Improving Reliability in Lossy Wireless Networks Using Network Coding

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***Abstract*—Wireless communications between devices can be lossy owing to a number of issues, such as channel fading, interference or mobility of devices. In some scenarios, the lossy characteristic of wireless communications can be random hence better characterized from a stochastic perspective. In view of this, lossy wireless networks have been studied recently, where the transmission between each pair of nodes is successful with a certain probability. This paper investigates the reliability of broadcast in lossy networks, where the reliability is measured by the probability that every node in the network receives the packets of every other node. To improve the reliability, nodes can cooperate with each other using network coding techniques. In this paper, a neighbor network coding scheme is proposed and network reliability under this coding scheme is investigated analytically. This paper shows that reliability of networks can be improved considerably by using the proposed neighbor coding scheme. Further, closed-form upper and lower bounds on the network reliability are presented. Moreover, an optimal neighbor coding scheme that maximizes the reliability of a given network is discussed.**

***Index Terms*—Network coding; Cooperative; Broadcast; Reliability**

I. Introduction

Wireless communication can be affected by a number of issues, such as channel fading, interference or mobility of devices, making reliability a major challenge in wireless communications [1]–[4]. This work examines the reliability of cooperative broadcast using network coding techniques, where the reliability is measured by the probability that every node in the network receives the packets of every other node.

Network coding is a technique that allows nodes in a network not only to store and forward received messages but also to process and combine several inputs into a single output. It was first proposed in [5]; showing that in single-source multicast wired networks, network coding brings benefits in capacity. Afterwards, network coding has been applied in wireless networks to improve throughput.

Another important benefit of network coding is to improve network reliability. Conventionally, in a non-cooperative wireless network, the successful reception of a packet relies on multiple retransmissions of the same information from the source node. Therefore, in literature, reliability is usually improved by increasing the number of retransmissions. Lots of research has been conducted to reduce the number of retransmissions while maintaining a certain reliability. Most recently, network coding is also employed to reduce the number of retransmissions.

This work proposes a neighbor network coding scheme for all-to-all broadcasting networks allowing nodes to cooperate with each other. More specifically, a node can assist its neighbor by broadcasting a network coded packet including packets of its own and its neighbor. Then, a Markov chain is established, using which, exact results of the reliability are obtained. Additionally, upper and lower bounds are provided and the optimal neighbor coding scheme which maximizes the reliability of a given network is discussed.

The rest of the paper is organized as follows. Section II reviews related work. Section III introduces the system model. Theoretical analysis of the reliability is given in Section IV, followed by the closed-form upper and lower bounds on the reliability in Section V. Section VI presents the simulation and numerical results. Section VII concludes the paper and proposes future work.

II. Related work

In lossy wireless networks, network coding improves reliability while reducing the number of retransmissions [1]–[4].

The network coding aided ARQ is studied in access point (AP) based networks in [1], [2]. Network coding is utilized in a source node to broadcast a selected combination of unsuccessfully received packets to different receivers. In [1], all users listen to all the packets, and intended users may decode the network-coded packet using the overheard packets. Ref. [2] considers the fairness of all users in terms of the service time and goodput. The paper also implements the network coding aided retransmission scheme in a real environment and demonstrates its effectiveness.

In [3], [4], network coding is applied to networks with tree topologies where each multicast tree has equal number of children. The expected numbers of retransmissions by the source node under different error control protocols are computed. Based on numerical comparison, it is conjectured that network coding achieves a logarithmic reliability gain with respect to the number of receivers in a multicast group compared with a simple ARQ scheme. This hypothesis is then proved in the latter work [4].

Different from previous work [1]–[4], this paper considers an all-to-all broadcasting model. Further, this work gives the exact probabilities that every node in the network receives the packets successfully of every other node, i.e. reliability, after each retransmission, which is more general compared with previous work [3], [4] that only consider the *expected* number of retransmissions to achieve reliability of 100 percent.

It is worth to note that in [6], Nistor et al. study the delay probability distribution of message broadcasting in wireless networks using random linear network coding [7]. The probability of successful decoding at individual delay is similar to the reliability considered in this paper. They consider a one-to-all case where a single source broadcasts multiple packets to multiple receivers over erasure channels. They use Markov chain to analyze a network with two receivers only and a brute-force method for three receivers. In contrast, this work considers arbitrary number of receivers.

