A Tight Upper Bound for Heterogeneous ON-OFF Sources

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Abstract—**ON-OFF traffic source models are extensively used in Connection Admission Control (CAC) schemes because of their simplicity and their ability to model real traffic sources. However, the calculation of probability mass function of heterogeneous ON-OFF sources entails convolution operation which cannot be carried out in real time. Based on bufferless fluid flow model this paper proposes the cell loss rate function (clrf) and uses it for studying cell loss in bufferless systems. The properties of heterogeneous ON-OFF sources in bufferless systems are extensively discussed using clrf. A tight cell loss upper bound for heterogeneous ON-OFF sources is proposed which results in great computational savings. The upper bound is applied in the CAC scheme. Simulation studies are presented which indicate that the upper bound is tight and the CAC scheme has very good performance.**

I. Introduction

ON-OFF traffic source models are widely used in CAC schemes [1], [2], [3], [4] for their simplicity. They have been successfully used to characterize the ON/OFF nature of an individual source or source element, like packetized voice and video [3], [5].

Many current modeling techniques model each traffic source or source element by an ON-OFF source. These techniques cannot be applied to high speed networks, simply because of the exploded input state space when a large number of diverse sources are multiplexed on each link. Efforts have been made to reduce the input state space. Rasmussen et al. [4] propose an upper bound for heterogeneous ON-OFF sources based on fluid flow model. They conjecture that for N heterogeneous ON-OFF sources which have peak cell rates (pcr) $pcr_1, \cdots, pcr_N$, and, the sum of their mean cell rates (mcr) is S, then the case of N homogeneous ON-OFF sources with pcr $pcr = max\{pcr_1, \cdots, pcr_N\}$ and activity parameter $p = S/(N \times pcr)$ will constitute its upper bound in terms of cell loss. His approach results in great computational savings. However, if sources with large bandwidth demands and sources with small bandwidth demands are multiplexed together, the upper bound will be far from the actual cell loss. Hwang et al. [6] propose a method of input state space reduction. Their study shows that queuing performance is mainly determined by low frequency components of traffic sources. In their approach, 2-state Markov Chains are built to statistically match with the power spectrum function and the traffic distribution of the aggregate traffic. Their approach may result in decrease in input state space but is difficult to be implemented in real time.

In this paper, based on bufferless fluid flow model (bffm) we first propose cell loss rate function (clrf) for estimating cell loss. Then, the properties of heterogeneous ON-OFF sources in bufferless systems are studied using the clrf. The conjecture proposed by Rasmussen et al. [4] is proved. Furthermore we propose a tight cell loss upper bound for heterogeneous ON-OFF sources and apply the upper bound to the CAC scheme. The state space in our scheme is greatly reduced which makes our CAC scheme suit real time requirements of ATM network admission control.

II. Tight Cell Loss Upper Bound for Heterogeneous ON-OFF Sources

An ON-OFF source generates cells at peak cell rate denoted by pcr in active periods. In idle periods no cells are generated. Let mcr denote the mcr of the ON-OFF source. The activity parameter of an ON-OFF source is defined as the ratio of mcr to pcr $p \triangleq mcr/pcr$. Then the probability that an ON-OFF source is active or idle is given by p or $1-p$ respectively.

Assume there are N independent ON-OFF sources on the link, and that ON-OFF source i has pcr pcr_i and mcr mcr_i. And its activity parameter is $p_i = mcr_i/pcr_i$. Denote the probability mass function (pmf) of source i as $f_{1,(pcr_i)}^{(p_i)}$, then the pmf of the aggregation of N ON-OFF sources can be expressed as:

$$f_{N,(pcr_1,\cdots,pcr_N)}^{(p_1,\cdots,p_N)}(x) = f_{1,(pcr_1)}^{(p_1)} * \cdots * f_{1,(pcr_N)}^{(p_N)}(x)$$

where $*$ denotes convolution operation. In this paper we use subscript n, subscript (pcr) and superscript (p) to denote the number of ON-OFF sources, pcr of sources and their activity parameters respectively when we need to accentuate the dependence of a function on these parameters.

