Improving Reliability in Lossy Wireless Networks Using Network Coding

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Abstract—Wireless communications between devices can be lossy owing to a number of issues, such as channel fading, interference or mobility of devices. In some scenarios, the lossy characteristic of wireless communications can be random hence better characterized from a stochastic perspective. In view of this, lossy wireless networks have been studied recently, where the transmission between each pair of nodes is successful with a certain probability. This paper investigates the reliability of broadcast in lossy networks, where the reliability is measured by the probability that every node in the network receives the packets of every other node. To improve the reliability, nodes can cooperate with each other using network coding techniques. In this paper, a neighbor network coding scheme is proposed and network reliability under this coding scheme is investigated analytically. This paper shows that reliability of networks can be improved considerably by using the proposed neighbor coding scheme. Further, closed-form upper and lower bounds on the network reliability are presented. Moreover, an optimal neighbor coding scheme that maximizes the reliability of a given network is discussed.

Index Terms—Network coding; Cooperative; Broadcast; Reliability

I. INTRODUCTION

Wireless communication can be affected by a number of issues, such as channel fading, interference or mobility of devices, making reliability a major challenge in wireless communications [1]–[4]. This work examines the reliability of cooperative broadcast using network coding techniques, where the reliability is measured by the probability that every node in the network receives the packets of every other node. Network coding is a technique that allows nodes in a network not only to store and forward received messages but also to process and combine several inputs into a single output. It was first proposed in [5]; showing that in single-source multicast wired networks, network coding brings benefits in capacity. Afterwards, network coding has been applied in wireless networks to improve throughput. Another important benefit of network coding is to improve network reliability. Conventionally, in a non-cooperative wireless network, the successful reception of a packet relies on multiple retransmissions of the same information from the source node. Therefore, in literature, reliability is usually improved by increasing the number of retransmissions. Lots of research has been conducted to reduce the number of retransmissions while maintaining a certain reliability. Most recently, network coding is also employed to reduce the number of retransmissions.

This work proposes a neighbor network coding scheme for all-to-all broadcasting networks allowing nodes to cooperate with each other. More specifically, a node can assist its neighbor by broadcasting a network coded packet including packets of its own and its neighbor. Then, a Markov chain is established, using which, exact results of the reliability are obtained. Additionally, upper and lower bounds are provided and the optimal neighbor coding scheme which maximizes the reliability of a given network is discussed.

The rest of the paper is organized as follows. Section II reviews related work. Section III introduces the system model. Theoretical analysis of the reliability is given in Section IV, followed by the closed-form upper and lower bounds on the reliability in Section V. Section VI presents the simulation and numerical results. Section VII concludes the paper and proposes future work.

II. RELATED WORK

In lossy wireless networks, network coding improves reliability while reducing the number of retransmissions [1]–[4]. The network coding aided ARQ is studied in access point (AP) based networks in [1], [2]. Network coding is utilized in a source node to broadcast a selected combination of unsuccessfully received packets to different receivers. In [1], all users listen to all the packets, and intended users may decode the network-coded packet using the overhead packets. Ref. [2] considers the fairness of all users in terms of the service time and goodput. The paper also implements the network coding aided retransmission scheme in a real environment and demonstrates its effectiveness.

In [3], [4], network coding is applied to networks with tree topologies where each multicast tree has equal number of children. The expected numbers of retransmissions by the source node under different error control protocols are computed. Based on numerical comparison, it is conjectured that network coding achieves a logarithmic reliability gain with respect to the number of receivers in a multicast group compared with a simple ARQ scheme. This hypothesis is then proved in the latter work [4]. Different from previous work [1]–[4], this paper considers an all-to-all broadcasting model. Further, this work gives the exact probabilities that every node in the network receives the packets successfully of every other node, i.e. reliability, after each retransmission, which is more general compared with previous work [3], [4] that only consider the expected number of retransmissions to achieve reliability of 100 percent.
It is worth to note that in [6], Nistor et al. study the delay probability distribution of message broadcasting in wireless networks using random linear network coding [7]. The probability of successful decoding at individual delay is similar to the reliability considered in this paper. They consider a one-to-all case where a single source broadcasts multiple packets to multiple receivers over erasure channels. They use Markov chain to analyze a network with two receivers only and a brute-force method for three receivers. In contrast, this work considers arbitrary number of receivers.