III. System model

For a network consisting of n nodes, denote the k^{th} node by N_k. Each node acts as a source node and has a packet to broadcast to all other nodes in the network. Denote the original packet that N_k broadcasts by X_k. Further, it is assumed that time is slotted and in each time slot only one source node (say N_j) broadcasts a single packet.

Due to lossy nature of wireless communications, the packet broadcast from a source node may not be able to reach every other node in one time slot. Let p_{ji}, where $p_{ji} \in (0, 1]$, be the probability that a packet sent from N_j reaches N_i successfully in one time slot. We refer readers to [8] for approaches on obtaining p_{ji} for all pairs of nodes in a network, known as the *probabilistic connectivity matrix*. Since this work focuses on the impact of network coding on the reliability, it is assumed that p_{ji} for every $i, j \in \{1, 2, ..., n\}$ is known.

In the case that a packet does not reach all nodes in one time slot, the source node has to broadcast more than once. Assume that all nodes in the network transmit in a round robin manner and a successful transmission is not acknowledged, which is a common scenario for broadcasting. A *round* is defined as a sequence of time slots during which every source node broadcasts exactly once. Consequently, the *reliability* of the network at round R (time slot $t = nR$) is defined as the probability that every node in the network has a copy of the packets of all other nodes at R.

It is worth noting that without cooperation, a source node can only re-broadcast its original packet. With the use of network coding, the source node may broadcast a combination of its own packet and received packets. This work considers the *neighbor network coding scheme*. Specifically, each node (say N_j) selects another node (say N_h), namely *coding neighbor*, to perform the XOR coding. Note that the only constraints on the selection of coding neighbor are: $j \neq h$ and a pair of nodes cannot mutually select each other. Therefore our analysis is generally applicable to arbitrary neighbor selection rules, where the optimal rule is discussed in Section VI.

A buffer is used at each node to store the received packets (duplicated packets are dropped). Decoding is performed at each node after receiving every packet. Note that if N_j has X_h, it broadcasts $X_j \oplus X_h$; otherwise, it broadcasts X_j. Therefore, the packet that node N_j broadcasts at time t depends on the packets received by N_j from other nodes up to time t. This creates challenge to the theoretical analysis, as shown in the next section.

Index	1	2	3	4
State	[100]	[101]	[102]	[103]
Packets	X_1	X_1, X_3	$X_1, X_3 \oplus X_2$	$X_1, X_3, X_3 \oplus X_2$
Index	5	6	7	8
State	[110]	[111]	[112]	[113]
Packets	X_1, X_2	X_1, X_2 X_3	X_1, X_2 $X_3 \oplus X_2$	X_1, X_2 $X_3, X_3 \oplus X_2$
Index	9	10	11	12
State	[120]	[121]	[122]	[123]
Packets	$X_1, X_2 \oplus X_1$	$X_1, X_2 \oplus X_1$ X_3	$X_1, X_2 \oplus X_1$ $X_3 \oplus X_2$	$X_1, X_2 \oplus X_1$ $X_3, X_3 \oplus X_2$
Index	13	14	15	16
State	[130]	[131]	[132]	[133]
Packets	X_1 X_2 $X_2 \oplus X_1$	X_1 $X_2, X_2 \oplus X_1$ X_3	X_1 $X_2, X_2 \oplus X_1$ $X_3 \oplus X_2$	X_1 $X_2, X_2 \oplus X_1$ $X_3, X_3 \oplus X_2$

TABLE I
The states of N_1 and corresponding packets for a network with three nodes, where the coding neighbor for N_1, N_2 and N_3 are N_3, N_1 and N_2 respectively. For example, the 5^{th} state is [110], which represents that N_1 has packets X_1 and X_2.

IV. Theoretical analysis

In this section, we study the reliability by examining the packets received by a node (say N_i) from an arbitrary source node (say N_j).

Suppose that the coding neighbor of N_j is N_h. Then, N_j may broadcast either X_j or $X_j \oplus X_h$, depending on the packets that N_j has. It follows that the state of node N_i in a time slot, viz. the packets received by N_i, depends only on the states of N_i and N_j in the previous time slot *and* the packet reception in this time slot. Therefore, the transmission can be modeled by a Markov chain.

