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Distributed source localization of multi-agent systems with bearing angle measurements

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***Abstract*—This note deals with the distributed source localization problem by considering a group of unicycle-type agents. Without the need of GPS and compass, we develop a distributed source localization scheme based on bearing angle measurements about neighbors. It is shown that if the sensing and communication graph is connected and the relative motion of every pair of neighboring agents satisfies a persistent excitation condition, then every agent is able to estimate the relative coordinate of the source asymptotically.**

***Index Terms*—Multi-agent system, source localization, bearing information.**

I. INTRODUCTION

This note is concerned with a network of mobile robots that are sent out to perform source localization. This can be found in many applications such as search and rescue, surveillance, etc. That is, if the source is within the field-of-view of a robot, then that robot will get relative measurements and localize the source in its local frame. Otherwise, the robot may need assistance from its neighboring robots to compute the estimate of the source's location. However, due to the absence of a commonly agreed global coordinate system such as applications underwater or inside buildings where GPS is unavailable, a necessity for distributed collaborative source localization is that each agent needs also to determine the relative positions of its neighboring agents in its local frame of reference. Moreover, it is often desired that no central fusion center or leader agent exists to collect all relative measurements and combine them together to compute the estimate of the source. In this note, we are interested in developing a distributed source localization algorithm for a network of agents, which collaborate by exchanging appropriate messages between neighboring agents, such that every agent is able to determine the relative position of the source in its local frame.

Related problems have been considered in the past, including acoustic source localization [1], [2], single landmark based localization [3], collaborative target tracking [4], [5], cooperative localization and formation [6], [7], circumnavigation [8], [9], etc. Depending on the type of relative measurements, [1] and [2] consider TDOA measurements for a static sensor network to identify the acoustic source location, while [10] and [11] consider distance measurements for a mobile robot or a network of mobile robots to localize the target of interest. As an alternative, bearing information, referring to the measurement of the angle of arrival with respect to the local frame on an agent, can also be used in source localization [8], [12]–[14]. Bearing measurement techniques are passive methods, which are particularly preferred in the scenarios where the agents must maintain radio silence.

In this note, we aim to solve the distributed source localization problem based on bearing measurements. We consider the nonholonomic unicycle model, which has been employed in many studies of distributed robotic systems to model a differentially driven mobile robot [15] and aerial vehicle [16]. Differing from the point-mass model assumed in the aforementioned works, the local frame on each unicycle-type agent rotates according to its kinematic motion, which causes extra challenges in relative localization, in particular in the scenarios where no GPS and no compass are available to provide each agent its own absolute position and orientation. However, we do succeed in developing a simple and provably convergent distributed source localization algorithm. It is shown that if the graph describing the communication and sensing relationship among the agents and the source is connected and if the relative motion of any pair of neighboring agents satisfies a persistent excitation condition, then the estimate by every agent can asymptotically converge to the true relative coordinate of the source in its local frame. The novelty of our work is in the development of a distributed estimation scheme that takes into consideration of the more complicated nonholonomic-constraint motion and does not require a commonly known coordinate system, which has not been addressed in [8], [12], [13]. Compared to our earlier work [14], this note develops a new estimator that does not require to know the changing rate of the bearing angle, which is not easy to be obtained precisely. Moreover, based on this new estimator, the agents are able to localize the source in the case when the agents perform uniform linear motions, for which the localization scheme in [14] does not work.

II. PROBLEMS AND PRELIMINARIES

This section presents a graph model for a network of agents and then formulates the distributed source localization problem.

A. Graph Modeling

We consider a setup with a stationary source labeled as 0, and N mobile agents labeled as $1, 2, \ldots, N$.

The work was supported by Zhejiang Provincial Natural Science Foundation of China LR13F030002, the Distinguished Visiting Scholars Scheme of The University of Technology Sydney, and the Open Research Project (No. ICT1403) of the State Key Laboratory of Industrial Control Technology, Zhejiang University.

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This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TAC.2015.2457112, IEEE Transactions on Automatic Control

First, we use an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ to describe the interconnection structure of the N agents. The node set $\mathcal{V} = \{1, 2, \ldots, N\}$ is defined such that each node i corresponds to an agent, and the edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is defined such that $(i, j) \in \mathcal{E}$ if and only if agents i and j are neighbors in the sense that they can mutually measure the bearing angle of the other and can communicate to each other. We denote by $\mathcal{N}_i$ the neighbor set of node i in $\mathcal{G}$. Moreover, we let $A = [a_{ij}] \in \mathbb{R}^{N \times N}$ be the adjacency matrix associated with $\mathcal{G}$, for which $a_{ij} = a_{ji} = 1$ if (i, j) is an edge in $\mathcal{G}$ and $a_{ij} = 0$ otherwise. A diagonal matrix $D = \operatorname{diag}\{d_1, d_2, \ldots, d_N\} \in \mathbb{R}^{N \times N}$ is called the degree matrix of $\mathcal{G}$, whose diagonal elements $d_i = \sum_{j \in \mathcal{N}_i} a_{ij}$ for $i = 1, \ldots, N$. The Laplacian of the graph $\mathcal{G}$ is defined as $L = D - A$.

