# Robust Distributed Sensor Network Localization Based on Analysis of Flip Ambiguities

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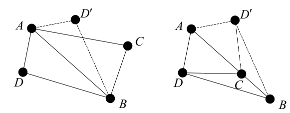
***Abstract*— A major problem in wireless sensor network localization is erroneous local geometric realizations in some parts of the network due to the sensitivity to certain distance measurement errors, which may in turn affect the reliability of the localization of the whole or a major portion of the sensor network. This phenomenon is well-described using the notion of "flip ambiguity" in rigid graph theory. In a recent study by the coauthors, an initial formal geometric analysis of flip ambiguity problems has been provided. The ultimate aim of that study was to quantify the likelihood of flip ambiguities in arbitrary sensor neighborhood geometries. In this paper we propose a more general robustness criterion to detect flip ambiguities in arbitrary sensor neighborhood geometries in planar sensor networks. This criterion enhances the recent study by the coauthors by removing the assumptions of accurately knowing some inter-sensor distances. The established robustness criterion is found to be useful in two aspects: (a) Analyzing the effects of flip ambiguity and (b) Enhancing the reliability of the location estimates of the prevailing localization algorithms by incorporating this robustness criterion to eliminate neighborhoods with flip ambiguity from being included in the localization process.**

## I. Introduction

Accurate self-localization capability is a highly desirable characteristic of wireless sensor networks [1]. Distance based distributed localization algorithms [1]–[3] rely on self-localization of each node in a sensor network using the inter-sensor distance and position information they collect from one-hop or multi-hop neighbors. A major problem affecting the unique realizability of the distance-based localization is erroneous local geometric realizations in some parts of the network due to certain distance measurement errors together with the actual geometries of these parts causing the realizations to be sensitive to such errors [1][2].

*Flip ambiguity* (Fig. 1) is a phenomenon that can prevent network realization from being unique [2]–[4] (in the sense that of differing from the other possible realization of the underlying graph of the network at most by translation, rotation or reflection). When the underlying graph is rigid but not globally rigid [1], it may lead to the possibility of neighbors forming a mirror through which a node can be reflected (See Fig. 1a). But even in a network with globally rigid underlying graph [1], almost collinear neighbors of a sensor may lead to the possibility of neighbors forming a mirror through which the sensor node can be reflected (See Fig. 1b).




(a) (b)

(a) Flip ambiguity in rigid but not globally rigid graph.
(b) Flip ambiguity in globally rigid graph with collinear neighbors.

Fig. 1. Flip ambiguity: Reflecting $D$ through a mirror formed by neighbors, a new realization $D'$ is obtained without violating the distance constraints.

In this paper, we present an analysis on the likelihood of flip ambiguity in the presence of distance measurement errors, thereby establishing a robustness criterion to quantify the likelihood of flip ambiguity in completely connected sensor quadruples in a planar sensor network. We then use this robustness criterion to eliminate consideration (in the localization process) of the nodes that contribute to flip ambiguity from the neighborhood. Thus the robustness criterion helps the localization algorithm to select neighborhoods with all neighbors that do not contribute to any flip ambiguity. The paper also presents a number of simulation results demonstrating the use of the established robustness criterion.

## II. Flip Ambiguity Effects in Sensor Network Localizations

There exist different distributed localization algorithms in the literature [1], [5], which can be generally categorized into two groups: (i) sequential/incremental localizations [6]–[10] and (ii) cluster localization followed by stitching/patching [3], [11]–[13].

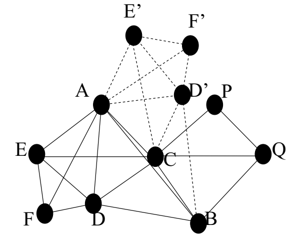



Fig. 2. A single flip ambiguity and the avalanche effect in the error propagation. Initially, locations of $A, B, C, P$ and $Q$ are known and $D, E$ and $F$ are unknown. At the first iteration, $D$ uses locations of collinear neighbors $A, B, C$ and localize itself at $D'$ due to flip ambiguity. In the second iteration, $E$ uses locations of neighbors $A, C, D$ and localize itself at $E'$. In the third iteration, $F$ uses locations of neighbors $A, D, E$ and localize itself at $F'$.

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[2]Although these problems may also exist in localization based on other types of measurement techniques, we focus only on distance based localization.