A. Construction of the states

Let the *state* of a node (say N_i) be the combination of packets it has. More specifically, a state is expressed by a $1 \times n$ vector, $[\xi_1, ..., \xi_n]$, where an entry ξ_k indicates the packets received and stored from node N_k. There are four possible values for each ξ_k where $k \neq i$, which are: $\xi_k = 0$, 1, 2 and 3 representing the cases that the node N_i has received no packet, original packet, XORed packet, and both original and XORed packets from N_k respectively. Note that $\xi_i = 1$ in every state of N_i, because N_i always has its own packet. The total number of states L for each node is equal to 4^{n-1}.

Once a source node has the packet of its designated coding neighbor, it starts to broadcast the XORed packet. Consequently, it is impossible for a node to receive the original packet (X_j) from a source node (N_j) after receiving the XORed packet $X_j \oplus X_h$. Therefore, there exist some absorbing states which cannot exit after entering. The absorbing states of node N_i have the characteristic that $\xi_k = 2$ or 3 for all $k \in \{1, 2, \ldots, n\} \setminus \{i\}$.

Take a network with three nodes as an example. Suppose that the coding neighbors for N_1, N_2 and N_3 are N_3, N_1 and N_2 respectively. There are $L = 16$ states for each node. The states of N_1 and their corresponding packets are listed in Table I.

B. Transition matrices

Consider that in time slot t, N_j broadcasts. Next we examine the transitions of the states of a receiving node, say

N_i. Denote a as the state of N_i in time slot t and b as the state of N_i in time slot $t+1$.

Denote by $Q_{ji}(t)$ the *transition matrix* governing the transitions of the states of N_i when N_j broadcasts. It is worth noting that $Q_{ji}(t)$ depends on the packet N_j broadcasts, which can be either its original packet or the XORed packet. Consequently, denote $M_{ji}^{\mu_1}$ and $M_{ji}^{\mu_2}$ as the conditional transition matrices representing the transition matrices of the state of N_i *conditioned on* the event that N_j broadcasts its original packet and the XORed packet respectively. $M_{ji}^{\mu_1}$ and $M_{ji}^{\mu_2}$ are $L \times L$ matrices. Denote elements of $M_{ji}^{\mu_1}$ and $M_{ji}^{\mu_2}$ as $P_M^{\mu_1}(b|a)$ and $P_M^{\mu_2}(b|a)$ respectively.

Then, $Q_{ji}(t)$ can be computed as follows according to total probability theory:

$$Q_{ji}(t) = \mu_1(t) M_{ji}^{\mu_1} + \mu_2(t) M_{ji}^{\mu_2}, \quad (1)$$

where $\mu_1(t)$ (resp. $\mu_2(t)$) is the probability that N_j transmits its original packet (resp. the XORed packet) in time slot t.

The probabilities $\mu_1(t)$ and $\mu_2(t)$ will be discussed in the next sub-section. The conditional transition matrices are time-invariant and can be constructed according to the following algorithms.

Each element of $M_{ji}^{\mu_1}$, say $P_M^{\mu_1}(b|a)$, can be constructed by comparing states a and b, according to Algorithm 1. In the algorithm, $a\{k\}$ denotes the k^{th} element of state a and we say $a = b$ if $a\{k\} = b\{k\}$ for all $k \in \{1, 2, \ldots, n\}$. Similarly, each element of $M_{ji}^{\mu_2}$, say $P_M^{\mu_2}(b|a)$, can be constructed by comparing the states a and b, according to Algorithm 2.

Algorithm 1 Construct $M_{ji}^{\mu_1}$

for each $P_M^{\mu_1}(b|a)$ in $M_{ji}^{\mu_1}$ **do**
  if $a = b$ and $a\{j\} = b\{j\} = 0$ **then** N_i does not receive the packet from N_j, which happens with probability $P_M^{\mu_1}(b|a) = 1 - p_{ji}$;
  else if $a\{j\} = 0$, $b\{j\} = 1$, while $a\{k\} = b\{k\}$ for all $k \in \{1, 2, \ldots, n\} \setminus \{j\}$ **then** N_i receives the packet from N_j, which happens with probability $P_M^{\mu_1}(b|a) = p_{ji}$;
  else if $a = b$ and $a\{j\} = b\{j\} \neq 0$ **then** the state transition does not depend on whether or not N_i receives the packet from N_j, hence $P_M^{\mu_1}(b|a) = 1$;
  else let $P_M^{\mu_1}(b|a) = 0$.
  end if
end for