III. System model

For a network consisting of \( n \) nodes, denote the \( k \)th node by \( N_k \). Each node acts as a source node and has a packet to broadcast to all other nodes in the network. Denote the original packet that \( N_k \) broadcasts by \( X_1 \). Further, it is assumed that time is slotted and in each time slot only one source node (say \( N_j \)) broadcasts a single packet.

Due to lossy nature of wireless communications, the packet broadcast from a source node may not be able to reach every other node in one time slot. Let \( p_{ji} \), where \( p_{ji} \in (0,1] \), be the probability that a packet sent from \( N_j \) reaches \( N_i \) successfully in one time slot. We refer readers to [8] for approaches on obtaining \( p_{ji} \) for all pairs of nodes in a network, known as the probabilistic connectivity matrix. Since this work focuses on the impact of network coding on the reliability, it is assumed that \( p_{ji} \) for every \( i, j \in \{1, 2, ..., n\} \) is known.

In the case that a packet does not reach all nodes in one time slot, the source node has to broadcast more than once. Assume that all nodes in the network transmit in a round robin manner and a successful transmission is not acknowledged, which is a common scenario for broadcasting. A round is defined as a sequence of time slots during which every source node broadcasts exactly once. Consequently, the reliability of the network at round \( R \) (time slot \( t = nR \)) is defined as the probability that every node in the network has a copy of the packets of all other nodes at \( R \).

It is worth noting that without cooperation, a source node can only re-broadcast its original packet. With the use of network coding, the source node may broadcast a combination of its own packet and received packets. This work considers the neighbor network coding scheme. Specifically, each node (say \( N_j \)) selects another node (say \( N_h \)), namely coding neighbor, to perform the XOR coding. Note that the only constraints on the selection of coding neighbor are: \( j \neq h \) and a pair of nodes cannot mutually select each other. Therefore our analysis is generally applicable to arbitrary neighbor selection rules, where the optimal rule is discussed in Section VI.

A buffer is used at each node to store the received packets (duplicated packets are dropped). Decoding is performed at each node after receiving every packet. Note that if \( N_j \) has \( X_h \), it broadcasts \( X_h \oplus X_i \); otherwise, it broadcasts \( X_i \). Therefore, the packet that node \( N_j \) broadcasts at time \( t \) depends on the packets received by \( N_j \) from other nodes up to time \( t \). This creates challenge to the theoretical analysis, as shown in the next section.

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### IV. Theoretical analysis

In this section, we study the reliability by examining the packets received by a node (say \( N_j \)) from an arbitrary source node (say \( N_i \)).

Suppose that the coding neighbor of \( N_j \) is \( N_h \). Then, \( N_j \) may broadcast either \( X_h \) or \( X_h \oplus X_i \), depending on the packets that \( N_h \) has. It follows that the state of node \( N_j \) in a time slot, viz. the packets received by \( N_j \), depends only on the states of \( N_i \) and \( N_h \) in the previous time slot and the packet reception in this time slot. Therefore, the transmission can be modeled by a Markov chain.

A. Construction of the states

Let the state of a node (say \( N_j \)) be the combination of packets it has. More specifically, a state is expressed by a \( 1 \times n \) vector, \([\xi_1, ..., \xi_n]\), where an entry \( \xi_k \) indicates the packets received and stored from node \( N_k \). There are four possible values for each \( \xi_k \) where \( k \neq i \), which are: \( \xi_k = 0, 1, 2 \) and \( 3 \) representing the cases that the node \( N_i \) has received no packet, original packet, XORed packet, and both original and XORed packets from \( N_k \) respectively. Note that \( \xi_i = 1 \) in every state of \( N_i \), because \( N_i \) always has its own packet. The total number of states \( L \) for each node is equal to \( 4^n - 1 \).

