On the Maximum Throughput of A Single Chain Wireless Multi-Hop Path

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***Abstract*—In this paper, the maximum achievable throughput of a single chain wireless multi-hop path is investigated analytically. First, we prove that in an error free radio environment, a wireless multi-hop path (the number of hops $k \geq 3$), in which all nodes are optimally placed and all links have an identical normalized capacity of 1, has a maximum achievable throughput of $\frac{1}{3}$. Moreover, we show that this maximum throughput can only be achieved when all links of the path are separated into three maximal independent sets and each set of links become active alternatively for the same amount of time. Second, we extend our analysis to an erroneous radio environment, and prove that the maximum throughput of the k-hop path is determined by its bottleneck consecutive three-hop segment. That is, the maximum throughput of the k-hop path is the minimum of the maximum throughputs of all its consecutive three-hop segments. The findings in this paper lay guidelines for designing optimum scheduling algorithms.**

I. Introduction

Wireless multi-hop networks, e.g., ad-hoc and sensor networks, have attracted significant research interest recently. The capacity of wireless multi-hop networks is one of the most fundamental research issues in this area. A single chain wireless multi-hop path constitutes a basic element of wireless multi-hop networks. Investigating the capacity or the maximum achievable throughput of a single chain wireless multi-hop path has both theoretical and practical significance.

In this paper, we analyze the maximum achievable throughput of a single chain wireless multi-hop path in both an error free and an erroneous radio environment. Compared with most previous work in the area, whose outcomes are either validated using simulations or established using computing software (e.g., based on linear programming (LP) formulations) [1]–[6], this paper features a thorough analytical study of the problem. Therefore, this paper offers better insight understanding of the throughput performance of wireless multi-hop networks and the relationship among parameters determining the throughput, which is difficult to obtain by examining simulation results or the outcomes of computing software. For example, for a k-hop path ($k \geq 3$) under an error free radio environment, our analysis not only gives the maximum throughput, but also verifies that this maximum throughput can only be achieved by separating the links into three maximal independent sets, a concept which will be defined later, and scheduling each set of links to be active alternatively for the same amount of time. Also, our analysis shows that in an erroneous radio environment the maximum throughput of a k-hop path is determined by its bottleneck consecutive three-hop segment. These results are not obvious by examining simulation results or outcomes of computing software.

The rest of this paper is organized as follows: in Section II, related work is discussed; in Section III, the system model and assumptions used in our analysis is introduced; in Section IV, LP formulations and conflict graph [2] is briefly discussed; the maximum throughput of a single chain k-hop path in an error free environment is analyzed in Section V; in Section VI, earlier analysis in an error free environment is extended to an erroneous radio environment; finally, Section VII concludes this paper.

II. Related Work

Extensive work has been done in this area, In this section, we limit our discussion only to those closely related [1]–[6].

In [1], Li *et al.* assert that the maximum throughput of a single chain wireless multi-hop path is $\frac{1}{n}$ of the capacity of the constituent links where each link has an identical capacity, considering that n consecutive links can mutually interfere if they are simultaneously active. However, they did not prove this assertion and their IEEE 802.11 based simulation study also did not verify this assertion. In [2], Jain *et al.* use a conflict graph approach to analyze the interference between adjacent links in a wireless multi-hop path. With conflict graph, the maximum throughput of the multi-hop path is evaluated using LP approach. In [4]–[6], the authors also use a similar LP approach to analyze the throughput of wireless multi-hop networks. Moreover, in [4], [5], the authors consider power consumption issue in their LP formations. Therefore the outcome of their LP formations is the maximum throughput with the least power consumption. In [6], in addition to the throughput and power consumption, Lin *et al.* include the optimal data rate setting for each link in the proposed LP formation. Accordingly, the outcome of the LP formation in [6] is the maximum throughput with the least power consumption, as well as the optimal data rate setting. Unfortunately as more constraints are considered, as shown in [4]–[6], the computational complexity of the LP formation

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increases significantly. In [3], some computation optimization approach is introduced to reduce the computational complexity of LP approach.