C. The probability vectors

Denote the *probability vector* of node N_i in time slot t as $S_i(t)$. A probability vector is a $1 \times L$ row vector whose l^{th} entry represents the probability that N_i is at the l^{th} state in time slot t. Suppose that N_j broadcasts in time slot t, then using Eq. (1), the probability vector of N_i in time slot $t+1$ can be calculated by:

$$\begin{aligned} S_i(t+1) &= S_i(t) Q_{ji}(t) \\ &= S_i(t)\left(\mu_1(t) M_{ji}^{\mu_1} + \mu_2(t) M_{ji}^{\mu_2}\right). \end{aligned} \quad (2)$$

Next, we need to obtain $\mu_1(t)$ and $\mu_2(t)$. Denote B_j as a $L\times1$ indicator vector, whose l^{th} entry is set to one if N_j broadcasts the XORed packet in the l^{th} state; otherwise it is set to zero. On the other hand, let A_j be a $L \times 1$ indicator vector, whose l^{th} entry is set to one if N_j broadcasts its original packet in the l^{th} state; otherwise it is set to zero. Then we have:

$$\begin{aligned} \mu_1(t) &= S_j(t) A_j, \\ \mu_2(t) &= S_j(t) B_j. \end{aligned} \quad (3)$$

Algorithm 2 Construct $M_{ji}^{\mu_2}$

for each $P_M^{\mu_2}(b|a)$ in $M_{ji}^{\mu_2}$ **do**
  if $a = b$ and $a\{j\} = b\{j\} = 0$ or 1 **then** N_i does not receive the packet from N_j, which happens with probability $P_M^{\mu_2}(b|a) = 1 - p_{ji}$;
  else if $a\{j\} = 0$ and $b\{j\} = 2$ **or** $a\{j\} = 1$ and $b\{j\} = 3$, while $a\{k\} = b\{k\}$ for all $k \in \{1, 2, \ldots, n\} \setminus \{j\}$ **then** N_i receives the packet from N_j, which happens with probability $P_M^{\mu_2}(b|a) = p_{ji}$;
  else if $a = b$ and $a\{j\} = 2$ or 3 **then** the state transition does not depend on whether or not N_i receives the packet from N_j, hence $P_M^{\mu_2}(b|a) = 1$;
  else let $P_M^{\mu_2}(b|a) = 0$.
  end if
end for

Consequently, the probability vector of N_i in time slot $t+1$ can be generated by a recursive formula including the probability vectors of N_i and N_j in time slot t:

$$S_i(t+1) = S_i(t)\left(S_j(t) \times A_j \times M_{ji}^{\mu_1} + S_j(t) \times B_j \times M_{ji}^{\mu_2}\right). \quad (4)$$

The initial state of N_i contains packet X_i only. Then, in the initial probability vector $S_i(0)$, the initial state is assigned with probability one and all other states are with probability zero. For example, if the states of N_1 are arranged as shown in Table I, the initial state is [100]. Therefore, $S_1(0)$ is a 1×16 vector whose first entry is one and all other entries are zero.

D. Reliability

Denote by $\psi_i(t)$ the probability that N_i has packets of every other node in time slot t. Then, it can be calculated by:

$$\psi_i(t) = \sum_{x \in \chi} S_i^x(t), \quad (5)$$

where $S_i^x(t)$ is the x^{th} entry of $S_i(t)$, the set χ includes the indexes of states in which N_i has the packets from every other node. Take $i = 1$ as an example, as shown in Table I, we have $\chi = \{4, 6, 7, 8, 10, 11, 12, 14, 15, 16\}$.

Finally, the reliability of the network in time slot t, i.e., the probability that every node receives packets of every other node can be expressed by:

$$\psi(t) = \prod_{i \in \{1,2,\ldots,n\}} \psi_i(t). \quad (6)$$

V. Bounds on the reliability

The theoretical results presented in the previous section are exact results but the computation can be complicated. To shed more insights into the impact of fundamental network parameters, e.g. the connectivity between nodes p_{ij} and the selection of coding neighbor, on the network reliability, we present closed-form results of upper and lower bounds on the network reliability in this section.