Under the bffm, cell loss due to overflow occurs if and only if the sum of the pcr of all active connections denoted by R exceeds the link capacity C. Define the function $F(m)$ as follows:

$$F(m) \triangleq E[(R-m)^+] \triangleq \sum_x (x-m)^+ f(x) \qquad (1)$$

where $(\bullet)^+$ denotes $max\{0, \bullet\}$. We call $F(m)$ the clrf of $f(x)$. The clrf has many attractive features which facilitates our analysis of cell loss in bffm. For example, $F(C)$ denotes the cell loss rate of the traffic with pmf $f(x)$. From the definition of clrf we can easily derive that:

$$F(0) = \rho = \sum_i mcr_i \qquad (2)$$

$$F(m) = F(0) - m \quad \text{for} \quad m < 0 \qquad (3)$$

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where ρ denotes the traffic load and mcr_i denotes the mcr of connection i on the link. The cell loss ratio (clr) can be calculated as:

$$clr = \frac{F(C)}{\rho} = \frac{F(C)}{F(0)} \tag{4}$$

We can derive several properties of clrf.

Property 1: If $f(x)$ and $g(x)$ denote the pmf of independent traffic sources X_1 and X_2 respectively then the clrf of $f * g(x)$ equals $F * g(x)$.

Proof: Construct a function $h(x)$ such that:

$$h(x) = \begin{cases} -x & x < 0 \\ 0 & x \geq 0 \end{cases}$$

It can be shown that: $F(m) = h * f(m)$.

Thus the clrf $FG(m)$ of $f(x) * g(x)$ is:

$$\begin{aligned} FG(m) &= h * (f * g(m)) \\ &= (h * f(m)) * g(m) = F * g(m) \end{aligned}$$

■

Property 2: Let $f(x)$, $g(x)$ and $q(x)$ denote the pmf of independent traffic sources X_1, X_2 and X_3 respectively. Denote the clrf of $f(x)$, $g(x)$, $f * g(x)$ and $g * q(x)$ by $F(m)$, $G(m)$, $FG(m)$ and $GQ(m)$ respectively. If for $F(m)$ and $G(m)$ we have $F(m) \geq G(m)$ for any m, then $FQ(m) \geq GQ(m)$ for any m.

Proof: Note that clrf is always non-negative. The proof of this property is straightforward. ■

The two properties enable us to decompose the complex analysis of the aggregation of several traffic sources into the simpler analysis of individual sources. Note that the two properties are not limited to ON-OFF sources.

Theorem 1: Let $X_1, \cdots, X_N$ be N independent Bernoulli sources[1] with activity parameters $p_1, \cdots, p_N$ respectively. Their activity parameters are subject to $p_1 + \cdots + p_N = P$. Then in the bffm, maximum cell loss will occur when $p_1 = \cdots = p_N$.[2]

Proof: It suffices to prove that the clrf of N homogeneous Bernoulli sources with activity parameter $p = P/N$, is greater than or equal to the clrf of the heterogeneous sources in the theorem for any m and N.

1. First let us consider the case where $N = 2$. We need to prove that the clrf of two heterogeneous Bernoulli sources with activity parameter p_1 and p_2 and the same peak cell rate pcr is less than or equal to the clrf of two homogeneous Bernoulli sources with activity parameter $p = (p_1 + p_2)/2$ and peak cell rate pcr. Their pmf are:

$$f_2^{(p_1,p_2)}(x) = \begin{cases} p_1 p_2 & x = 2pcr \\ (1-p_1)p_2 + (1-p_2)p_1 & x = pcr \\ (1-p_1)(1-p_2) & x = 0 \\ 0 & else \end{cases}$$

$$f_2^{(p)}(x) = \begin{cases} p^2 & x = 2pcr \\ 2p(1-p) & x = pcr \\ (1-p)^2 & x = 0 \\ 0 & else \end{cases}$$