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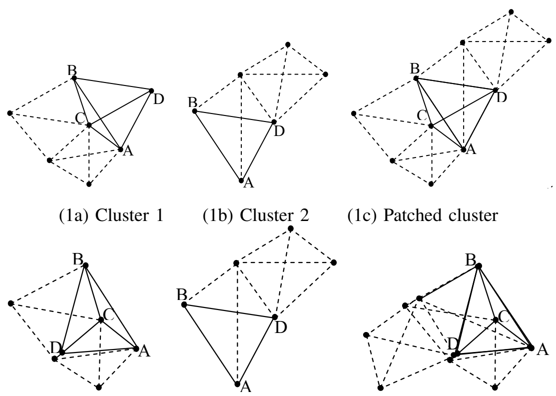
In sequential localization [6]–[10], at each iteration, all unknown nodes which have at least 3 known neighbors are localized. These known neighbors can be anchors or some nodes defining the local coordinate system. When unknown nodes are localized, they are elevated to anchor status, thereby increasing the chances of unknown nodes being localized in the subsequent iterations. However the presence of flip ambiguities at each iteration may have avalanche effects[3] causing large errors in subsequent location estimates, which is demonstrated in Fig. 2 via a simple example.




(1a) Cluster 1 (1b) Cluster 2 (1c) Patched cluster

(2a) Cluster 1 (2b) Cluster 2 (2c) Patched cluster

Fig. 3. Localization of the node $D$ using the positions of nodes $A$, $B$ and $C$ and inter sensor measurements $\overline{d}_{AD}$, $\overline{d}_{BD}$ and $\overline{d}_{CD}$ in cluster 1 resulted in flip ambiguity in (2a) but not in (1a). Hence stitching cluster 1 and cluster 2 created avalanche effect in (2c) but not in (1c).

Most of the cluster localization [3], [11]–[13] which claim to reduce propagation of estimation errors is done in three phases: In Phase 1 of cluster localization, each unknown node having 3 or more neighbors forms a local coordinate system and all of its one-hop neighbors are localized in that local coordinate system. Flip ambiguity may occur in this phase affecting the location estimate of the cluster members, although the avalanche effect may not be prominent at this stage. In Phase 2, local clusters having 3 or more common nodes are stitched together forming one or more disjoint large clusters. In Phase 3, the stitched clusters with 3 or more anchor nodes are translated to the central coordinate system. If a flipped realized node in Phase 1 is used in the stitching in Phase 2 or in the transformation in Phase 3, avalanche effects may occur similarly to sequential localization schemes. This is illustrated in Fig. 3.

It is necessary to identify the possible occurrence of flip ambiguity in any neighborhood $N_i$ in order to avoid the avalanche error propagation. In section III we propose a robustness criterion to make a binary decision of whether to accept or to reject consideration of any neighbor $j \in N_i$ in the localization process. To keep the analysis simple, we are only considering fully connected quadrilaterals (FCQs) as a neighborhood, which represents a quadruple of sensors, all of which are neighbors of each other, i.e. the distance between any pair is measurable[4]. Later we use this criterion to select all members $j \in N_i$ which do not contribute to flip ambiguity to form the robust neighborhood $RN_i$, so that the localization algorithm uses information from all known neighbors which do not contribute to flip ambiguity.

## III. Robustness Criterion to Identify Possible Flip Ambiguities

Consider an ordered FCQ $ABCD$ where locations of sensors $A, B, C$ and the respective measured distances $\overline{d}_{AD}$, $\overline{d}_{BD}$ and $\overline{d}_{CD}$ from sensor $D$ to sensors $A$, $B$ and $C$ are known. Assuming the distance measurement error has a known Gaussian distribution with a zero mean and $\sigma$ standard deviation [14], a threshold $\overline{\epsilon} > 0$ can be chosen such that the absolute value of the distance measurement error is smaller than the threshold with a certain probability. For example, if $\overline{\epsilon} = 3\sigma$, then the probability of the absolute value of the distance measurement error is less than $\overline{\epsilon}$ is 99%. For a given $\overline{\epsilon}$, the relationship between the true distances $|AD|$, $|BD|$ and $|CD|$ and the measured distances is:

$$|jD| \quad \in \quad [\overline{d}_{jD} - \overline{\epsilon}, \overline{d}_{jD} + \overline{\epsilon}] \; \forall j \in \{A, B, C\} \qquad (1)$$

Without loss of generality, let the coordinates of $A$ and $B$ lie on the x-axis with the midpoint of $AB$ at the origin. Then $(-\frac{|AB|}{2}, 0)$ and $(\frac{|AB|}{2}, 0)$ are the coordinates of $A$ and $B$, respectively. Let $(x_C, y_C)$ be the coordinate of sensor $C$. Without considering the distance measurement errors in $\overline{d}_{AD}$ and $\overline{d}_{BD}$, sensor $D$ could be localized at either of the two possible points $\hat{D}$ and $\hat{D}'$ as shown in Fig. 4, which are symmetric w.r.t $AB$. The third distance measurement $\overline{d}_{CD}$ is used to disambiguate the two possible locations of $D$.