The analysis starts with the reception of a single packet X_j at a node N_i. Assume that node N_j selects N_h as its coding neighbor and N_j is selected by N_d as coding neighbor. Then, there are two possible *processes* for the packet X_j to reach N_i. The first one is through the path N_j to N_i, via the reception of packets X_j or $X_j \oplus X_h$; and the second one is through the path N_j to N_d and then through the path N_d to N_i, via the reception of packet $X_d \oplus X_j$.

Denote $F_{ji}(R)$ as the probability that N_i receives and decodes X_j *by* round R and denote $f_{ji}(R)$ as the probability that N_i receives and decodes X_j *at* round R.

A. The upper bound

Theorem 1. *Suppose that N_d selects N_j as coding neighbor. The probability that node N_i receives X_j in R^{th} round satisfies:*

$$F_{ji}(R) \leq (1-p_{ji})^R \sum_{\alpha=1}^{R}\left(1-(1-p_{di})^{R-\alpha}\right)(1-p_{jd})^{\alpha-1}p_{jd} + \left(1-(1-p_{ji})^R\right) \triangleq U_{ji}(R). \quad (7)$$

Proof: To obtain an upper bound on the probability $F_{ji}(R)$, we consider that N_i can decode X_j upon receiving any packet from N_j.

Denote by Ξ_R (resp. Γ_R) the event that a packet containing X_j (either X_j or a XORed packet containing X_j) reaches N_i by round R via the first (resp. the second) process.

Then, it is straightforward that $\Pr(\Xi_R) = 1-(1-p_{ji})^R$,

$$\Pr(\Gamma_R) = \sum_{\alpha=1}^{R}\left(1-(1-p_{di})^{R-\alpha}\right) f_{jd}(\alpha), \quad (8)$$

where α is the round at which the packet broadcast by N_j reaches N_d for the first time. It is evident that $f_{jd}(\alpha)$ follows a geometric distribution with success probability p_{jd}. Therefore, Eq. (8) becomes:

$$\Pr(\Gamma_R) = \sum_{\alpha=1}^{R}\left(1-(1-p_{di})^{R-\alpha}\right)(1-p_{jd})^{\alpha-1}p_{jd}. \quad (9)$$

Then, Eq. (7) can be obtained using $F_{ji}(R) \leq \Pr(\Xi_R \cup \Gamma_R) = \Pr(\Xi_R) + (1-\Pr(\Xi_R))\Pr(\Gamma_R)$. ■

Finally, the upper bound of the reliability of the network at the R^{th} round, denoted by $U(R)$, can be calculated by:

$$U(R) = \prod_{i,j\in\{1,2,\ldots,n\}} U_{ji}(R), \quad (10)$$

where $U_{ji}(R)$ is given by Theorem 1.

B. The lower bound

Theorem 2. *Suppose that the coding neighbors of N_j, N_d and N_h are N_h, N_j and N_g respectively. The probability that node N_i has packet X_j at the R^{th} round satisfies:*

$$F_{ji}(R) \geq \sum_{\beta=1}^{R}\left(\Pr(\Omega_R|\beta) + \Pr(\Psi_R|\beta) - \Pr(\Omega_R|\beta)\Pr(\Psi_R|\beta)\right) \times f_{hj}^L(\beta) \triangleq L_{ji}(R), \quad (11)$$

where $\Pr(\Omega_R|\beta)$, $\Pr(\Psi_R|\beta)$ and $f_{hj}^L(\beta)$ are given by Eq. (12), Eq. (13) and Eq. (15) respectively.

Proof: Similarly to the proof of Theorem 1, we separately investigate the two processes described at the beginning of this section. Denote α as the round at which N_d first has X_j and begins to broadcast $X_d \oplus X_j$. Further, denote β as the round at which N_j first receives X_h from N_h and begins to broadcast $X_j \oplus X_h$. To obtain a lower bound on the network reliability, we consider only the cases when N_h broadcasts its original packet in the first β rounds and omits the probability that N_h broadcasts coded packets.