Using the upper case to denote their clrf, we have:

- when $m < 0$,

$$F_2^{(p_1,p_2)}(m) = F_2^{(p)}(m) \quad = pcr \times (p_1 + p_2) - m$$

- when $0 \leq m < pcr$,

$$F_2^{(p)}(m) - F_2^{(p_1,p_2)}(m) = 3m \left[\frac{(p_1+p_2)^2}{4} - p_1 p_2 \right] \geq 0$$

- when $pcr \leq m < 2pcr$,

$$\begin{aligned} & F_2^{(p)}(m) - F_2^{(p_1,p_2)}(m) \\ = & \quad (2pcr - m) \left[\frac{(p_1+p_2)^2}{4} - p_1 p_2 \right] \geq 0 \end{aligned}$$

- when $2pcr \leq m$,

$$F_2^{(p)}(m) = F_2^{(p_1,p_2)}(m) = 0$$

From the above discussion, we have:

$$F_2^{(p)}(m) \geq F_2^{(p_1,p_2)}(m) \quad \text{for any } m$$

2. Suppose $F_N^{(p)}(m) \geqslant F_N^{(p_1,\cdots,p_N)}(m)$ holds for the case when $N = n$, i.e.

$$F_n^{(p)}(m) \geq F_n^{(p_1,\cdots,p_n)}(m) \quad \text{for any } m$$

where $p = (p_1 + \cdots + p_n)/n$. Let us consider the case when $N = n + 1$. We have:

$$\begin{aligned} & F_{n+1}^{(p_1,\cdots,p_{n+1})}(m) \\ = & \; F_n^{(p_1,\cdots,p_n)} * f_1^{(p_{n+1})}(m) \\ \leqslant & \; F_n^{(\frac{p_1+\cdots+p_n}{n})} * f_1^{(p_{n+1})}(m) \\ = & \left[F_{n-1}^{(\frac{p_1+\cdots+p_n}{n})} * f_1^{(\frac{p_1+\cdots+p_n}{n})}(m) \right] * f_1^{(p_{n+1})}(m) \\ = & \left[F_{n-1}^{(\frac{p_1+\cdots+p_n}{n})} * f_1^{(p_{n+1})}(m) \right] * f_1^{(\frac{p_1+\cdots+p_n}{n})}(m) \\ \leqslant & \; F_n^{(\frac{(n-1)\frac{p_1+\cdots+p_n}{n}+p_{n+1}}{n})} * f_1^{(\frac{p_1+\cdots+p_n}{n})}(m) \\ & \cdots\cdots\cdots\cdots\cdots\cdots \end{aligned}$$

[1] We call ON-OFF sources with the same peak cell rate as Bernoulli sources.
[2] This theorem was first proposed as a conjecture by Rasmussen et al. [4] and numerically validated in many literatures including [1].

Define a sequence a_k so that the above procedure can be expressed as:

$$\begin{aligned} F_{n+1}^{(p_1,\cdots,p_{n+1})}(m) &\leqslant F_n^{(a_1)} * f_1^{(a_0)}(m) \\ &\cdots\cdots\cdots\cdots \\ &\leqslant F_n^{(a_k)} * f_1^{(a_{k-1})}(m) \\ &\cdots\cdots\cdots\cdots \end{aligned}$$

We can derive that:

$$a_k = \frac{(n-1)a_{k-1}+a_{k-2}}{n}$$
$$a_0 = p_{n+1} \quad \text{and} \quad a_1 = \frac{p_1+\cdots+p_n}{n}$$