All LP approaches in [2]–[5] are solved using computing software. Using computing software to solve LP problem may appear convenient, especially when dealing with complex LP formulations. However, it is quite difficult to obtain any insight by examining the outcomes of computing software and/or the simulation results. This motivates us to study the throughput of the single chain multi-hop wireless path using an analytic approach.

Finally, we want to point out that the scenario considered in this paper is distinctly different from some other previous work (see [7], [8] and references therein) on analyzing the asymptotic capacity of a 2D wireless network in which an infinite number of wireless nodes are distributed randomly or arbitrarily in a bounded region. Our focus is on obtaining an accurate end-to-end capacity for a finite number of wireless nodes.

III. System Model and Assumptions

In this paper, we analyze the maximum throughput of a single chain wireless k-hop path. A generic form of the multi-hop path being analyzed is shown in Fig. 1. Each node is numbered and the link between node j and node $j+1$ is denoted as L_j.

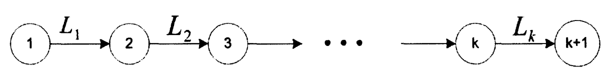


Fig. 1. A generic form of a single chain wireless k-hop path.

The following assumptions are used:

- Each node is equipped with a single transceiver and is optimally located, so that its omnidirectional transmission can only be heard by its adjacent neighbors. Consequently at any time there can only be one link active among three consecutive links.
- The transmission of an i^{th} node does not cause any interference at the $(i \pm 3)^{th}$ node. This assumption is only necessary when analyzing the throughput in an error-free environment and is no longer required when we consider the throughput in an erroneous environment later.
- The traffic load at the first node of the multi-hop path is saturated. Each node along the multi-hop path has an infinite queue so that there is no packet loss due to traffic congestion and queue overflow.
- All links have an identical normalized capacity of 1. In an erroneous environment, each link has a different and independent successful transmission probability, denoted by P_i ($0 < P_i \leq 1$). P_i can also be considered as the effective capacity of each link. Thus our results in erroneous case can be readily extended to links with unequal capacities.
- The activity of each node is ideally scheduled such that there is no collision. This assumption allows us to establish the maximum achievable throughput of the path.

IV. LP Formulations and Conflict Graph

In this section, we introduce the fundamental principle of our analysis and define the symbols and terms used in the rest of the paper. First, a LP formulation for calculating the maximum throughput is proposed and this LP problem is solved analytically. Second, the conflict graph analysis is introduced. Finally, the proposed LP formulation is enhanced by using the conflict graph analysis.

A. A LP formulation for computing the maximum throughput

In this paper, we consider the end-to-end throughput of a k-hop path from time 0 to t as $t \to \infty$ and the throughput is defined as the time average network traffic transported during this time period. Accordingly, two symbols are defined:

t_j: the proportion of time that link L_j is active during [0, t].

T_j: the throughput of link L_j.

In an error free environment $T_j = t_j$ and in an erroneous environment $T_j = t_j P_j$.

To achieve the maximum throughput of a k-hop path, a LP formulation can be used, given by

$$\text{Maximize:} \quad T_k \tag{1}$$
$$\text{subject to:} \quad 0 \leq t_j \leq 1, j = 1, \ldots, k \tag{2}$$
$$T_j \geq T_{j+1}, j = 1, ..., k-1 \tag{3}$$
$$t_j + t_{j+1} + t_{j+2} \leq 1, j = 1, ..., k-2. \tag{4}$$

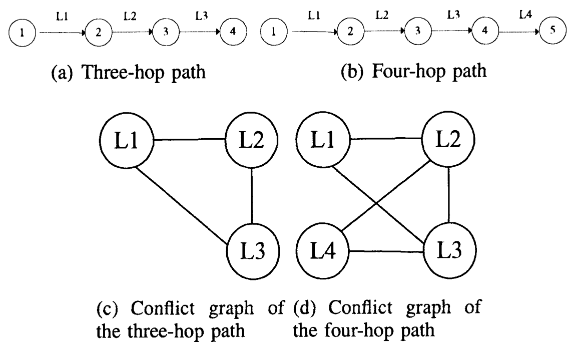
This maximization problem maximizes the end-to-end throughput of the k-hop path, which is equal to the throughput of the last hop. The first constraint represents that the proportion of time in which a link is active must be between 0 and 1. The second constraint indicates that the throughput of a link cannot exceed that of its previous link. The third constraint indicates that the proportions of the time in which any three consecutive links are active cannot exceed 1 because, as mentioned earlier, at any time only one of the three consecutive links can be active.