Regarding the first process, it is obvious that the XORed packet broadcast by N_j, i.e., $X_j \oplus X_h$, can be decoded by N_i if N_i has packet X_h. Denote by Ω_R^A the event that N_i receives the packet X_j via the first process by round R. Further, denote by Ω_R^B the event that N_i receives the packet $X_j \oplus X_h$ via the first process by round R *but* N_i only stores the packets received from N_h in the first β rounds.

Denote by $\Pr(\Omega_R^A|\beta)$ the probability that event Ω_R^A occurs conditioned on the event that β is the round at which node N_j receives X_h for the first time. It is straightforward that $\Pr(\Omega_R^A|\beta) = 1-(1-p_{ji})^\beta$. Similarly, we have that $\Pr(\Omega_R^B|\beta) = \left(1-(1-p_{ji})^{R-\beta}\right)\left(1-(1-p_{hi})^\beta + (1-p_{hi})^\beta\left(1-(1-p_{ji})^\beta\right)\right)$. Further, because events Ω_R^A and Ω_R^B are correlated, we have $\Pr(\Omega_R^A \cap \Omega_R^B) = \Pr(\Omega_R^B|\Omega_R^A)\Pr(\Omega_R^A) = \left(1-(1-p_{ji})^{R-\beta}\right)\left(1-(1-p_{ji})^\beta\right)$. Finally, it is evident that the probability $\Pr(\Omega_R|\beta)$, defined as:

$$\begin{aligned}\Pr(\Omega_R|\beta) &\triangleq \Pr(\Omega_R^A \cup \Omega_R^B|\beta) \quad (12)\\ &= \Pr(\Omega_R^A|\beta) + \Pr(\Omega_R^B|\beta) - \Pr(\Omega_R^A \cap \Omega_R^B|\beta),\end{aligned}$$

provides a lower bound on the probability that N_i receives and decodes X_j by round R via the first process.

Regarding the second process, denote by Ψ_R the event that N_i receives X_j via the second process by round R *but* N_i only receives X_d from N_d when N_d broadcasts its original packet. Then, the probability that event Ψ_R occurs conditioned on the event that N_j receives X_h for the first time at round β is:

$$\begin{aligned}\Pr(\Psi_R|\beta) &= \sum_{\alpha=1}^{\beta}(1-(1-p_{di})^\alpha)\left(1-(1-p_{di})^{R-\alpha}\right)f_{jd}(\alpha) \quad (13)\\ &= \sum_{\alpha=1}^{\beta}(1-(1-p_{di})^\alpha)\left(1-(1-p_{di})^{R-\alpha}\right)\left(\left(1-p_{jd}\right)^{\alpha-1}p_{jd}\right),\end{aligned}$$

Therefore, the probability that node N_i receives X_j in the R^{th} round satisfies:

$$\begin{aligned}F_{ji}(R) &\geq \sum_{\beta=1}^{R}\phi_{hj}(\beta)\Pr(\Omega_R \cup \Psi_R|\beta) \quad (14)\\ &= \sum_{\beta=1}^{R}\phi_{hj}(\beta)\left(\Pr(\Omega_R|\beta) + \Pr(\Psi_R|\beta) - \Pr(\Omega_R|\beta)\Pr(\Psi_R|\beta)\right),\end{aligned}$$

where $\phi_{hj}(\beta)$ is the probability that N_j receives X_h from N_h at round β for the first time, which satisfies:

$$\phi_{hj}(\beta) \geq (1-p_{hj})^{\beta-1}p_{hj}(1-F_{gh}(\beta)) \triangleq f_{hj}^L(\beta), \quad (15)$$

■

VI. Numerical results and discussion

In this section, simulations are conducted to validate our theoretical analysis. Moreover, the benefits in reliability of neighbor network coding over non-coded networks are shown, followed by discussions about the relation between the selection of coding neighbors and network reliability.

The probabilistic connectivity matrices indicating channel conditions can be arbitrary. In this section, the entries are chosen randomly to generate numerical results. The matrix used in each figure is given in the respective caption.

The reliability of networks with arbitrary number of nodes at arbitrary round can be calculated using Eq. (6). In Fig. 1, the theoretical results for networks with 3, 4 and 5 nodes. The coding scheme is that N_k chooses $N_{(k+1 \mod n)}$ as coding neighbor. It shows that the theoretical results match with the simulation results well, which in turn validates theoretical analysis.