Solving for a_k, when $k \geqslant 2$, we get:

$$\begin{aligned} a_k &= \frac{p_1+\cdots+p_n}{n} \\ &-\frac{1-\left(-\frac{1}{n}\right)^{k-1}}{n+1}\left(\frac{p_1+\cdots+p_n}{n}-p_{n+1}\right) \end{aligned}$$

Thus we have:

$$\lim_{k\to\infty} a_k = \lim_{k\to\infty} a_{k-1} = \frac{p_1+\cdots+p_{n+1}}{n+1}$$

This means that when the above process goes on and on, the activity parameters of the n homogeneous Bernoulli sources and the single Bernoulli source in the above equations will converge to $\frac{p_1+\cdots+p_{n+1}}{n+1}$. So we conclude that:

$$\begin{aligned} & F_{n+1}^{(p_1,\cdots,p_{n+1})}(m) \\ \leqslant\ & F_n^{(\frac{p_1+\cdots+p_{n+1}}{n+1})} * f_1^{(\frac{p_1+\cdots+p_{n+1}}{n+1})}(m) \\ =\ & F_{n+1}^{(\frac{p_1+\cdots+p_{n+1}}{n+1})}(m) \end{aligned}$$

Hence from the supposition that $F_N^{(p)}(m) \geqslant F_N^{(p_1,\cdots,p_N)}(m)$ holds for the case when $N=n$, we derive that the inequality holds for the case when $N=n+1$.
Combining step 1 and step 2 we conclude that:

$$F_N^{(p)}(m) \geqslant F_N^{(p_1,\cdots,p_N)}(m) \quad \text{for any } m$$

holds for any N. Thus the theorem is proved. ∎

The significance of the theorem is pointing out that the cell loss of heterogeneous ON-OFF sources with the same pcr is upper bounded by its corresponding homogeneous ON-OFF sources. In real networks, many traffic sources of the same type have the same pcr, however because of their specific application environments, they have different mcr. Theorem 1 is very useful for analyzing this kind of traffic sources.

Theorem 2: Let X_1 and X_2 be two independent ON-OFF sources with the same mcr denoted by mcr. X_1 and X_2 have pcr pcr_1, pcr_2 respectively. Let X_3 be another independent traffic source (not necessarily an ON-OFF source). If $pcr_1 \leqslant pcr_2$, then in the bffm, the cell loss generated by the aggregation of X_2 and X_3 is more than or equal to the cell loss generated by the aggregation of X_1 and X_3.

Proof: The activity parameters of X_1 and X_2 are $\frac{mcr}{pcr_1}$ and $\frac{mcr}{pcr_2}$ respectively. We shall calculate the clrf of X_1 and X_2:

$$F_{1,(pcr_1)}^{(\frac{mcr}{pcr_1})}(m) = \begin{cases} mcr-m & m \leqslant 0 \\ mcr\left(1-\frac{m}{pcr_1}\right) & 0<m\leqslant pcr_1 \\ 0 & m>pcr_1 \end{cases}$$

$$F_{1,(pcr_2)}^{(\frac{mcr}{pcr_2})}(m) = \begin{cases} mcr-m & m \leqslant 0 \\ mcr\left(1-\frac{m}{pcr_2}\right) & 0<m\leqslant pcr_2 \\ 0 & m>pcr_2 \end{cases}$$

It is easy to derive that for any m:

$$F_{1,(pcr_1)}^{(\frac{mcr}{pcr_1})}(m) \leqslant F_{1,(pcr_2)}^{(\frac{mcr}{pcr_2})}(m) \tag{5}$$

∎

Theorem 2 was used by Lee et al. in their CAC scheme [2].

Theorem 3: Let X_1 be N independent homogeneous ON-OFF sources where each ON-OFF source has pcr pcr and mcr mcr. The sum of their mcr is denoted by S where $S = N \times mcr$. Let X_2 be another independent traffic source (not necessarily an ON-OFF source) and u be any positive integer. Define X_3 to be $\lceil \frac{N}{u} \rceil$ independent homogeneous ON-OFF sources, each source has pcr $u \times pcr$, and the sum of their mcr equals to S. Then in bufferless systems the cell loss resulting from the aggregation of X_1 and X_2 is less than or equal to the cell loss resulting from the aggregation of X_3 and X_2.