In the presence of distance measurement errors, the location of $D$, could be located inside two possible regions $S_1$ and $S_2$ with a certain probability defined by $\overline{\epsilon}$ as shown in Fig. 4. These two regions $S_1$ and $S_2$ containing the two points $\hat{D}$ and $\hat{D}'$, are also symmetric with respect to $AB$ and are bounded by the following equations:

$$\begin{aligned} (\overline{d}_{AD} - \overline{\epsilon})^2 \le (x + \frac{|AB|}{2})^2 + y^2 \le (\overline{d}_{AD} + \overline{\epsilon})^2 \\ (\overline{d}_{BD} - \overline{\epsilon})^2 \le (x - \frac{|AB|}{2})^2 + y^2 \le (\overline{d}_{BD} + \overline{\epsilon})^2 \end{aligned} \qquad (2)$$

Initially we only consider the situation where the two regions $S_1$ and $S_2$ are disjoint. The situations with joint regions are discussed separately later in this section.

The third erroneous distance measurement $\overline{d}_{CD}$ is used to disambiguate the possible locations of $D$. If the difference of the estimated distances between sensor $C$ and the possible locations of $D$, $\Big||C\hat{D}| - |C\hat{D}'|\Big|$ is small, the likelihood of flip occurring is large and vice-versa. In fact, if $\frac{1}{2}||C\hat{D}| - |C\hat{D}'|| > \overline{\epsilon}$, the likelihood of an incorrect flip is small. Hence an ordered FCQ $ABCD$ is considered $\overline{\epsilon}$-robust if,

$$\min_{\substack{\hat{D} \in S_1 \\ \hat{D}' \text{symmetric to } \hat{D} \text{ w.r.t. AB}}} \quad e^2 \triangleq (|C\hat{D}| - |C\hat{D}'|)^2 \; > \; 4\overline{\epsilon}^2 \qquad (3)$$

[3]Avalanche error propagation is a phenomenon of estimation error multiplication that can allow very large estimation errors to propagate in the network staring off with a flip ambiguity.

[4]Abusing the notation, we denote such quadruples of sensors with FCQ as well.

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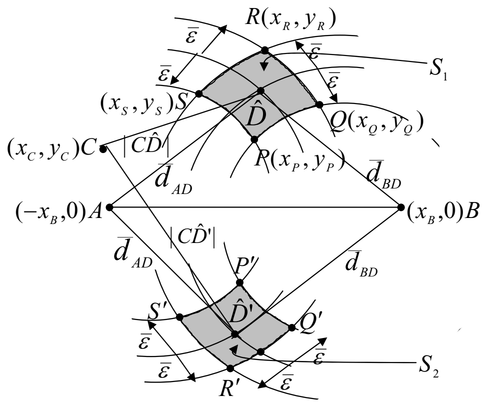



Fig. 4. Possible locations of sensor $D$ using the accurate position of sensors $A$ and $B$ and inter sensor distance measurements $\overline{d}_{AD}$ and $\overline{d}_{BD}$.

Note that (3) simultaneously considers three measurement errors in $\overline{d}_{AD}$, $\overline{d}_{BD}$ and $\overline{d}_{CD}$. The error in $\overline{d}_{CD}$ is considered explicitly in the inequality while the errors in $\overline{d}_{AD}$ and $\overline{d}_{BD}$ are considered through minimizing (3) over the region $S_1$ (and $S_2$). From Fig. 4 and Eq. 2, we can see that when $(x_S, y_S)$, $(x_Q, y_Q)$, $(x_R, y_R)$, $(x_P, y_P)$ are the coordinates of the intersection points $S$, $Q$, $R$ and $P$ respectively, the range of the coordinates $(x, y)$ and $(x, -y)$ of $\hat{D}$ and $\hat{D}'$ respectively satisfying (2) is bounded as follows.

$$x_S \le x \le x_Q \;\text{ and }\; y_P \le y \le y_R, \tag{4}$$

Substituting the coordinates of $\hat{D}, \hat{D}'$ and $C$ in (3) will give[5]

$$e^2 = 2\left((x_C-x)^2+y_C^2+y^2-\sqrt{((x_C-x)^2+y_C^2+y^2)^2-4y_C^2y^2}\right)$$