Second, we add the source as a new node to the graph $\mathcal{G}$ and construct an augmented graph, denoted by $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$. The node set is $\bar{\mathcal{V}} = \{0\} \cup \mathcal{V}$ and the edge set is $\bar{\mathcal{E}} = \mathcal{E}_o \cup \mathcal{E}$ where an edge $(i, 0)$ is in $\mathcal{E}_o$ if and only if the source is a neighbor of agent i in the sense that agent i can measure the bearing angle of the source. Though no bidirectional information exchange exists between an agent and the source, we still use an undirected edge connecting node i and 0 for simplicity. The graph $\bar{\mathcal{G}}$ is connected if there exist at least a path between each pair of nodes. We then denote by $\bar{\mathcal{N}}_i$ the set of agent i's neighbors in $\bar{\mathcal{G}}$. Furthermore, we define a diagonal matrix $B \in \mathbb{R}^{N \times N}$ to be the source adjacency matrix associated with $\bar{\mathcal{G}}$ with diagonal elements b_i, $i = 1, \ldots, N$, where $b_i = 1$ if node $0 \in \bar{\mathcal{N}}_i$ and $b_i = 0$ otherwise. Next we recall a preliminary result about the matrix $H = L + B$.

Lemma 2.1 ([17]): The matrix $H = L + B$ is positive definite if and only if $\bar{\mathcal{G}}$ is connected.

We give a simple example consisting of four agents and one source. The graphs $\bar{\mathcal{G}}$ and $\mathcal{G}$ associated with the system are shown in Fig. 1.For this example, $\bar{\mathcal{G}}$ is connected. It can be checked that the matrix H in Lemma 2.1 is

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & -1 & -1 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

and is positive definite.

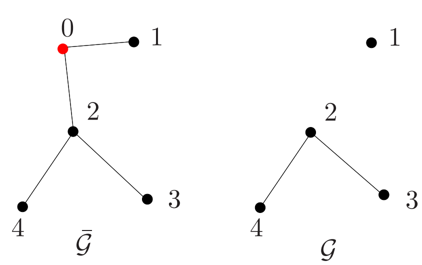

Fig. 1. An example of $\bar{\mathcal{G}}$ and $\mathcal{G}$.

B. Problem Statement

This note addresses the *distributed source localization* problem, for which the goal is to let each agent asymptotically estimate the relative coordinate of the stationary source in its own local frame.

In this note, we use $p_i^g = [x_i^g \quad y_i^g]^\top$ to represent the coordinate of agent i in the global frame Σ_g. Assume each agent is governed by a unicycle model, where θ_i is its orientation within Σ_g. With these three states x_i^g, y_i^g and θ_i, the motion equation of agent i in the global frame Σ_g is

$$\begin{cases} \dot{x}_i^g = v_i \cos(\theta_i) \\ \dot{y}_i^g = v_i \sin(\theta_i) \\ \dot{\theta}_i = \omega_i \end{cases} \tag{1}$$

where v_i and ω_i are the linear and angular speed, respectively.

We consider a local frame Σ_i attached to agent i, with its positive x-axis coincident with the heading of agent i. We suppose that the bearing angle α_{ij} can be measured by agent i if j is its neighbor. Note that α_{ij} is defined within the local frame Σ_i as no global frame is known by the agents (see Fig. 2 for an example). Moreover, we suppose that the linear speed v_i and the angular speed ω_i are available to agent i by its equipped speedometer. The combination of linear speed v_i, angular speed ω_i and bearing angle α_{ij} is called the *measurement data* by agent i. In addition, if $j \in \mathcal{N}_i$ (i.e. neighbor j is a mobile agent rather than the source), then the measurement data by agent j can be communicated to agent i, which is called the *communication data* available by agent i.

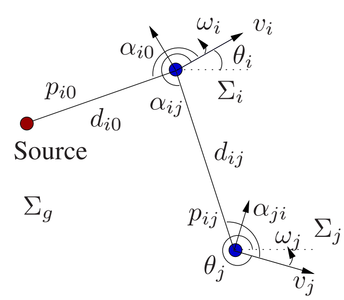

Fig. 2. Local frame and bearing angle measurement.