Proof: Using the properties of clrf, we only need to prove that the clrf of X_1 is less than or equal to the clrf of X_3 for any value of m and N.
1. Let us first consider the simple case when $N = u$. For this case we need to prove that:

$$F_{N,(pcr)}^{(\frac{mcr}{pcr})}(m) \leqslant F_{1,(N\times pcr)}^{(\frac{mcr}{pcr})}(m) \tag{6}$$

We can easily show that:

$$F_{1,(Npcr)}^{(\frac{mcr}{pcr})}(m) = \begin{cases} Nmcr-m & m \leqslant 0 \\ Nmcr\left(1-\frac{m}{Npcr}\right) & 0<m\leqslant Npcr \\ 0 & m>Npcr \end{cases}$$

Let us now consider two cases:
(a) $N = u = 1$: It is apparent that inequality 6 holds for any value of m because X_1 and X_3 are the same traffic sources.
(b) $N = u = k$: We shall prove that if inequality 6 holds for $N = u = k$, then it will also hold for $N = u = k+1$. We have:

$$\begin{aligned} F_{k+1,(pcr)}^{(\frac{mcr}{pcr})}(m) &= F_{k,(pcr)}^{(\frac{mcr}{pcr})} * f_{1,(pcr)}^{(\frac{mcr}{pcr})}(m) \\ &\leqslant F_{1,(k\times pcr)}^{(\frac{mcr}{pcr})} * f_{1,(pcr)}^{(\frac{mcr}{pcr})}(m) \end{aligned}$$

Calculating the convolution $F_{1,(kpcr)}^{(\frac{mcr}{pcr})}(m) * f_{1,(pcr)}^{(\frac{mcr}{pcr})}(m)$ and comparing it with $F_{1,((k+1)pcr)}^{(\frac{mcr}{pcr})}(m)$ we get:

$$F_{k+1,(pcr)}^{(\frac{mcr}{pcr})}(m) \leqslant F_{1,(kpcr)}^{(\frac{mcr}{pcr})} * f_{1,(pcr)}^{(\frac{mcr}{pcr})}(m) \leqslant F_{1,((k+1)pcr)}^{(\frac{mcr}{pcr})}(m)$$

Combing 1a and 1b above we conclude that the theorem holds for the case when $N = u$.

2. Let us now consider the case when $N < u$. We have:

$$\begin{aligned} F_{N,(pcr)}^{(\frac{mcr}{pcr})}(m) &\leqslant F_{1,(Npcr)}^{(\frac{mcr}{pcr})}(m) & (7) \\ &\leqslant F_{1,(u\times pcr)}^{(\frac{Nmcr}{u\times pcr})}(m) & (8) \end{aligned}$$

Inequality 5 was used in deriving 7. Inequality 8 comes from inequality 5 which was shown in the proof of theorem 2.

3. For the case when $N > u$ and N is an integer multiple of u, we have:

$$\begin{aligned} & F_{N,(pcr)}^{(\frac{mcr}{pcr})}(m) \\ = \; & F_{u,(pcr)}^{(\frac{mcr}{pcr})} * \overbrace{f_{u,(pcr)}^{(\frac{mcr}{pcr})} * \cdots * f_{u,(pcr)}^{(\frac{mcr}{pcr})}}^{\frac{N}{u}-1}(m) \\ \leqslant \; & F_{1,(u\times pcr)}^{(\frac{mcr}{pcr})} * \overbrace{f_{u,(pcr)}^{(\frac{mcr}{pcr})} * \cdots * f_{u,(pcr)}^{(\frac{mcr}{pcr})}}^{\frac{N}{u}-1}(m) \\ \leqslant \; & \cdots\cdots\cdots\cdots\cdots\cdots \\ \leqslant \; & F_{1,(u\times pcr)}^{(\frac{mcr}{pcr})} * \overbrace{f_{1,(u\times pcr)}^{(\frac{mcr}{pcr})} * \cdots * f_{1,(u\times pcr)}^{(\frac{mcr}{pcr})}}^{\frac{N}{u}-1}(m) \\ = \; & F_{\frac{N}{u},(u\times pcr)}^{(\frac{mcr}{pcr})}(m) \end{aligned}$$