B. Conflict Graph

In this section we introduce the concept of conflict graph, which is used to improve the earlier problem formulation. A conflict graph consists of a series of vertices and edges. Each vertex represents a link. If two links in a multi-hop path can not be active at the same time due to interference, an edge will be drawn between the corresponding two vertices. Two examples of the conflict graph are drawn for a three-hop path and a four-hop path respectively, as shown in Fig. 2.

Using the conflict graph, all possible sets of links that can be active simultaneously without interfering each other can be found. Such a set of links is called an "independent set". A maximal independent set is an independent set that includes as many links as possible. Each independent set is represented by a distinct number and the links in the set. For example, $C_i : (L_j, L_k)$ represents the i^{th} independent set and this set includes two links, L_j and L_k.

As shown in Fig. 2(c), the conflict graph of a three-hop path is a complete graph. Accordingly, three independent sets (also



(a) Three-hop path (b) Four-hop path

(c) Conflict graph of the three-hop path (d) Conflict graph of the four-hop path

Fig. 2. Examples of conflict graph.

the maximal independent sets for the three-hop path) exist for the three-hop path: $C_1 : (L_1)$, $C_2 : (L_2)$, and $C_3 : (L_3)$.

Similarly, we may obtain all independent sets for the four-hop path: $C_1 : (L_1)$, $C_2 : (L_2)$, $C_3 : (L_3)$, $C_4 : (L_4)$, and $C_5 : (L_1, L_4)$. Among them, C_2, C_3, and C_5 are the maximal independent sets. It should be noted that C_1 and C_4 are not the maximal independent sets because they are sub-sets of C_5. It is obvious that three maximal independent sets exist for any k-hop path: $C_1 : (L_1, L_4, ..., L_{3i-2}, ...)$, $C_2 : (L_2, L_5, ..., L_{3i-1}, ...)$, and $C_3 : (L_3, L_6, ..., L_{3i}, ...)$.

C. An Enhanced LP formulation

Using the concept of the conflict graph, we can obtain an enhanced LP formulation of the problem.

First, all simultaneously active links must belong to the same independent set. At any time, at most one independent set is allowed to be active. Note that if two or more independent sets of links are active at the same time, the union of them must form a larger independent set. Thus they are considered as a single independent set.

A new symbol, x_i, is defined which represents the proportion of time that an independent set C_i is active. If C_i includes more than one link, all member links will be active simultaneously.

The value of t_j is related to x_i by

$$t_j = \sum_i \mathbf{1}_{ij} x_i, \tag{5}$$

where $\mathbf{1}_{ij}$ is an indicator function: $\mathbf{1}_{ij} = 1$ if $L_j \in C_i$; otherwise $\mathbf{1}_{ij} = 0$. Note that a link can belong to more than one independent sets. The value of T_j is related to x_i by

$$T_j = \sum_i \mathbf{1}_{ij} x_i P_j. \tag{6}$$

Finally, the LP formulation can be revised as

$$\begin{aligned}
\text{Maximize:} \quad & \textstyle\sum_i \mathbf{1}_{ik} x_i P_k & (7)\\
\text{subject to:} \quad & 0 \le x_i \le 1, i = 1, ... & (8)\\
& \textstyle\sum_i \mathbf{1}_{ij} x_i P_j \ge \sum_i \mathbf{1}_{ij+1} x_i P_{j+1}, j = 1, ..., k-1 & (9)\\
& \textstyle\sum_i x_i \le 1. & (10)
\end{aligned}$$

V. The Maximum Throughput of a Single Chain Wireless k-hop Path in an Error Free Radio Environment

Theorem 1: In an error free radio environment ($P_i = 1$), a single chain wireless k-hop ($k \ge 3$) path where all links have an identical capacity 1, can achieve a maximum throughput of $\frac{1}{3}$. The only way to achieve this maximum throughput is by dividing the links into three maximal independent sets and schedule each set of links to be active alternatively for the same amount of time.