Let p_{i0} be the relative coordinate of the source in agent i's local frame Σ_i, that is,$p_{i0} = R(-\theta_i)(p_0^g - p_i^g)$, where p_0^g is the absolute coordinate of the source in the global frame Σ_g and $R(\cdot) = \begin{bmatrix} \cos(\cdot) & -\sin(\cdot) \\ \sin(\cdot) & \cos(\cdot) \end{bmatrix}$ is the rotation matrix. The distributed source localization problem is stated as follows.

Problem 2.1: Design a distributed source localization algorithm such that each agent only utilizes the *measurement data* and *communication data* available to it, yet it is able to asymptotically estimate the relative coordinate of the source (i.e., the estimate converges to p_{i0} as $t \to \infty$).

To make the distributed source localization problem solvable and make the potential solutions practically meaningful, we make the following assumptions.

Assumption 2.1: The linear speed $v_i(t)$ and the angular speed $\omega_i(t)$ of each agent $i = 1, \ldots, N$ are continuously differentiable and bounded.

Assumption 2.2: The agents do not collide with each other, i.e., the distance d_{ij} between any pair of agents is greater than the safe distance d_{safe} for any i and j.

Assumption 2.3: All the agents do not communicate and exchange measurement data with the source.

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III. DISTRIBUTED SOURCE LOCALIZATION

This section is going to present a solution to the distributed source localization problem.

A. Relative Motion Dynamics

As we assume in this note, there is no global coordinate system available and all measurements and estimation are made on local frames attached to the mobile agents. So in this subsection, we will derive the relative motion dynamics of each agent relative to its neighbor.

Consider an agent i and a neighbor of agent i, say agent j. Then $p_{ij}^g = p_j^g - p_i^g$ is the relative coordinate of agent j and agent i in the global frame Σ_g. Within the local frame Σ_i attached to agent i, the relative coordinate of agent j and agent i can then be written as $p_{ij} = R(-\theta_i)p_{ij}^g$, where $R(\cdot)$ is the rotation matrix in the plane. An illustration is shown in Fig. 3.

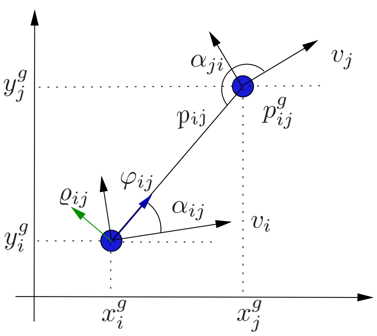

Fig. 3. Relative motion.

Denote the relative orientation angle θ_{ij} between the local frames Σ_i and Σ_j as $\theta_{ij} = \theta_j - \theta_i$. In terms of the relative coordinate p_{ij} in the local frame Σ_i, the relative motion dynamics can be obtained from (1) as follows.

$$\begin{aligned} \dot{p}_{ij} &= \dot{R}(-\theta_i)p_{ij}^g + R(-\theta_i)\dot{p}_{ij}^g \\ &= \omega_i \frac{dR(-\theta_i)}{d\theta_i} R(\theta_i)p_{ij} + R(-\theta_i)\dot{p}_{ij}^g \\ &= \begin{bmatrix} 0 & \omega_i \\ -\omega_i & 0 \end{bmatrix} p_{ij} + \begin{bmatrix} v_j\cos(\theta_{ij}) - v_i \\ v_j \sin(\theta_{ij}) \end{bmatrix}. \end{aligned}$$

Writing the relative motion dynamics in the *polar coordinate system* in terms of the distance d_{ij} between agent j and i and the bearing angle α_{ij}, we have

$$\begin{cases} \dot{d}_{ij} = -v_i\cos(\alpha_{ij}) - v_j\cos(\alpha_{ji}) \\ \dot{\alpha}_{ij} = \frac{v_i\sin(\alpha_{ij}) + v_j\sin(\alpha_{ji})}{d_{ij}} - \omega_i. \end{cases} \tag{2}$$

Define the unit vector along the line of sight as $\varphi_{ij} := [\cos(\alpha_{ij}) \quad \sin(\alpha_{ij})]^\intercal$, and define the unit vector rotated counterclockwise $\pi/2$ from φ_{ij} (see Fig. 3.) as $\varrho_{ij} := [-\sin(\alpha_{ij}) \quad \cos(\alpha_{ij})]^\intercal$. Then it is clear that $0 = \varrho_{ij}^\intercal p_{ij}$.