Once a source node has the packet of its designated coding neighbor, it starts to broadcast the XORed packet. Consequently, it is impossible for a node to receive the original packet (\( X_h \)) from a source node (\( N_h \)) after receiving the XORed packet \( X_h \oplus X_i \). Therefore, there exist some absorbing states which cannot exit after entering. The absorbing states of node \( N_i \) have the characteristic that \( \xi_k = 2 \) or \( 3 \) for all \( k \in \{1, 2, ..., n\} \setminus \{i\} \).

Take a network with three nodes as an example. Suppose that the coding neighbors for \( N_1 \), \( N_2 \), and \( N_3 \) are \( N_2 \), \( N_1 \), and \( N_2 \) respectively. There are \( L = 16 \) states for each node. The states of \( N_1 \) and their corresponding packets are listed in Table 1.

B. Transition matrices

Consider that in time slot \( t \), \( N_j \) broadcasts. Next we examine the transitions of the states of a receiving node, say
Denote $a$ as the state of $N_t$ in time slot $t$ and $b$ as the state of $N_t$ in time slot $t + 1$.

Denote by $Q_{ji}(t)$ the transition matrix governing the transitions of the states of $N_t$ when $N_t$ broadcasts. It is worth noting that $Q_{ji}(t)$ depends on the packet $N_t$ broadcasts, which can be either its original packet or the XORRed packet. Consequently, denote $M^{0i}_{ji}$ and $M^{1i}_{ji}$ as the conditional transition matrices representing the transition matrices of the state of $N_t$ conditioned on the event that $N_t$ broadcasts its original packet and the XORRed packet respectively. $M^{0i}_{ji}$ and $M^{1i}_{ji}$ are matrices and can be constructed according to the following algorithms.

Each element of $M^{0i}_{ji}$, say $P_{ji}^{0i}(b|a)$, can be constructed by comparing states $a$ and $b$, according to Algorithm 1. In the algorithm, $a(k)$ denotes the $k$th element of state $a$ and we say $a = b$ if $a[k] = b[k]$ for all $k \in \{1, 2, \ldots, n\}$. Similarly, each element of $M^{1i}_{ji}$, say $P_{ji}^{1i}(b|a)$, can be constructed by comparing the states $a$ and $b$, according to Algorithm 2.

Algorithm 1 Construct $M^{0i}_{ji}$

for each $P_{ji}^{0i}(b|a)$ in $M^{0i}_{ji}$ do
  if $a = b$ and $a[j] = b[j] = 0$ then $N_t$ does not receive the packet from $N_j$, which happens with probability $P_{ji}^{0i}(b|a) = 1 - p_{ji}$;
  else if $a[j] = 0$, $b[j] = 1$, while $a[k] = b[k]$ for all $k \in \{1, 2, \ldots, n\} \setminus \{j\}$ then $N_t$ receives the packet from $N_j$, which happens with probability $P_{ji}^{0i}(b|a) = p_{ji}$;
  else if $a = b$ and $a[j] = b[j] \neq 0$ then the state transition does not depend on whether or not $N_t$ receives the packet from $N_j$, hence $P_{ji}^{0i}(b|a) = 1$;
  else let $P_{ji}^{0i}(b|a) = 0$.
end for

C. The probability vectors

Denote the probability vector of node $N_t$ in time slot $t$ as $S_i(t)$. A probability vector is a $1 \times L$ row vector whose $t^{th}$ entry represents the probability that $N_t$ is at the $t^{th}$ state in time slot $t$. Suppose that $N_t$ broadcasts in time slot $t$, then using Eq. (1), the probability vector of $N_t$ in time slot $t + 1$ can be calculated by:

$S_i(t + 1) = S_i(t)Q_i(t)$

$= S_i(t) \left( \mu_1(t)M^{0i}_{ji} + \mu_2(t)M^{1i}_{ji} \right)$.  

Next, we need to obtain $\mu_1(t)$ and $\mu_2(t)$. Denote $B_i$ as a $L \times 1$ indicator vector, whose $t^{th}$ entry is set to one if $N_t$ broadcasts the XORRed packet in the $t^{th}$ state; otherwise it is set to zero. On the other hand, let $A_i$ be a $1 \times 1$ indicator vector, whose $t^{th}$ entry is set to one if $N_t$ broadcasts its original packet in the $t^{th}$ state; otherwise it is set to zero. Then we have:

$\mu_1(t) = S_i(t)A_i$,

$\mu_2(t) = S_i(t)B_i$.  