Taking partial derivative of $e^2$ with respect to $x$ is

$$\frac{\partial e^2}{\partial x} = 4(x_C-x)\left(\frac{(x_C-x)^2+y_C^2+y^2}{\sqrt{((x_C-x)^2+y_C^2+y^2)^2-4y_C^2y^2}}-1\right)$$

since

$$((x_C-x)^2+y_C^2+y^2)^2-4y_C^2y^2 > 0$$

and

$$\sqrt{((x_C-x)^2+y_C^2+y^2)^2-4y_C^2y^2} < (x_C-x)^2+y_C^2+y^2$$

it is easily seen that

$$0 \;<\; \left(\frac{(x_C-x)^2+y_C^2+y^2}{\sqrt{((x_C-x)^2+y_C^2+y^2)^2-4y_C^2y^2}}-1\right)$$

Therefore, $e^2$ has only one extremum and it occurs at $x = x_C$. It is also seen that when $x < x_C$, $\frac{\partial e^2}{\partial x} > 0$ and when $x > x_C$, $\frac{\partial e^2}{\partial x} < 0$, hence the extremum is a maximum. That is, for a fixed value of $y$, the minimum of $e^2$ will occur when $\hat{D}$ is on the boundaries (arcs $SP$, $PQ$, $QR$ and $RS$ in Fig.4) of $S_1$ where $|x_C - x|$ is at its maximum.

By rearranging $e^2$ as

$$e^2 = \frac{8y_C^2}{\frac{((x_C-x)^2+y_C^2)}{y^2}+1+\sqrt{\frac{((x_C-x)^2+y_C^2)^2}{y^4}+\frac{2((x_C-x)^2-y_C^2)}{y^2}+1}} \tag{5}$$

[5]The detail derivation regarding the analysis in this section can be obtained from the authors in preprint form.

it can be shown that

$$\lim_{y\to+\infty} e^2 = 4y_C^2;\;\; \lim_{y\to 0} e^2 = 0 \text{ for a fixed value of } x.$$

Given that $y_P \le y \le y_R$, irrespective of location $C$, the minimum of $e^2$ will occur when $\hat{D}$ is on the lower boundary (arcs $SP$ and $PQ$ in Fig.4) of $S_1$ for a fixed value of $x$.

Depending on the location of sensor $C$, minimum of $e^2$ will occur either when $\hat{D}$ is on the arc $SP$ or on the arc $QP$ in Fig.4. Without loss of generality, let's assume that minimum of $e^2$ occur when $\hat{D}$ is on the segment $SP$. Since arc $SP$ is a part of the circle centered at $(-\frac{|AB|}{2}, 0)$ with a radius $(\overline{d}_{AD} - \overline{\epsilon})$, substituting $y^2 = (\overline{d}_{AD} - \epsilon)^2 - (x + \frac{|AB|}{2})^2$ into $e^2$ and taking the second derivative w.r.t $x$ shows that $e^2$ is a concave function of $x$. The property of the concave function readily leads to the conclusion that when $\hat{D}$ is on the segment $SP$, the minimum of $e^2$ only occurs at the boundaries, i.e. when $\hat{D}$ is either at point $S$ or at point $P$. Thus, we have established that the minimum of $e^2$ for $\hat{D} \in S_1$ is only achieved when $\hat{D}$ is at one of the three boundary points $S$, $Q$ or $P$ of $S_1$ and the $\overline{\epsilon}$-robustness criterion could be re-defined as

$$\min_{\substack{\hat{D}\in\{S,Q,P\} \\ \hat{D}'\text{symmetric to } \hat{D} \text{ w.r.t. AB}}} e^2 \triangleq (|C\hat{D}| - |C\hat{D}'|)^2 \;>\; 4\overline{\epsilon}^2 \tag{6}$$

In the above analysis, we are considering that the point $D$ has access to distance measurements from $A$ and $B$, then analyzing the minimum of $e^2 = (|C\hat{D}| - |C\hat{D}'|)^2$ for $\hat{D} \in S_1$. If, instead, we consider that point $D$ has access to distance measurements from $A$ and $C$, then find the minimum of $e^2 = (|B\hat{D}| - |B\hat{D}'|)^2$, the confined regions $S_1$ and $S_2$ will be determined by the measured distances $\overline{d}_{AD}$, $\overline{d}_{CD}$ and the error term $\overline{\epsilon}$, and $\hat{D}'$ will be symmetric to $\hat{D}$ with respect to $AC$. This minimum is different from (6). Hence we should find the minimum $e^2$ for all three permutations of the sequence $A$, $B$, $C$ while analyzing the likelihood of flip.