In this derivation we have used inequality 6, as well as property 2 of the clrf were used in the derivation.

4. Now let us consider the case when $N > u$ and N is not an integer multiple of u:

$$\begin{aligned} & F_{N,(pcr)}^{(\frac{mcr}{pcr})}(m) \\ = \; & F_{u\lfloor\frac{N}{u}\rfloor,(pcr)}^{(\frac{mcr}{pcr})} * f_{N-u\lfloor\frac{N}{u}\rfloor,(pcr)}^{(\frac{mcr}{pcr})}(m) & (9) \\ \leqslant \; & F_{\lfloor\frac{N}{u}\rfloor,(u\times pcr)}^{(\frac{mcr}{pcr})} * f_{N-u\lfloor\frac{N}{u}\rfloor,(pcr)}^{(\frac{mcr}{pcr})}(m) & (10) \\ = \; & F_{N-u\lfloor\frac{N}{u}\rfloor,(pcr)}^{(\frac{mcr}{pcr})} * f_{\lfloor\frac{N}{u}\rfloor,(u\times pcr)}^{(\frac{mcr}{pcr})}(m) & (11) \\ \leqslant \; & F_{1,((N-u\lfloor\frac{N}{u}\rfloor)pcr)}^{(\frac{mcr}{pcr})} * f_{\lfloor\frac{N}{u}\rfloor,(u\times pcr)}^{(\frac{mcr}{pcr})}(m) & (12) \\ \leqslant \; & F_{1,(u\times pcr)}^{(\frac{(N-u\lfloor\frac{N}{u}\rfloor)mcr}{u\times pcr})} * f_{\lfloor\frac{N}{u}\rfloor,(u\times pcr)}^{(\frac{mcr}{pcr})}(m) & (13) \\ \leqslant \; & F_{\lceil\frac{N}{u}\rceil,(u\times pcr)}^{(\frac{N\times mcr}{\lceil\frac{N}{u}\rceil u\times pcr})}(m) & (14) \end{aligned}$$

In the above equations the symbol $\lfloor \bullet \rfloor$ denotes the largest integer less than or equal to $\bullet$. When $N > u$ and N is not an integer multiple of u, we have $\lceil \frac{N}{u} \rceil - \lfloor \frac{N}{u} \rfloor = 1$. This property is used in the derivation of inequalities 12 to 13 as well as theorem 2; in the derivation of inequalities 9 to 10, the conclusion proved for the case when N is integer multiple of u and property 2 are used; in the derivation of inequalities 11 to 12, the conclusion derived for the case when $N = u$ and property 2 are used; in the derivation of inequalities 12 to 13, theorem 2 is used; in the derivation of inequalities 13 to 14, theorem 1 and property 1 are used.

■

The three theorems provide us with a tight cell loss upper bound for ON-OFF sources.