Proof: A mathematical induction approach is used to prove the theorem. First, we prove that Theorem 1 is valid for a three-hop path; second, based on the assumption that the theorem is valid for a $k-1$-hop path, we show that it is also valid for a k-hop path. It is trivial to consider the throughput of a k-hop path with $k < 3$.

1) The conflict graph of a three-hop path is a complete graph. Each independent set of the three-hop path contains a single link only and they are also the maximal independent sets. Therefore, we have $T_1 = t_1 = x_1$, $T_2 = t_2 = x_2$, and $T_3 = t_3 = x_3$, respectively. The maximization of the throughput of the three-hop path can be formulated as the following:

$$\begin{aligned}
\text{Maximize:} \quad & x_3 & (11)\\
\text{subject to:} \quad & 0 \le x_i \le 1, i = 1, 2, 3, & (12)\\
& x_i \ge x_{i+1}, i = 1, 2, & (13)\\
& \textstyle\sum x_i \le 1, i = 1, 2, 3. & (14)
\end{aligned}$$

Rearranging the constraints in the above equations, we can readily obtain three new constraints:

$$\begin{aligned}
0 \le \quad x_2 + x_3 \quad & \le 1, & (15)\\
x_3 \quad & \le \frac{1}{2}(x_1 + x_2), & (16)\\
x_3 \quad & \le 1 - (x_1 + x_2). & (17)
\end{aligned}$$

These three new constraints, together with the constraint in (12), can be used to draw a figure to illustrate the relationship between x_3 and $x_1 + x_2$, as shown in Fig. 3.

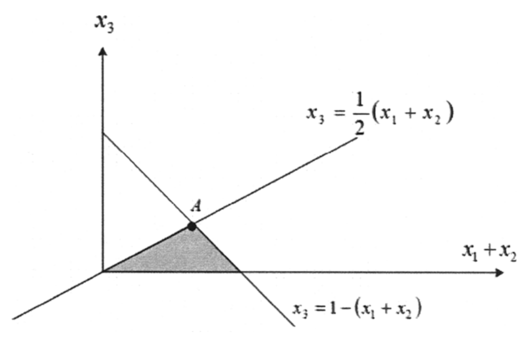


Fig. 3. The relationship between x_3 and $x_1 + x_2$.

In Fig. 3, the vertical axe represents the value of x_3, and the horizontal axe represents the value of $x_1 + x_2$. Two straight lines are drawn from two functions, $x_3 =$

$0.5(x_1+x_2)$ and $x_3 = 1-(x_1+x_2)$, respectively. Based on the aforementioned constraints, the possible values of x_3 and x_1+x_2 are limited in the shaded triangular area. The only possible values of x_1, x_2 and x_3 which both yield the maximum x_3 and satisfy the constraints are $x_1 = x_2 = x_3 = \frac{1}{3}$. Thus Theorem 1 is proved for the three-hop path.

2) Given that Theorem 1 is valid for a $k-1$-hop path, we shall now show that it is also valid for a k-hop path. Denote the three maximal independent link sets of the $j-1$-hop path by C_1, C_2 and C_3. Without losing generality, we consider that the final link L_{k-1} is within C_3. Therefore, when a new node (link), i.e., the $k+1^{th}$ node (k^{th} link), is added to the $k-1$-hop path, the new link L_k together with C_1 forms a maximal independent set of the k-hop path. It is trivial to show that the three maximal independent sets of the k-hop path are: $\{C_1, L_k\}$, C_2 and C_3.
   Obviously, the maximum throughput of the k-hop path is less than or equal to the maximum throughput of the $k-1$-hop path, which is equal to $1/3$. Therefore, if we can show that a throughput of $\frac{1}{3}$ is achievable for the k-hop path, the maximum achievable throughput of the k-hop path is also $\frac{1}{3}$. For the k-hop path, an end-to-end throughput of $\frac{1}{3}$ can be achieved by scheduling L_k to be active at the same time of C_1; and scheduling the three maximal independent sets of the k-hop path: $\{C_1, L_k\}$, C_2 and C_3 to be active alternatively for $1/3$ of the total time.
   Next we will show this is also the only way for the k-hop path to achieve a throughput of $1/3$. Given our assumption for the $k-1$-hop path, for the consecutive three-hop segment, $L_{k-3} - L_{k-2} - L_{k-1}$ (note $L_{k-1} \in C_3$, $L_{k-2} \in C_2$ and $L_{k-3} \in C_1$), the proportion of time these three links are active must be chosen such that $t_{k-3} = t_{k-2} = t_{k-1} = \frac{1}{3}$ in order to achieve the maximum throughput of $\frac{1}{3}$ for the $k-1$-hop path. Note that that their sum is 1 and t_k must satisfy $t_k + t_{k-1} + t_{k-2} \leq 1$, therefore link L_k has to be simultaneously active with link L_k as well as all other links in C_1 in order to achieve the maximum throughput of $1/3$ for the k-hop path and $t_k = 1/3$. These finally lead to the conclusion that Theorem 1 is also valid for the k-hop path. This part of the proof can also be done by contradiction.