For neat expression, we denote $A_i = \begin{bmatrix} 0 & \omega_i \\ -\omega_i & 0 \end{bmatrix}$ and denote $v_{ij} = [v_j\cos(\theta_{ij}) - v_i \quad v_j\sin(\theta_{ij})]^\intercal$, that is the relative velocity of agent i with respect to agent j. Treating $0 = \varrho_{ij}^\intercal p_{ij}$ as the measurement equation, then we have

$$\begin{cases} \dot{p}_{ij} = A_i p_{ij} + v_{ij} \\ 0 = \varrho_{ij}^\intercal p_{ij}. \end{cases} \tag{3}$$

In particular, when neighbor j is the source ($v_j = 0$), then the system (3) becomes

$$\begin{cases} \dot{p}_{i0} = A_i p_{i0} + v_{i0} \\ 0 = \varrho_{i0}^\intercal p_{i0} \end{cases} \tag{4}$$

where $v_{i0} = [-v_i \ 0]^\intercal$.

Remark 3.1: For the relative orientation angle θ_{ij}, it can be calculated in terms of the bearing angle measurements, i.e., $\theta_{ij} = \mathrm{PV}(\pi + \alpha_{ij} - \alpha_{ji})$, where $\mathrm{PV}(x) \triangleq [(x+\pi) \bmod 2\pi] - \pi$ [18]. Therefore, the relative velocity v_{ij} in (3) for $j \in \mathcal{N}_i$ can be known by agent i based on its measurement data and communication data from agent j. But v_{i0} in (4) can be known by agent i based only on its measurement data and so agent i does not need to communicate with the source. Moreover, it is worth to point out that A_i in (3) and (4) relies only on the angular speed ω_i, while ϱ_{ij} relies only on the bearing angle α_{ij}. In brief, all the system parameters are available locally by agent i such that it is possible to make an estimation in a distributed way.

B. Distributed Source Localization Algorithm

In this subsection, we are going to develop a distributed source localization algorithm such that every agent i asymptotically estimates the relative coordinate of the source in its own local frame no matter whether it is able to detect the source or not.

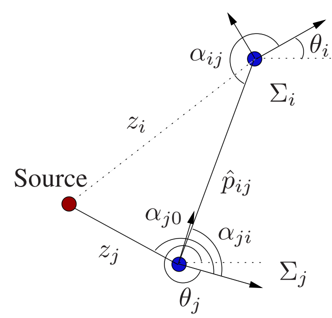

Fig. 4. The setup of distributed source localization.

As illustrated in Fig. 4, we denote by z_i, $i = 1, \ldots, N$, the estimate of the relative coordinate of the source by agent i in its local frame Σ_i. For notation consistency, we also let $z_0 = 0$. Moreover, we denote by $\hat{p}_{ij}$, $i = 1, \ldots, N$ and $j \in \bar{\mathcal{N}}_i$, the *pairwise estimate* of the relative coordinate of neighbor agent j by agent i in its local frame Σ_i. Here, we want to clarify that if the source is a neighbor of agent i, the pairwise estimate $\hat{p}_{i0}$ is different from the estimate z_i as the latter may be updated using more pairwise estimate information from other neighbors.

To solve the source localization problem, we propose the following *distributed source localization algorithm*

$$\dot{\hat{p}}_{ij} = \underbrace{A_i\hat{p}_{ij} + v_{ij}}_{\text{Prediction}} - \underbrace{\varrho_{ij}\varrho_{ij}^\intercal\hat{p}_{ij}}_{\text{Innovation}}, \quad \forall j \in \bar{\mathcal{N}}_i \tag{5a}$$

$$\dot{z}_i = \underbrace{A_i z_i + v_{i0}}_{\text{Prediction}} + \underbrace{\sum_{j\in\bar{\mathcal{N}}_i} [\hat{p}_{ij} - (z_i - R(\theta_{ij})z_j)]}_{\text{Innovation}} \tag{5b}$$

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where $i = 1, \ldots, N$.

Remark 3.2: Eq. (5a) gives a pairwise estimation scheme to estimate the relative coordinate of a neighbor and Eq. (5b) provides an estimation scheme to estimate the relative coordinate of the source by using all the pairwise estimates of its neighbors. Both of them use the prediction + innovation structure. In Eq. (5a), the first part is the prediction term according to the dynamical model of the relative motion between each pair of agents. The second part $\varrho_{ij}\varrho_{ij}^{\mathsf{T}}\hat{p}_{ij}$ is the orthogonal projection of $\hat{p}_{ij}$ onto the orthogonal compliment of the bearing. This part is zero only if the estimated relative bearing is consistent to the measured bearing. The similar idea was also appeared in [8] for point mass agents. In Eq. (5b), the first part $A_i z_i + v_{i0}$ is the prediction term according to the relative motion dynamics with respect to the source and the second part is the innovation term that compares with the pairwise estimates $\hat{p}_{ij}$ of agent i's neighbors obtained from (5a) and the relative coordinates $z_i - R(\theta_{ij})z_j$ of agent i's neighbors derived from the estimates about the source by agent i in agent i's local frame as well as by agent i's neighbors in their own local frames. The presence of the rotation matrix $R(\theta_{ij})$ is to make the coordinates all in agent i's local frame Σ_i.