As an example let us consider a scenario where X_1 represents N independent homogeneous ON-OFF sources, each source has pcr pcr_1 and mcr mcr_1. Let $f_{N,(pcr_1)}^{(\frac{mcr_1}{pcr_1})}(x)$ denotes the pmf of X_1. The sum of the mcr is given by $S_1 = N \times mcr_1$ and the sum of the pcr by $P_1 = N \times pcr_1$. Furthermore, let X_2 represent M independent homogeneous ON-OFF sources with pcr pcr_2 and mcr mcr_2. We denote the pmf of X_2 by $f_{M,(pcr_2)}^{(\frac{mcr_2}{pcr_2})}(x)$, the sum of the mcr by $S_2 = M \times mcr_2$ and the sum of pcr by $P_2 = M \times pcr_2$. If we assume $\frac{pcr_2}{pcr_1} = u$ and u is any positive integer, then when X_1 and X_2 are multiplexed together, their clrf is evaluated as:

$$\begin{aligned} F(m) &= F_{N,(pcr_1)}^{(\frac{mcr_1}{pcr_1})} * f_{M,(pcr_2)}^{(\frac{mcr_2}{pcr_2})}(m) \\ &\leqslant F_{\lceil\frac{N}{u}\rceil,(u\times pcr_1)}^{(\frac{N\times mcr_1}{\lceil\frac{N}{u}\rceil u\times pcr_1})} * f_{M,(pcr_2)}^{(\frac{mcr_2}{pcr_2})}(m) \\ &= F_{\lceil\frac{Npcr_1}{pcr_2}\rceil,(pcr_2)}^{(\frac{S_1}{\lceil\frac{Npcr_1}{pcr_2}\rceil pcr_2})} * f_{M,(pcr_2)}^{(\frac{mcr_2}{pcr_2})}(m) \\ &= F_{\lceil\frac{P_1}{pcr_2}\rceil,(pcr_2)}^{(\frac{S_1}{\lceil\frac{P_1}{pcr_2}\rceil pcr_2})} * f_{M,(pcr_2)}^{(\frac{mcr_2}{pcr_2})}(m) \\ &\leqslant F_{\lceil\frac{P_1}{pcr_2}\rceil+M,(pcr_2)}^{(\frac{S_1+S_2}{(\lceil\frac{P_1}{pcr_2}\rceil+M)pcr_2})}(m) = F_{\lceil\frac{P_1+P_2}{pcr_2}\rceil,(pcr_2)}^{(\frac{S_1+S_2}{(\lceil\frac{P_1+P_2}{pcr_2}\rceil)pcr_2})}(m) \end{aligned}$$

Thus using the theorems we show that $\left\lceil \frac{P_1+P_2}{pcr_2} \right\rceil$ homogeneous ON-OFF sources with pcr $pcr_2 \geqslant pcr_1$ and the sum of mcr equals $S_1 + S_2$ is the upper bound, in terms of cell loss, of the aggregation of X_1 and X_2. In the above process we limit u to be a positive integer because this is the limit that was imposed in theorem 3. However, we believe that theorem 3 can be extended to any value of $u \geqslant 1$. Hence from the above argument, we propose the following conjecture:

Conjecture 1: Let $X_1, \cdots, X_N$ be N independent ON-OFF sources with peak cell rate $pcr_1, \cdots, pcr_N$ and mcr $mcr_1, \cdots, mcr_N$ respectively. Let S denote the sum of their mcr: $S = mcr_1 + \cdots + mcr_N$, P denote the sum of their pcr: $P = pcr_1 + \cdots + pcr_N$ and pcr denote any number satisfying $pcr \geqslant max\{pcr_1, \cdots pcr_N\}$. Then in a bufferless system, the cell loss resulting from $\left\lceil \frac{P}{pcr} \right\rceil$ independent homo-

TABLE I

PARAMETERS OF THE THREE TRAFFIC TYPES

	$\lambda(s^{-1})$	$pcr(kb/s)$	$activity$	$L(cells)$
type 1	10	100	0.1	100
type 2	50	50	0.2	50
type 3	100	10	0.5	20

geneous ON-OFF sources, where each source has pcr *pcr* and mcr $S \Big/ \left\lceil \frac{P}{pcr} \right\rceil$, is greater than or equal to the cell loss resulting from the aggregation of $X_1, \cdots, X_N$.