Consequently, the probability vector of $N_t$ in time slot $t + 1$ can be generated by a recursive formula including the probability vectors of $N_t$ and $N_t$ in time slot $t$:

$S_i(t + 1) = S_i(t) \left( S_i(t) \times A_i \times M^{0i}_{ji} + S_j(t) \times B_j \times M^{1i}_{ji} \right)$.  

The initial state of $N_t$ contains packet $X_t$ only. Then, in the initial probability vector $S_i(0)$, the initial state is assigned with probability one and all other states are with probability zero. For example, if the states of $N_t$ are arranged as shown in Table I, the initial state is $[100]$. Therefore, $S_i(0)$ is a $1 \times 16$ vector whose first entry is one and all other entries are zero.

D. Reliability

Denote by $\psi_i(t)$ the probability that $N_i$ has packets of every other node in time slot $t$. Then, it can be calculated by:

$\psi_i(t) = \sum_{x \in X} S^x_i(t)$,  

where $S^x_i(t)$ is the $x^{th}$ entry of $S_i(t)$, the set $X$ includes the indexes of states in which $N_i$ has the packets from every other node. Take $i = 1$ as an example, as shown in Table I, we have $X = \{4, 6, 7, 8, 10, 11, 12, 14, 15, 16\}$.

Finally, the reliability of the network in time slot $t$, i.e., the probability that every node receives packets of every other node can be expressed by:

$\psi(t) = \prod_{i=1}^{16} \psi_i(t)$.  

V. Bounds on the reliability

The theoretical results presented in the previous section are exact results but the computation can be complicated. To shed more insights into the impact of fundamental network parameters, e.g. the connectivity between nodes $p_{ij}$ and the
selection of coding neighbor, on the network reliability, we present closed-form results of upper and lower bounds on the network reliability in this section.

The analysis starts with the reception of a single packet \( X_j \) at a node \( N_i \). Assume that node \( N_i \) selects \( N_h \) as its coding neighbor and \( N_j \) is selected by \( N_d \) as coding neighbor. Then, there are two possible processes for the packet \( X_j \) to reach \( N_i \).
The first one is through the path \( N_j \rightarrow N_h \), and the second one is through the path \( N_j \rightarrow N_d \rightarrow N_h \), via the reception of packet \( X_d \). Therefore, Eq. (8) becomes:

\[
F_{ji}(R) \leq (1 - p_{ji})^R \sum_{a=1}^{R} \left( 1 - (1 - p_{ji})^{R-a} \right) (1 - p_{jd})^{a-1} p_{jd} + (1 - (1 - p_{ji})^R) \pm U_{ji}(R).
\]  

Proof: To obtain an upper bound on the probability \( F_{ji}(R) \), we consider that \( N_i \) can decode \( X_j \) upon receiving any packet from \( N_j \).

Denote by \( \Xi_R \) (resp. \( \Gamma_R \)) the event that a packet containing \( X_j \) (either \( X_d \) or an XORed packet containing \( X_j \)) reaches \( N_i \) by round \( R \) via the first (resp. the second) process.

Then, it is straightforward that \( \Pr(\Xi_R) = 1 - (1 - p_{ji})^R \),

\[
\Pr(\Gamma_R) = \sum_{a=1}^{R} \left( 1 - (1 - p_{ji})^{R-a} \right) f_{jd}(\alpha),
\]

where \( \alpha \) is the round at which the packet broadcast by \( N_j \) reaches \( N_d \) for the first time. It is evident that \( f_{jd}(\alpha) \) follows a geometric distribution with success probability \( p_{jd} \).

Therefore, Eq. (8) becomes:

\[
\Pr(\Gamma_R) = \sum_{a=1}^{R} \left( 1 - (1 - p_{ji})^{R-a} \right) (1 - p_{jd})^{a-1} p_{jd}.
\]  

Then, Eq. (7) can be obtained using \( F_{ji}(R) \leq \Pr(\Xi_R \cup \Gamma_R) = \Pr(\Xi_R) + (1 - \Pr(\Xi_R)) \Pr(\Gamma_R) \).