Note that in the distributed source localization algorithm (5), the required information includes the measurement data (agent i's angular speed ω_i and linear speed v_i, and the bearing angle measurement α_{ij} measured by agent i) and the communication data from agent i's neighbors (the linear speed v_j, the bearing angle measurement α_{ji} measured by agent j, and agent j's estimate z_j for $j \in \bar{\mathcal{N}}_i$). Note that the agents do not need to communicate with the source.

Next, we present our main result on the convergence of the distributed source localization algorithm.

Theorem 3.1: Suppose that $\bar{\mathcal{G}}$ is connected. If there exist $\epsilon > 0$ and $T > 0$ such that for all $t > 0$, $i \in \mathcal{V}$ and $j \in \bar{\mathcal{N}}_i$,

$$\int_t^{t+T} \left| \frac{v_i \sin(\alpha_{ij}) + v_j \sin(\alpha_{ji})}{d_{ij}} \right| d\tau \geq \epsilon, \tag{6}$$

then the estimate $z_i(t)$ in (5) converges to the true relative coordinate $p_{i0}(t)$ of the source.

Proof: First, we show that if (6) holds, then the estimate $\hat{p}_{ij}$ in (5a) converges to the true relative coordinate p_{ij} of its neighbor. To this end, we define the estimation error $\tilde{p}_{ij} = \hat{p}_{ij} - p_{ij}$. Then the error dynamics is obtained from (3) and (5a) as

$$\dot{\tilde{p}}_{ij} = A_i \tilde{p}_{ij} - \varrho_{ij}\varrho_{ij}^{\mathsf{T}}\tilde{p}_{ij}. \tag{7}$$

Consider the positive definite function $V(\tilde{p}_{ij}) = \frac{1}{2}\tilde{p}_{ij}^{\mathsf{T}}\tilde{p}_{ij}$. Taking the derivative of V along the solution of system (7), we have

$$\begin{aligned} \dot{V}(\tilde{p}_{ij}) &= \tfrac{1}{2}\tilde{p}_{ij}^{\mathsf{T}}\left[-2\varrho_{ij}(t)\varrho_{ij}^{\mathsf{T}}(t) + A_i(t) + A_i^{\mathsf{T}}(t)\right]\tilde{p}_{ij} \\ &= \tfrac{1}{2}\tilde{p}_{ij}^{\mathsf{T}}\left[-2\varrho_{ij}(t)\varrho_{ij}^{\mathsf{T}}(t)\right]\tilde{p}_{ij} = -\left[\varrho_{ij}^{\mathsf{T}}(t)\tilde{p}_{ij}\right]^2, \end{aligned}$$

which is negative semi-definite. So we know that $V(\tilde{p}_{ij})$ and $\tilde{p}_{ij}$ are upper bounded and $V(\tilde{p}_{ij}(t))$ has a limit as $t \to \infty$. Next we check the boundedness of $\ddot{V}(\tilde{p}_{ij}) = -2[\varrho_{ij}^{\mathsf{T}}\tilde{p}_{ij}][\dot{\varrho}_{ij}^{\mathsf{T}}\tilde{p}_{ij} + \varrho_{ij}^{\mathsf{T}}\dot{\tilde{p}}_{ij}]$. Since $\tilde{p}_{ij}$ is upper bounded, considering the formula of $\dot{\tilde{p}}_{ij}$ in (7) and Assumption 2.1, we know that $\dot{\tilde{p}}_{ij}$ is upper bounded. Moreover, since $\dot{\varrho}_{ij} = -\dot{\alpha}_{ij}\varphi_{ij}$, by considering Assumption 2.1 and 2.2, we conclude from (2) that $\dot{\alpha}_{ij}$ is upper bounded and so is $\dot{\varrho}$. Hence, it follows from the formula of $\ddot{V}(\tilde{p}_{ij})$ that $\ddot{V}(\tilde{p}_{ij})$ is upper bounded, which implies that $\dot{V}(\tilde{p}_{ij})$ is uniformly continuous. Thus, we apply Barbalat's lemma and obtain that $\dot{V}(\tilde{p}_{ij}(t)) \to 0$ as $t \to \infty$. This is equivalent to