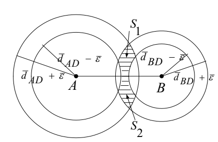



Fig. 5. Illustration of the two regions $S_1$ and $S_2$ joint together.

Above analysis only considered the situation that the two regions $S_1$ and $S_2$ are disjoint. If instead they are joint as shown in Fig. 5, the minimum of $e^2$ will always be zero when $\hat{D}$ and $\hat{D}'$ both coincide at the same point on the common boundary of $S_1$ and $S_2$. In such situations, the FCQs will always be detected as non-robust. This scenario could happen more frequently when the noise in the distance measurement or the chosen error bound $\overline{\epsilon}$ is larger, and may cause increase in false alarms.

## IV. SUBSTANTIAL FLIP AMBIGUITY AND NUMERICAL ANALYSIS OF THE ROBUSTNESS CRITERION

If the localization error is at most a predefined accuracy level $\delta_S$, then any flip ambiguity satisfying (6) with $\left|\hat{D}\hat{D}'\right| <$

$\delta_S$ is negligible in terms of the localization task. Hence a location estimate can be identified as having substantial flip ambiguity when 1) the estimated location has changed sides w.r.t the line defined by any two neighbors $A, B \in N_i$ and the true location of the unknown node $i$ and 2) the distance from the estimated location to the line joining the nodes $A$ and $B$ is larger than $\frac{\delta_s}{2}$.

We test our robustness criterion in (6) via various simulations. In these simulations 10000 FCQs are used, which are selected from a pool of 4-nodes sensor networks composed of nodes that are uniformly distributed in a region of $100m \times 100m$ with a transmission range of $10m$. If the true distance $|ji|$ between any two nodes $j$ and $i$ is smaller than the transmission range $R$, then the nodes are set to be neighbors. The measured distance between these neighbor nodes is blurred by a Gaussian noise [14] as shown below:

$$\overline{d}_{ji} \quad = \quad \overline{d}_{ij} = |ji| + \mathcal{N}(0, \sigma^2) \tag{7}$$

In our simulations, Gaussian noise is truncated such that $0 \leq \overline{d}_{ji}$ and has a mean of zero and variance $\sigma^2$ where $\sigma$ varied from $0.05m$ to $0.5m$.

Once the sensors are placed and the distance measurements are obtained, localization is done by minimizing the cost function:

$$(\hat{x}_i, \hat{y}_i) \quad = \quad \arg\min_{(x,y)} \sum_{j \in N_i} (\overline{d}_{ji} - d_{ji})^2 \tag{8}$$

where $(\hat{x}_i, \hat{y}_i)$ is the estimated location of node $i$; $\overline{d}_{ji}$ and $d_{ji} = \sqrt{(x - x_j)^2 + (y - y_j)^2}$ are the measured and estimated distances between node $i$ and its single hop or multi hop neighbor $j$ respectively; If $M_i$ is the set of all neighbors $j$ of node $i$ whose inter-sensor measurements $\overline{d}_{ji}$ are known, then $N_i (\subseteq M_i)$ is a set whose locations are also known.

The estimated location is checked to determine whether a flip has occurred. Using this procedure, the robustness criterion (6) is compared with another criterion proposed in [3]. Fig. 6(a) and Fig. 6(b) show that the rate of correct flip detection is 100% and the missed flip detection is 0% when the value $\overline{\epsilon} > 2\sigma$. Fig. 6(c) and Fig. 6(d) show that if the error bound is increased, the false alarm rate is high. This is due to more frequent joining of regions $S_1$ and $S_2$ with larger error bounds.

We also studied the effect of different noise levels ($\sigma$) on the correct detection of the location estimates for the fixed threshold value ($\overline{\epsilon} = 3\sigma$). The results are shown in Fig. 7. From Fig. 6 and 7, it can be seen that our criteria makes the correct decision more accurately than the criteria in [3]. The drawback of both criteria is that under conditions of high measurement noise, the algorithm may be unable to localize a significant number of nodes.

## V. Flip Ambiguity Frequencies and Effects on Estimation Errors

In order to do an empirical study on the frequency of flip ambiguity occurrences and their corresponding estimation errors, fully connected random neighborhoods $N_i$ are placed