Finally, the upper bound of the reliability of the network at the \( R^\text{th} \) round, denoted by \( U(R) \), can be calculated by:

\[
U(R) = \prod_{i,j \in \{1,2,\ldots,a\}} U_{ji}(R),
\]

where \( U_{ji}(R) \) is given by Theorem 1.

B. The lower bound

Theorem 2. Suppose that the coding neighbors of \( N_i, N_d \), and \( N_h \) are \( N_j, N_i \) and \( N_h \), respectively. The probability that node \( N_i \) has packet \( X_j \) at the \( R^\text{th} \) round satisfies:

\[
F_{ji}(R) \geq \sum_{b=1}^{R} \Pr(\Omega_{ji}(\Omega_{ih}(\beta))) \Pr(\Psi_{ji}(\beta)) \times f_{ji}(\beta) \pm L_{ji}(R),
\]

where \( \Pr(\Omega_{ji}(\beta)), \Pr(\Psi_{ji}(\beta)) \) and \( f_{ji}(\beta) \) are given by Eq. (12), Eq. (13) and Eq. (15) respectively.

Proof: Similarly to the proof of Theorem 1, we separately investigate the two processes described at the beginning of this section. Denote \( \alpha \) as the round at which \( N_d \) first has \( X_i \) and begins to broadcast \( X_i \). Further, denote \( \beta \) as the round at which \( N_j \) first receives \( X_h \) from \( N_i \) and begins to broadcast \( X_j \). To obtain a lower bound on the network reliability, we consider only the cases when \( N_i \) broadcasts its original packet in the first \( \beta \) rounds and omits the probability that \( N_h \) broadcasts coded packets.

Regarding the first process, it is obvious that the XORed packet broadcast by \( N_j \), i.e., \( X_i \oplus X_h \), can be decoded by \( N_i \) if \( N_i \) has packet \( X_h \). Denote by \( \Omega_{ji}(\beta) \) the event that \( N_i \) receives the packet \( X_j \) via the first process by round \( R \). Further, denote by \( \Omega_{ih}(\beta) \) the event that \( N_i \) receives the packet \( X_j \oplus X_h \) via the first process by round \( R \) but \( N_i \) only stores the packets received from \( N_h \) in the first \( \beta \) rounds.

Denote by \( \Pr(\Omega_{ji}(\beta)) \) the probability that event \( \Omega_{ji}(\beta) \) occurs conditioned on the event that \( \beta \) is the round at which node \( N_j \) receives \( X_h \) for the first time. It is straightforward that \( \Pr(\Omega_{ji}(\beta)) = 1 - (1 - p_{ji})^\beta \). Similarly, we have that \( \Pr(\Omega_{ih}(\beta)) = (1 - (1 - p_{ji})^\beta) (1 - (1 - p_{ih})^\beta) (1 - (1 - p_{ji})^\beta) \).

Further, because events \( \Omega_{ji}(\beta) \) and \( \Omega_{ih}(\beta) \) are correlated, we have \( \Pr(\Omega_{ji}(\beta) \cap \Omega_{ih}(\beta)) = \Pr(\Omega_{ji}(\beta)) \Pr(\Omega_{ih}(\beta)) = (1 - (1 - p_{ji})^\beta) (1 - (1 - p_{ih})^\beta) (1 - (1 - p_{ji})^\beta) \).

Finally, it is evident that the probability \( \Pr(\Omega_{ji}(\beta)) \), defined as:

\[
\Pr(\Omega_{ji}(\beta)) = \Pr(\Omega_{ji}(\beta) \cup \Omega_{ih}(\beta)) = \Pr(\Omega_{ji}(\beta)) + \Pr(\Omega_{ih}(\beta)) - \Pr(\Omega_{ji}(\beta) \cap \Omega_{ih}(\beta)),
\]

provides a lower bound on the probability that \( N_i \) receives and decodes \( X_j \) by round \( R \) via the first process.

Regarding the second process, denote by \( \Psi_{ji}(\beta) \) the event that \( N_i \) receives \( X_j \) via the second process by round \( R \) but \( N_i \) only stores \( X_h \) from \( N_h \) when \( N_j \) broadcasts its original packet.