$$\lim_{t\to\infty} \varrho_{ij}^{\mathsf{T}}(t)\tilde{p}_{ij}(t) = 0. \tag{8}$$

Denote $\eta = \varrho_{ij}^{\mathsf{T}}\tilde{p}_{ij}$ and $\rho = \frac{v_i \sin(\alpha_{ij}) + v_j \sin(\alpha_{ji})}{d_{ij}}$. Taking the derivative of η and utilizing (2) and (7), it is obtained after several steps of mathematical manipulation that

$$\dot{\eta} = \dot{\varrho}_{ij}^{\mathsf{T}}\tilde{p}_{ij} + \varrho_{ij}^{\mathsf{T}}\dot{\tilde{p}}_{ij} = -\rho\varphi_{ij}^{\mathsf{T}}\tilde{p}_{ij} - \eta. \tag{9}$$

Similarly, we can know that $\ddot{\eta}$ is upper bounded under Assumption 2.1 and 2.2. So $\dot{\eta}$ is uniformly continuous. Then, applying Barbalat's lemma again, we obtain that $\dot{\eta} \to 0$ as $t \to \infty$, which implies from (9) that

$$\lim_{t\to\infty} \rho(t)\varphi_{ij}^{\mathsf{T}}(t)\tilde{p}_{ij}(t) = 0. \tag{10}$$

Let $P = \begin{bmatrix} -\sin(\alpha_{ij}) & \cos(\alpha_{ij}) \\ \rho\cos(\alpha_{ij}) & \rho\sin(\alpha_{ij}) \end{bmatrix}$ and let $\delta(t) = P(t)\tilde{p}_{ij}(t)$. Then eq. (8) and (10) can be rewritten in a compact form, i.e., $\lim_{t\to\infty}\delta(t) = \lim_{t\to\infty} P(t)\tilde{p}_{ij}(t) = 0$. Notice that the determinant of $P(t)$ is $\det(P(t)) = -\rho(t)$. So if condition (6) holds, there must exist an infinite sequence $[t_1, t_2, \ldots t_k, \ldots]$ satisfying $t_k \to \infty$ as $k \to \infty$, for which $|\det(P(t_k))| \geq \frac{\epsilon}{T}$. Hence, $P(t_k)$, $k = 1, 2, \ldots$, is non-singular and the entries in $P^{-1}(t_k)$ are uniformly bounded. So we can conclude that $\lim_{k\to\infty}\tilde{p}_{ij}(t_k) = \lim_{k\to\infty} P^{-1}(t_k)\delta(t_k) = 0$, implying that $\lim_{k\to\infty} V(\tilde{p}_{ij}(t_k)) = 0$. Furthermore, notice that $V(\tilde{p}_{ij}(t))$ is continuous and non-increasing. It follows that $V(\tilde{p}_{ij}(t)) \to 0$ as $t \to \infty$, or equivalently to say that the estimate $\hat{p}_{ij}$ converges to the true relative coordinate p_{ij}.

Second, we show that the estimate $z_i(t)$ in (5b) converges to the true relative coordinate $p_{i0}(t)$ of the source.

For $i = 0, 1, \ldots, N$, we define the estimation error as $\tilde{z}_i = z_i - p_{i0}$. For the source node, namely $i = 0$, we have $\tilde{z}_0 = 0$ as both z_0 and p_{00} equal to 0. Then the error dynamics of $\tilde{z}_i$ can be obtained from (5), which is

$$\dot{\tilde{z}}_i = \sum_{j\in\bar{\mathcal{N}}_i} [R(\theta_{ij})\tilde{z}_j - \tilde{z}_i] + A_i\tilde{z}_i + \sum_{j\in\bar{\mathcal{N}}_i}\tilde{p}_{ij}. \tag{11}$$

Notice that $\tilde{z}_i$, $i = 1, \ldots, N$, is the estimation error defined in agent i's local frame. For convenience of analysis, we transform every estimation error $\tilde{z}_i$ to the one in the global frame Σ_g. That is, we let $\tilde{z}_i^g = R(\theta_i)\tilde{z}_i$ and let $u_i^g = R(\theta_i)\sum_{j\in\bar{\mathcal{N}}_i}\tilde{p}_{ij}$. Expressing in terms of these global coordinates, (11) can be written as

$$\begin{aligned} \dot{\tilde{z}}_i^g &= \tfrac{d}{d\theta_i}R(\theta_i)\dot{\theta}_i\tilde{z}_i + R(\theta_i)\dot{\tilde{z}}_i \\ &= -A_i\tilde{z}_i^g + \textstyle\sum_{j\in\bar{\mathcal{N}}_i}(\tilde{z}_j^g - \tilde{z}_i^g) + A_i\tilde{z}_i^g + R(\theta_i)\sum_{j\in\bar{\mathcal{N}}_i}\tilde{p}_{ij} \\ &= \textstyle\sum_{j\in\bar{\mathcal{N}}_i}(\tilde{z}_j^g - \tilde{z}_i^g) + u_i^g. \end{aligned}$$

