $(x_B, 0)$, respectively.

Algorithmic criteria corresponding to Propositions 1 and 2 can be developed via writing the analysis leading to each of these propositions in algorithm form. Following is a possible algorithmic implementation of Proposition 1:

IF ($|D\hat{C}| \leq R$ and $|D\hat{C}'| \leq R$)

D is in the transmission range of both $\hat{C}$ and $\hat{C}'$

IF ($|DA| \leq R$ and $|DB| \leq R$ and $d_{error} \geq 2\bar{\epsilon}$)

There is no possibility for flip ambiguity. So accept the quad as robust

IF $|D\hat{C}| - \hat{d}_{DC} < |D\hat{C}'| - \hat{d}_{DC}$ accept $\hat{C}$

ELSE accept $\hat{C}'$

ELSE

There is possibility for flip ambiguity. So reject the quad as it is not robust

ELSE

D is not in the transmission range of both $\hat{C}$ and $\hat{C}'$

IF $|D\hat{C}| \leq R$ *Accept* $\hat{C}$

ELSE IF $|D\hat{C}'| \leq R$ *Accept* $\hat{C}'$

5. Conclusion

In this paper we have provided a formal geometric analysis of flip ambiguity problems, which are possible sources of computational corruption in distance-based localization. For a quadrilateral $ABCD$ with known node positions A and B, we have determined the region for the position of D such that the node C can be uniquely localized using the measurements of the distances $|AC|$,$|BC|$,$|DC|$.

This analysis could be extended by considering all the possible node process orders in light of Remark 1. It is part of our future work to develop a more robust technique to identify quadrilaterals that don't suffer from substantial flip ambiguity, considering the measurement errors in $\hat{d}_{AC}$ and $\hat{d}_{BC}$ as well as the estimated position errors of known nodes A, B and D. Still another direction for our future work is investigating propagation effects of these errors. This will improve scalability of the algorithm.

References

[1] S. Guolin, C. Jie, G. Wei, and K. Liu, "Signal processing techniques in network-aided positioning: a survey of state-of-the-art positioning designs," *IEEE Signal Proc. Magazine*, vol. 22, no. 4, pp. 12 – 23, 2005.

[2] A. Sayed, A. Tarighat, and N. Khajehnouri, "Network-based wireless location: challenges faced in developing techniques for accurate wireless location information," *IEEE Signal Processing Magazine*, vol. 22, no. 4, pp. 24 – 40, 2005.

[3] F. Gustafsson and F. Gunnarsson, "Mobile positioning using wireless networks: possibilities and fundamental limitations based on available wireless network measurements," *IEEE Signal Processing Magazine*, vol. 22, no. 4, pp. 41 – 53, 2005.

[4] G. Mao, B. Fidan, and B. Anderson, "Sensor network localization," in *Sensor Networks and Configuration: Fundamentals, Techniques, Platforms and Experiments*, N. Mahalik, Ed. Springer, 2006.

[5] Y. Shang, W. Ruml, Y. Zhang, and M. Fromherz, "Localization from connectivity in sensor networks," *IEEE Transactions on Parallel and Distributed Systems*, vol. 15, no. 11, pp. 961 – 974, 2004.

[6] P. Biswas and y. Ye, "Semidefinite programming for ad hoc wireless sensor network localization," in *Information Processing in Sensor Networks, 2004. IPSN 2004*, 2004, pp. 46 – 54.

[7] A. Kannan, G. Mao, and B. Vucetic, "Simulated annealing based wireless sensor network localization with flip ambiguity mitigation," in *IEEE Vehicular Technology Conference*, vol. 2, 2006, pp. 1022 – 1026.

[8] K. Langendoen and N. Reijers, "Distributed localization in wireless sensor networks: a quantative comparison," *Computer Networks*, vol. 43, pp. 499 – 518, 2003.

[9] V. Fox, J. Hightower, L. Lin, D. Schulz, and G. Borriello, "Bayesian filtering for location estimation," in *IEEE Pervasive Computing*, vol. 2, no. 3, 2003, pp. 24 – 33.

[10] A. Ihler, I. Fisher, R. Moses, and A. Willsky, "Nonparametric belief propagation for self-localization of sensor networks," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 4, pp. 809 – 819, 2005.

[11] T. Eren et. al., "Rigidity, computation, and randomization in network localisation," in *IEEE INFOCOM*, vol. 4, 2004, pp. 2673 – 2684.

[12] D. Moore, J. Leonard, D. Rus, and S. Teller, "Robust distributed network localization with noisy range measurements," in *2nd ACM Conf. on Embedded Networked Sensor Systems*, Nov. 2004, pp. 50 – 61.

[13] J. Aspnes, T. Eren, D. Goldenberg, A. Morse, W. Whiteley, Y. Yang, B. Anderson, and P. Belhumeur, "A theory of network localization," *submitted to IEEE Transactions on Mobile Computing*, vol. 5, no. 12, pp. 1663 – 1678, December 2006.

[14] R. Connelly, "Generic global rigidity," *Discrete and Computational Geometry*, vol. 33, pp. 549 – 563, 2005.

[15] B. Hendrickson, "Conditions for unique graph realizations," *SIAM J. Comput.*, vol. 21, no. 1, pp. 65 – 84, 1992.

[16] A. Kannan, B. Fidan, G. Mao, and B. Anderson, "Analysis of flip ambiguities in distributed network localization," National ICT Australia, tech. report PA006288, November 2006.