■

Here we would like to further comment that in the beginning of a packet transmission session, it may happen that although a link in the later hops of the multi-hop path is scheduled to be active, that link may remain silent because there is no packet queued in the corresponding node. We have ignored this effect in our analysis of the maximum throughput of the multi-hop path. As $t \to \infty$, it is expected that the contribution of this effect to the maximum throughput becomes negligibly small.

VI. The Maximum Throughput of a k-Hop Path in an Erroneous Environment

In this section, we extend our analysis in the last section to an erroneous radio environment ($0 < P_i < 1$). First, we obtain the maximum throughput of a generic consecutive three-hop segment. We then extend our analysis to a generic k-hop path.

A. *The maximum throughput of a consecutive three-hop segment*

Consider the consecutive three-hop segment of the k-hop path, $L_i - L_{i+1} - L_{i+2}$ ($1 \leq i \leq k-2$). Denote the proportion of time that the three links are active by t_i, t_{i+1} and t_{i+2} respectively. The throughputs of these links then become $T_i = P_i t_i$, $T_{i+1} = P_{i+1}t_{i+1}$, and $T_{i+2} = P_{i+2}t_{i+2}$, respectively. Here T_{i+2} is also the throughput of this consecutive three-hop segment. Accordingly, the LP formulation is given by

$$\text{Maximize:} \quad P_{i+2}t_{i+2} \tag{18}$$

$$\text{subject to:} \quad 0 \leq t_j \leq 1, j = i, i+1, i+2 \tag{19}$$

$$t_j P_j \geq t_{j+1}P_{j+1}, j = i, i+1 \tag{20}$$

$$t_i + t_{i+1} + t_{i+2} \leq 1 \tag{21}$$

Since P_{i+2} is a known constant, the maximization problem in (18) can be simplified to

$$\text{Maximize: } t_{i+2}. \tag{22}$$

Rearranging the constraints in the above equations (19)-(21), three new constraints on the relationship between t_{i+2} and $t_i + t_{i+1}$ can be obtained, given by

$$0 \leq \quad t_i + t_{i+1} \quad \leq 1, \tag{23}$$

$$t_{i+2} \quad \leq \quad \frac{t_i + t_{i+1}}{P_{i+2}(\frac{1}{P_i} + \frac{1}{P_{i+1}})}, \tag{24}$$

$$t_{i+2} \quad \leq \quad 1 - (t_i + t_{i+1}). \tag{25}$$

A figure is drawn to illustrate the relationship between t_{i+2} and $t_i + t_{i+1}$, as shown in Fig. 4.

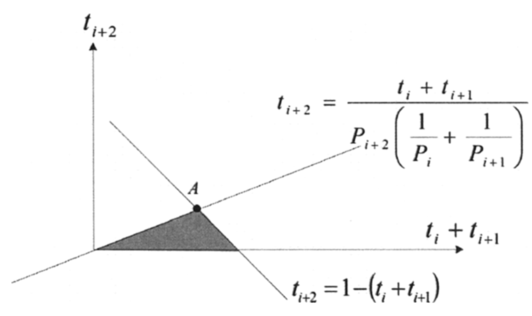


Fig. 4. The relationship between t_{i+2} and $t_i + t_{i+1}$.