Denote $\tilde{z}^g = [\tilde{z}_1^g, \ldots, \tilde{z}_N^g]^{\mathsf{T}}$ and $u^g = [u_1^g, u_2^g, \ldots, u_N^g]^{\mathsf{T}}$. Then we

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obtain the following aggregated system

$$\dot{\tilde{z}}^g = -(H \otimes I_2)\tilde{z}^g + u^g, \tag{12}$$

where H is the matrix defined in Subsection II-A, $\otimes$ represents the Kronecker product, and I_2 is the 2-by-2 identity matrix. Note that $\tilde{p}_{ij}(t) \to 0$ as we just showed, so $u^g(t)$ converges to 0 as well. On the other hand, by Lemma 2.1, H is positive definite, so the matrix $-(H \otimes I_2)$ is Hurwitz. For a linear system $\dot{x} = Fx + Gu$ with F Hurwitz, if x is a solution on $[0,\infty)$ corresponding to an input $u \in L^\infty$ with $u(t) \to 0$ as $t \to \infty$, then $x(t) \to 0$ as $t \to \infty$ [19]. Hence, we conclude that for the system (12), $\lim_{t\to\infty} \|\tilde{z}^g(t)\| = 0$. This implies that $\tilde{z}_i(t) = R(-\theta_i)\tilde{z}_i^g(t)$ converges to zero for all $i = 1,\ldots,N$. In other words, the estimate $z_i(t)$ in (5b) converges to the true relative coordinate $p_{i0}(t)$. ■

Remark 3.3: Now we come to understand the condition (6) in Theorem 3.1. First, the condition (6) implies that in order to correctly estimate the relative position of a neighbor, an agent cannot be too far away from its neighbor (i.e., d_{ij} in the denominator of (6) cannot be too large all the time). This is consistent to our intuition. Second, the condition (6) implies that in order to correctly estimate the relative position of a neighbor, an agent cannot remain relatively stationary with respect to its neighbor or move relatively straight towards its neighbor. This can be observed by looking at the numerator term $v_i \sin(\alpha_{ij}) + v_j \sin(\alpha_{ji})$ in (6). By several steps of mathematical manipulation, it can be shown that $v_i \sin(\alpha_{ij}) + v_j \sin(\alpha_{ji}) = \varrho_{ij}^\intercal v_{ij}$ where $v_{ij}(t)$ is the relative velocity of agent i relative to agent j, defined in Subsection III-A. As shown in Fig. 5, the unit vector ϱ_{ij} is orthogonal to the line of sight. So if the two agents remain relatively stationary (i.e. $v_{ij} = 0$) or the two agents move relatively straight towards each other (i.e. v_{ij} is coincident with the line of sight), then by intuition it is known that one agent is not able to estimate the relative position of the other by using only the bearing measurement. The condition (6) excludes such a case as well. From the above discussion, we can also see that the relative motion $\varrho_{ij}^\intercal v_{ij}$ and the relative distance d_{ij} are the two main factors, which affect the convergence rate of the estimator.

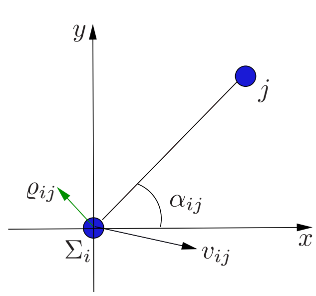

Fig. 5. Intuitions behind the condition (6).

Remark 3.4: If a neighbor j of agent i is just the source (labeled 0), which is stationary, then the condition (6) degenerates to

$$\int_t^{t+T} \left| \frac{v_i \sin \alpha_{i0}}{d_{i0}} \right| d\tau \geq \epsilon. \tag{13}$$

The condition (13) is indeed easier to understand as it requires the distance d_{i0} to the source cannot be too large, the agent cannot remain stationary (i.e. $v_i = 0$), and the agent cannot move straight towards the source (i.e. $\alpha_{i0} = 0$) all the time.

Remark 3.5: The condition (6) is mainly for each pair of neighbors. A similar integral condition was also developed in [20] for a double-integrator motion. It will be interesting if we can find a necessary and sufficient condition about the collective motion of the agents such that the source localization problem is solvable. This, however, may be possible by adopting the idea of cooperative persistent excitation from [21].

IV. Simulation

In this section, we present a simulation of four mobile agents (labeled $1, 2, \ldots, 4$) for the distributed source localization problem.