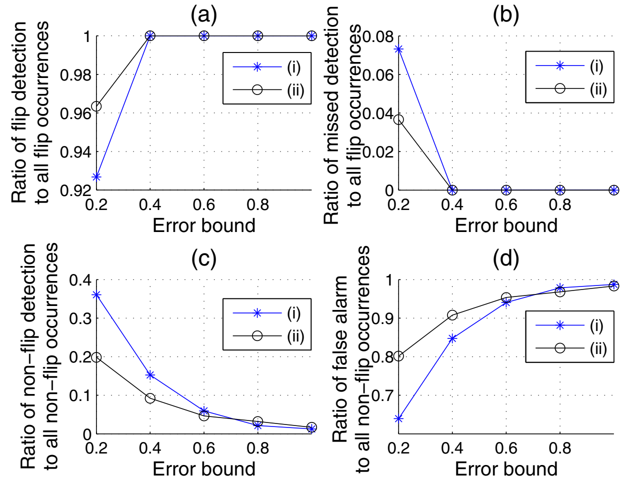



Fig. 6. (a) Ratio of correct flip detection to all the flip occurrences, (b) Ratio of missed flip detection to all the flip occurrences, (c) Ratio of correct detection when there is no flip to all the occurrences of estimation without flip and (d) Ratio of false alarm as flip when there is no flip to all the occurrences of estimation without flip. These results are obtained for different error bounds $\overline{\epsilon}$ for a given $\sigma = 0.2$: (i) test using criterion (6), (ii) test using criterion in [3]

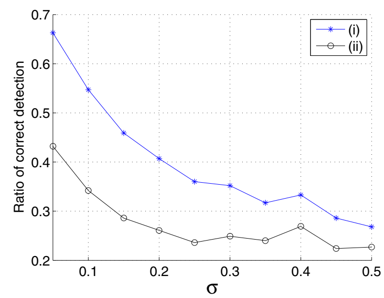



Fig. 7. Ratio of correct detection against $\sigma$ for a given $\epsilon = 2\sigma$: (i) Test using criterion (3), (ii) Test using criterion in [3]

as before, where the number of neighbors $|N_i|$ in the neighborhood is varied between $3 - 9$. The standard deviation in (7) is chosen as $0.2m$ and the location of the unknown node is estimated using (8).

Fig. 8 shows the percentage of the neighborhoods affected by flip ambiguities, demonstrating that when $|N_i|$ increases, the percentage of flipped realizations reduces. However, at the initial iterations of any localization algorithm, many neighbors $j \in M_i$ do not know their locations and localization is done with smaller $|N_i|$, causing more frequent occurrence of flip ambiguities. When the localization algorithm proceeds, it is more likely for $|N_i|$ to increase, reducing the frequency of flip ambiguities. However, if $N_i$ contains any flipped realized nodes, there can still be avalanche effect as described in section II.

We have also studied the estimation error pattern by defining the average root mean squared error (RMSE) of the location estimate as

$$RMSE = \frac{(\sum_{i=1}^{N_t} \sqrt{(\hat{x}_i - x_i)^2 + (\hat{y}_i - y_i)^2})}{N_t}$$

where $N_t$ is 10000. The RMSE created by all realizations

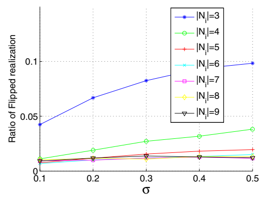
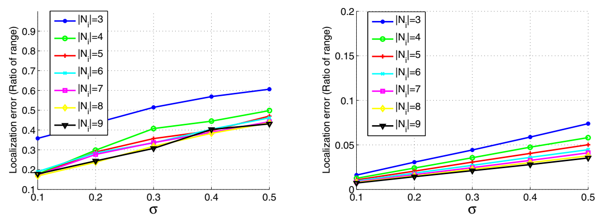



Fig. 8. Ratios of flipped realizations for different number of neighbors as a function of $\sigma$.

are shown in Fig. 9 as a ratio to transmission range. From this figure, it can be seen that the error when there is flip ambiguity is larger than when there is no flip ambiguity, and has a minimum value of 20% of the transmission range $R$.




(a) Average location estimation error when there is flip ambiguity

(b) Average location estimation error when there is no flip ambiguity

Fig. 9. Errors created by localization when there is flip and when there is no flip for different number of neighbors as a function of $\sigma$.

## VI. Performance Enhancement of Localization Algorithms Using the Robustness Criterion

In this section we use our robustness criterion (6) to enhance the performance of the localization algorithms in section II. To keep the system simple, we take one-hop neighbors as the only valid candidates to be in $N_i$. But the developed criterion could be used with any neighbor node, either one-hop or multi-hop, as long as the distance measurement and the corresponding node location are known.