In this paper, we proved the conjecture based on which Rasmussen et al. [4] propose their upper bound. Furthermore we showed that the cell loss resulting from the aggregation of $X_1, \cdots, X_N$ is upper-bounded by that of $\left\lceil \frac{P}{pcr} \right\rceil$ independent homogeneous ON-OFF source, where each source has peak cell rate *pcr* and mean cell rate $S \Big/ \left\lceil \frac{P}{pcr} \right\rceil$. It is not difficult, using the clrf, to prove that our upper bound is tighter than the upper bound proposed in [4].

The proposed upper bound can also be explained intuitively as follows: when substituting N independent heterogeneous ON-OFF sources with $\left\lceil \frac{P}{pcr} \right\rceil$ independent homogeneous ON-OFF sources, the sum of mcr does not change, therefore the traffic load remains the same. However, with the decrease in the number of multiplexed ON-OFF sources the aggregate traffic becomes more bursty. Therefore for the same utilization cell loss will increase.

III. CAC SCHEME AND SIMULATIONS

The proposed upper bound can be applied to either traffic descriptor-based CAC or measurement-based CAC. There are two traffic parameters needed for our CAC scheme, one is the sum of pcr and the other is the sum of mcr. We identify that in real networks it is reasonable to assume that the pcr can be tightly characterized by traffic sources. However it is very difficult, for all traffic sources to tightly characterize their mcr. Thus we take a hybrid approach where we derive the sum of pcr from traffic descriptors, and derive the sum of mcr from on-line measurement which is then the mean traffic rate of the network. The property of the clrf can also be utilized to simplify the calculations involved in our CAC scheme.

Extensive simulations have been carried out and one scenario is presented in this paper. In this scenario, three types of exponential ON-OFF sources are multiplexed onto a link with capacity $10Mb/s$. Each type of exponential ON-OFF sources have pcr *pcr*, an exponentially distributed burst length with mean L $cells$, an exponentially distributed call duration with mean 100 $seconds$, and an exponentially distributed call arrival rate with mean λ $calls/s$. The clr objective is set at 10^{-4}. Table I shows the parameters of the three types of ON-OFF sources.

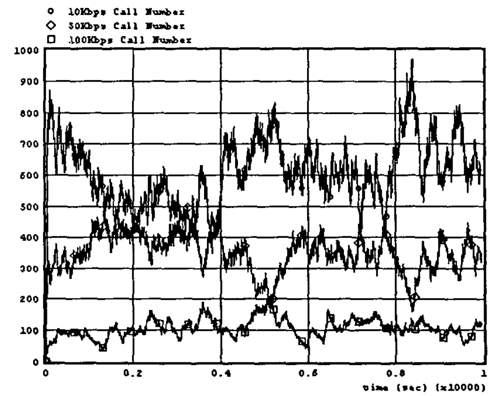

Fig. 1. The Number of the three type connections in the link

The simulation was run for 10,000 seconds. Fig. 1 shows the number of each connection type on the link. Our CAC scheme achieved an average utilization of 0.79 whilst maintaining robust QoS objective. All our simulations showed that the CAC scheme is able to achieve a high link utilization whilst maintaining robust QoS guarantees.

IV. CONCLUSION

In this paper we defined the cell loss rate function for studying cell loss in bufferless systems. Using the clrf we studied ON-OFF sources in bufferless systems. Several useful theorems were proposed and proved. We believe these theorems are very useful for studying ON-OFF sources in bufferless systems. A cell loss upper bound for heterogeneous ON-OFF sources in bufferless systems was proposed, and the upper bound was then applied in a CAC scheme using on-line measurements. Simulation studies were carried out which were indicative of very good performance of the CAC scheme. Our CAC scheme proved to be time efficient, robust and achieved high network utilization. We believe the proposed CAC scheme has many attractive features which makes it suitable for implementation in real ATM networks.

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