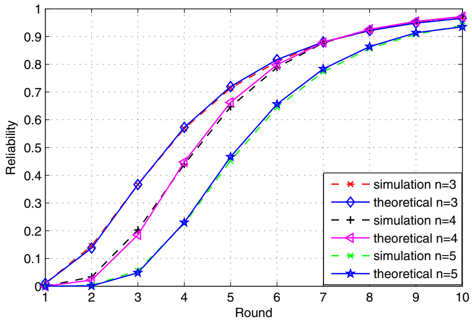

Fig. 1. Simulation and theoretical results of the reliability of networks when $n = 3, 4, 5$, where the probabilistic connectivity matrix is [1 0.3 0.6 0.5 0.4; 0.4 1 0.5 0.7 0.3; 0.7 0.4 1 0.3 0.5; 0.3 0.6 0.4 1 0.6; 0.6 0.5 0.3 0.4 1].

In order to examine the reliability benefits of the proposed neighbor network coding, the coded networks with different coding neighbors which give the best and the worst reliability are plotted together with the corresponding non-coded networks, as shown in Fig. 2. It shows that the coded networks have better reliability than the non-coded networks in every case, and the reliability gain can be considerable in some scenarios. For example, in the network of four nodes, the neighbor network coding brings reliability gain of more than 200 percent over the non-coded network at round $R = 10$.

Additionally, the selection of neighbors affects the network reliability. Based on numerous simulations, it is conjectured that if every node selects the node to which the connection probability is the lowest as coding neighbor, the reliability gain can be maximized.

Lastly, the bounds on the probability that X_1 is received by N_3, given by Theorem 1 and Theorem 2, are shown in Fig. 3. The coding scheme is the same as that in Table I. It can be seen that the bounds are valid. Moreover, the bounds can be further improved and be used to characterize the reliability gain and further to facilitate the proof of the aforementioned conjecture on the optimal neighbor selection rule, which is not a trivial task hence left as future work.

VII. Conclusion and future work

In this paper, a neighbor network coding scheme is proposed for cooperative broadcasting. Network reliability is

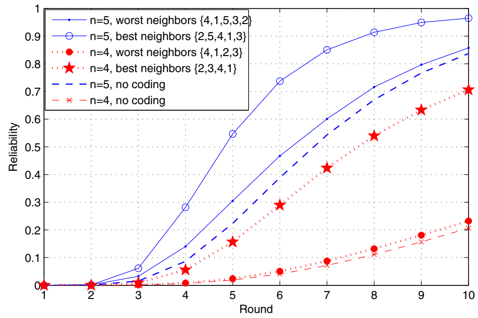

Fig. 2. Networks reliability on different neighbor selections, where probabilistic connectivity matrices for $n = 4$ and 5 are [1 0.1 0.5 0.4; 0.6 1 0.2 0.6; 0.7 0.3 1 0.1; 0.1 0.3 0.2 1] and [1 0.3 0.6 0.5 0.4; 0.4 1 0.5 0.7 0.3; 0.7 0.4 1 0.3 0.5; 0.3 0.6 0.4 1 0.6; 0.6 0.5 0.3 0.4 1] respectively.

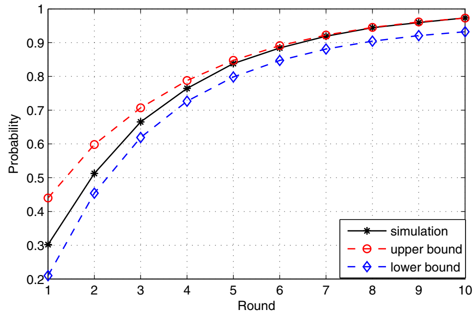

Fig. 3. Bounds on the probability that N_3 receives X_1 where $n = 3$ and the probabilistic connectivity matrix is [1 0.2 0.3; 0.4 1 0.1; 0.7 0.4 1].

investigated analytically and it has been shown that the proposed neighbor coding scheme can improve network reliability significantly. We also provide bounds on the reliability of the network applying the proposed coding scheme. In the future, the framework of analyzing network reliability established can be applied to study the reliability of networks applying different coding schemes. Moreover, it is important to develop a theoretical proof for the optimal coding scheme that maximizes the reliability of a given network.

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