The localization algorithms in section II either take any three members of $N_i$ and do trilateration or take all members of $N_i$ and do multilateration. Even though we used FCQs to develop our robustness criterion, we selected the robust neighborhood $RN_i$ in a particular way as follows: If an unknown node $i$ has $|N_i|$ neighbors, we obtain sets $\mathcal{C}_3^{|N_i|}$ of all possible FCQs with nodes $A, B, C \in N_i$ and $i$, and find the most robust FCQ to form the $RN_i$. Then by taking every neighbor $j \in N_i - RN_i$ (where $N_i - RN_i = \{j \in N_i; j \notin RN_i\}$), we check whether $j$ forms a robust FCQ with all combinations of nodes in $RN_i$. If it does, we enlarge $RN_i$ as $RN_i = RN_i \cup j$.

In order to evaluate the performance enhancement in the above algorithms using the proposed robustness criterion, 500 different simulated sensor networks with 100 randomly distributed nodes are constructed each with different random seed. Sensor nodes in each of the 500 sensor networks, nodes are uniformly distributed in a region of $100m \times 100m$. The first 10 sensor nodes were chosen as anchor nodes and are initialized with random coordinates within the boundary. The standard deviation in (7) is chosen as $0.2m$ and the location of the unknown nodes are estimated using (8). The number of nodes are kept fixed and the transmission range is adjusted in the simulations such that the average node degree varies between $4 - 25$.

The average mean squared error in location estimates is calculated and normalized to the transmission range $R$ as:

$$MSE = \frac{1}{|\{n|V_n \neq \emptyset\}|} \sum_{\substack{n=1 \\ |V_n| \neq 0}}^{500} \frac{1}{|V_n|} \frac{\sum_{i \in V_n} (x_i - \hat{x}_i)^2 + (y_i - \hat{y}_i)^2}{R^2}$$

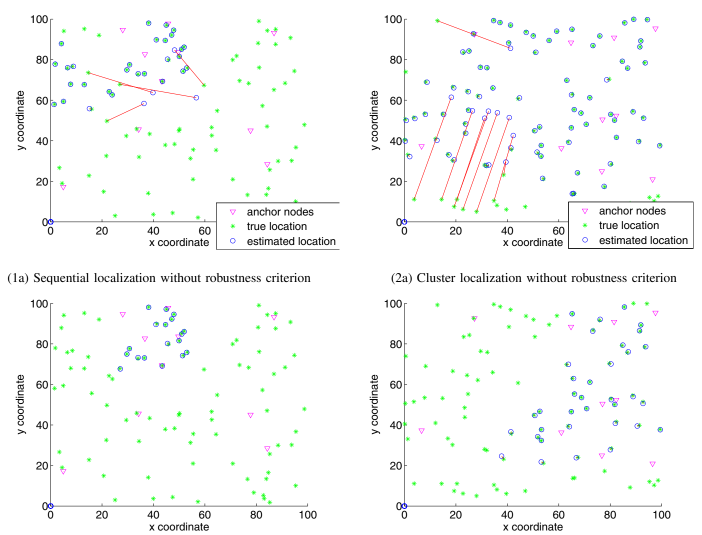
where $(\hat{x}_i, \hat{y}_i)$ and $(x_i, y_i)$ are the estimated and true location of sensor node $i$, $V_n$ is the set of nodes localized in the $n^{th}$ sensor network, and $|V_n|$ is the number of nodes in $V_n$.

To compare different scenarios, neighbors to be used in the location estimate are chosen in six different methods from the neighborhood $N_i$:

i Any 3 neighbors of $N_i$.
ii All neighbors of $N_i$.
iii Most robust 3 neighbors of $N_i$ by criterion (6).
iv All robust neighbors of $N_i$ by criterion (6).
v Most robust 3 neighbors of $N_i$ by criterion in [3].
vi All robust neighbors of $N_i$ by criterion in [3].

Sequential localization algorithm is simulated with all six different selection methods in order to compare both trilateration and multilateration. Since multilateration produce better estimation than its counterpart trilateration, we repeat the simulations for cluster localization algorithm with only the methods $(ii)$, $(iv)$ and $(vi)$ above.

From Fig. 10, we see that flip ambiguity is being efficiently removed when robustness criterion (6) is applied. From section




(1a) Sequential localization without robustness criterion

(2a) Cluster localization without robustness criterion

(1b)Sequential localization with robustness criterion (6)

(2b) Cluster localization with robustness criterion (6)

Fig. 10. Examples of network localization having flip ambiguity and removal of such flip ambiguity by the robustness criterion. Here, the red line segments show the error offset between original and estimated location of sensor node.