Without loss of generality, we set the stationary source (labeled 0) at the origin of a global coordinate system and let each agent take different types of motion in the plane. The linear speed, angular speed, and the initial position of the four agents are described in Table I. The trajectories resulting from these parameters are depicted in Fig. 6. It can be verified that the condition (6) in Theorem 3.1 holds.

TABLE I
PARAMETERS OF THE FOUR MOBILE AGENTS.

	$v_i[m/s]$	$\omega_i[rad/s]$	$p_i^g(0)$
1	$\cos(t)$	$0.4 + 0.1\cos(t)$	$(1, 0)$
2	2	$0.5 + 0.1\cos(t)$	$(-2, 0)$
3	3	0.65	$(-1.77, 1.77)$
4	$3.4 + 0.2\sin(t)$	$0.6 + 0.1\cos(t)$	$(-2.83, -2.83)$

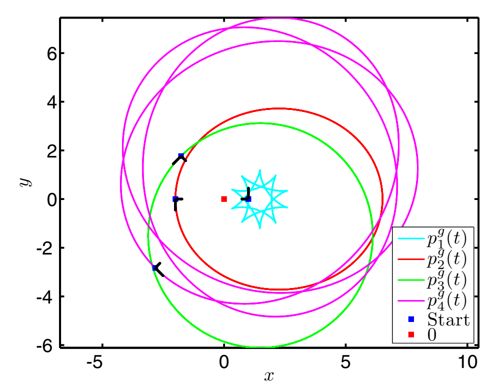

Fig. 6. The trajectories of the four agents in the plane.

The graph $\bar{\mathcal{G}}$ describing the interconnection relationship among the four agents and the source is shown in Fig. 1. That is, agent 1 and 2 can directly measure the bearing angles of the source in their own local frames, while agent 3 and 4 can not. But $\bar{\mathcal{G}}$ is connected as we can see.

We carry out two simulation studies by applying the distributed source localization algorithm (5) to estimate the relative coordinate of the source by each agent.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TAC.2015.2457112, IEEE Transactions on Automatic Control

First, we assume that the measurements of the bearing angle α_{ij}, the linear speed v_i and the angular speed ω_i are noiseless. In such a setup, the estimation errors $\|z_i(t) - p_{i0}\|$ $(i = 1, \ldots, 4)$ in the simulation are shown in Fig. 7 by taking $t = 100s$. From this simulation, we can see that the estimation errors converge to 0 as we expect.

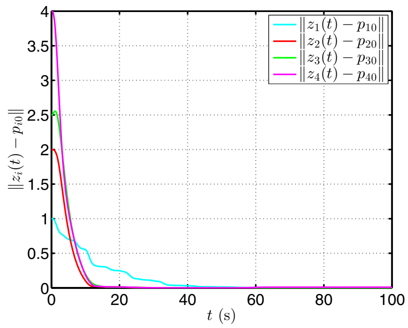

Fig. 7. The estimation errors $\|z_i(t) - p_{i0}\|$, $i = 1, \ldots, 4$, for the noiseless case.

Second, we assume that the measurements of the bearing angle α_{ij}, the linear speed v_i and the angular speed ω_i contain measurement noises. In this simulation, we add $0.05 *$ randn (the normally distributed random numbers in Matlab) to the bearing angle measurement α_{ij}, the linear speed measurement v_i as well as the angular speed measurement ω_i. In such a scenario with measurement noises, we run the distributed source localization algorithm (5) and the estimation errors are shown in Fig. 8. From this simulation, we can still see that with white measurement noises, the estimation errors still converge and approach close to 0.

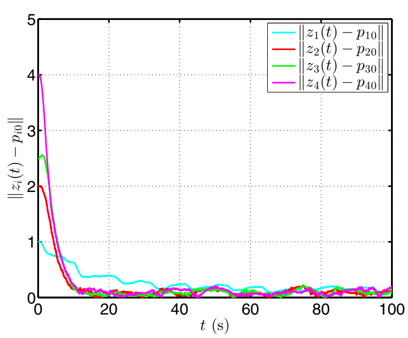

Fig. 8. The estimation errors $\|z_i(t) - p_{i0}\|$, $i = 1, \ldots, 4$, for the noisy measurement case.

V. CONCLUSION

This note studies the relative localization problem for a stationary source by considering a group of unicycle-type agents based on bearing information. A distributed source localization scheme is developed for the purpose by exchanging appropriate messages between neighboring agents. Many interesting problems arising from this research deserve further investigation. Examples include distributed localization for a mobile source or multiple sources, and distributed source localization under time-varying or position-dependent sensing graphs.

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