Then, the probability that event \( \Psi_{ji}(\beta) \) occurs conditioned on the event that \( N_i \) receives \( X_h \) for the first time at round \( \beta \) is:

\[
\Pr(\Psi_{ji}(\beta)) = \sum_{a=1}^{\beta} (1 - (1 - p_{ih})^a) (1 - (1 - p_{ih})^{R-a}) f_{jd}(\alpha) \]

\[
= \sum_{a=1}^{\beta} (1 - (1 - p_{ih})^a) (1 - (1 - p_{ih})^{R-a}) \left(1 - (1 - p_{jd})^{a-1}\right) p_{jd},
\]

Therefore, the probability that node \( N_i \) receives \( X_j \) in the \( R^\text{th} \) round satisfies:

\[
F_{ji}(R) \geq \sum_{b=1}^{R} \phi_{ji}(\beta) \Pr(\Omega_{ji} \cup \Psi_{ji}(\beta)) \]

\[
= \sum_{b=1}^{R} \phi_{ji}(\beta) \left( \Pr(\Omega_{ji}(\beta)) + \Pr(\Psi_{ji}(\beta)) - \Pr(\Omega_{ji}(\beta) \cup \Omega_{ih}(\beta)) \Pr(\Psi_{ji}(\beta)) \right),
\]

where \( \phi_{ji}(\beta) \) is the probability that \( N_i \) receives \( X_h \) from \( N_h \) at round \( \beta \) for the first time, which satisfies:

\[
\phi_{ji}(\beta) \geq (1 - p_{ji})^{\beta-1} p_{ji}(1 - F_{ji}(\beta)) \pm f_{ji}(\beta),
\]

325
VI. NUMERICAL RESULTS AND DISCUSSION

In this section, simulations are conducted to validate our theoretical analysis. Moreover, the benefits in reliability of neighbor network coding over non-coded networks are shown, followed by discussions about the relation between the selection of coding neighbors and network reliability.

The probabilistic connectivity matrices indicating channel conditions can be arbitrary. In this section, the entries are chosen randomly to generate numerical results. The matrix used in each figure is given in the respective caption.

The reliability of networks with arbitrary number of nodes at arbitrary round can be calculated using Eq. (6). In Fig. 1, the theoretical results for networks with 3, 4 and 5 nodes. The coding scheme is that \( N_i \) chooses \( N_{k+1 \mod n} \) as coding neighbor. It shows that the theoretical results match with the simulation results well, which in turn validates theoretical analysis.

In order to examine the reliability benefits of the proposed neighbor network coding, the coded networks with different coding neighbors which give the best and the worst reliability are plotted together with the corresponding non-coded networks, as shown in Fig. 2. It shows that the coded networks have better reliability than the non-coded networks in every case, and the reliability gain can be considerable in some scenarios. For example, in the network of four nodes, the neighbor network coding brings reliability gain of more than 200 percent over the non-coded network at round \( R = 10 \).

Additionally, the selection of neighbors affects the network reliability. Based on numerous simulations, it is conjectured that if every node selects the node to which the connection probability is the lowest as coding neighbor, the reliability gain can be maximized.

Lastly, the bounds on the probability that \( X_1 \) is received by \( N_3 \), given by Theorem 1 and Theorem 2, are shown in Fig. 3. The coding scheme is the same as that in Table I. It can be seen that the bounds are valid. Moreover, the bounds can be further improved and be used to characterize the reliability gain and further to facilitate the proof of the aforementioned conjecture on the optimal neighbor selection rule, which is not a trivial task hence left as future work.

VII. CONCLUSION AND FUTURE WORK

In this paper, a neighbor network coding scheme is proposed for cooperative broadcasting. Network reliability is investigated analytically and it has been shown that the proposed neighbor coding scheme can improve network reliability significantly. We also provide bounds on the reliability of the network applying the proposed coding scheme. In the future, the framework of analyzing network reliability established can be applied to study the reliability of networks applying different coding schemes. Moreover, it is important to develop a theoretical proof for the optimal coding scheme that maximizes the reliability of a given network.

REFERENCES