IV, we have that the robustness criterion removes all flip ambiguities when $\bar{\epsilon} > 2\sigma$ and hence this efficient removal of flip ambiguity is expected. But we have also observed in that section that there is considerable amount of false alarm created by the robustness criterion which eleminate some accurate

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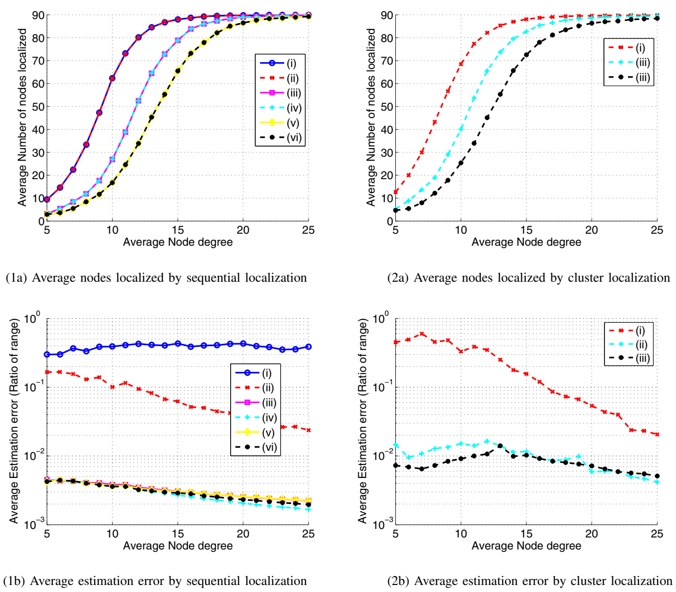
location estimates. In Fig. 11 it can be seen that average numbers of localized nodes with neighbor selection method (i) and (ii) are more than the average number of localized nodes with neighbor selection method (iii) and (iv). But it can also be seen that the average estimation error is ten times larger for neighbor selection method (i) and (ii) than the average estimation error of nodes localized using neighbor selection method (iii) and (iv). As we have explained in section V,




(1a) Average nodes localized by sequential localization

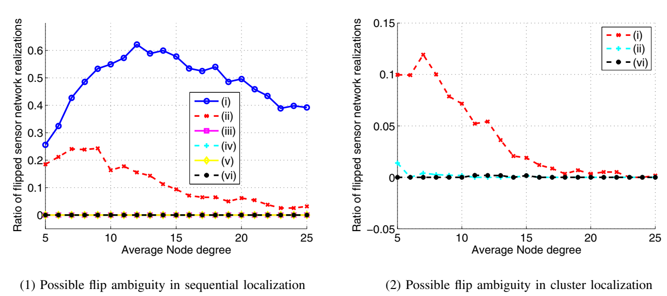
(2a) Average nodes localized by cluster localization

(1b) Average estimation error by sequential localization

(2b) Average estimation error by cluster localization

Fig. 11. Performance of different neighbor selection methods (i) to (vi).

the location estimation errors are generally low, but due to flipped realization and the following avalanche effects, there may be large variance in location estimation errors. Hence Fig. 11(1b) and (2b) imply that some of the localized nodes with neighbor selection method (i) and (ii) may have been realized in a flipped location. In order to simplify the flip detection, we treat any estimation error $> \frac{R}{5}$ as having substantial flip ambiguity. It can be seen from Fig. 12 that neighbor selection method (i) and (ii) cause more flipped realization in many sensor networks.




(1) Possible flip ambiguity in sequential localization

(2) Possible flip ambiguity in cluster localization

Fig. 12. Percentage of sensor networks with possible flip ambiguity while using different neighbor selection methods (i) to (vi).

Since the reliability of the location estimates is an absolute requirement of any localization algorithm, a robustness criterion for the neighborhood selection to remove those flip ambiguities is essential. For a robustness criterion to be effective, it needs to detect as much flip ambiguities as possible while making as little number of false alarm as possible. From these aspects, the above simulation results well-demonstrated the efficiency of the proposed robustness criterion in enhancing performance of localization algorithms.

## VII. Conclusion and Future Work

In this paper we have developed a robustness criterion which identifies the likelihood of flip ambiguity in fully connected quadrilaterals, quadruples of sensors all of which are neighbors of each other, via a formal optimality analysis. This robustness criterion is used in two different classes of localization algorithms to eliminate (in consideration in localization process) the nodes that contribute to flip ambiguities from any neighborhood. The simulation results demonstrate that the benefit of using the proposed robustness criterion is significant, and the criterion performs well in identifying flip ambiguity likelihood while creating very small number of false alarms.

As a future work, in order to improve the performance of the robustness criterion, the regions $S_1$ and $S_2$ in Fig.4 will be considered separately without assuming symmetry. Another future research direction is to calculate confidence factors of flip ambiguity rather than a binary decision and incorporate these confidence factors to improve localization performance.

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