Based on Fig. 4, it can be obtained that the maximum value of t_{i+2} while satisfying the constraints is $\frac{1}{P_{i+2}(\frac{1}{P_i}+\frac{1}{P_{i+1}}+\frac{1}{P_{i+2}})}$ and the maximum throughput of the three-hop segment is $\frac{1}{\frac{1}{P_i}+\frac{1}{P_{i+1}}+\frac{1}{P_{i+2}}}$. This maximum throughput is achieved when t_i and t_{i+1} assume the following values

$$t_j = \frac{1}{P_j(\frac{1}{P_i} + \frac{1}{P_{i+1}} + \frac{1}{P_{i+2}})}, j = i, i+1. \tag{26}$$

Note that the values of t_i, t_{i+1} and t_{i+2} corresponding to the maximum throughput satisfy: $t_i + t_{i+1} + t_{i+2} = 1$.

B. The maximum throughput of a k-hop path

Let us now consider the throughput of the generic k-hop path in an erroneous environment and we shall prove the following theorem.

Theorem 2: In an erroneous radio environment, the maximum throughput of a k-hop path ($k \geq 3$) is the minimum of the maximum throughputs of all consecutive three-hop segment within the k-hop path, that is,

$$\min_{1 \leq i \leq k-2} \frac{1}{\frac{1}{P_i} + \frac{1}{P_{i+1}} + \frac{1}{P_{i+2}}}. \tag{27}$$

Proof: Let us consider that a consecutive three-hop segment within the k-hop path, $L_j - L_{j+1} - L_{j+2}$, is the segment whose maximum throughput is smallest, i.e.,

$$\frac{1}{\frac{1}{P_j} + \frac{1}{P_{j+1}} + \frac{1}{P_{j+2}}} = \min_{1 \leq i \leq k-2} \frac{1}{\frac{1}{P_i} + \frac{1}{P_{i+1}} + \frac{1}{P_{i+2}}}. \tag{28}$$

Let us add an adjacent link to the above three-hop segment. Without loss of generality, let us assume the fourth link is L_{j+1}. Obviously the maximum throughput of the four-hop segment $L_j - L_{j+1} - L_{j+2} - L_{j+3}$ cannot exceed that of the three-hop segment $L_j - L_{j+1} - L_{j+2}$. Therefore, if we can show there exists a scheduling algorithm allowing the four-hop segment to achieve a throughput equal the maximum throughput of the three-hop segment, the maximum throughput of the four-hop segment must be the same as the throughput of the three-hop segment. It is trivial to show that a scheduling algorithm with

$$t_l = \frac{1}{P_l(\frac{1}{P_j} + \frac{1}{P_{j+1}} + \frac{1}{P_{j+2}})}, l = j, j+1, j+2, j+3 \tag{29}$$

satisfies our requirements. We will also show that such scheduling algorithm satisfies the constraints in (2)-(4). Let us start with constraint (4). It has been shown earlier that $t_j + t_{j+1} + t_{j+2} = 1$. Given (28), it can also be shown that

$$t_{j+1} + t_{j+2} + t_{j+3} = \frac{\frac{1}{\frac{1}{P_{j+1}} + \frac{1}{P_{j+2}} + \frac{1}{P_{j+3}}}}{\frac{1}{P_j} + \frac{1}{P_{j+1}} + \frac{1}{P_{j+2}}} \tag{30}$$

$$\leq 1 \tag{31}$$

Thus constraint (4) is satisfied. It naturally follows that constraint (2) is also satisfied. Constraint (3) is also satisfied because the scheduling algorithm readily leads to $T_j = T_{j+1} = T_{j+2} = T_{j+3}$. Therefore the maximum throughput of the four-hop segment is also $\frac{1}{\frac{1}{P_j} + \frac{1}{P_{j+1}} + \frac{1}{P_{j+2}}}$.

Repeat the above process and add adjacent links one by one to the consecutive three-hop segment with the minimum maximum throughput, it can be shown that the maximum throughput of a generic k-hop path is equal to the minimum of the maximum throughputs of all consecutive three-hop segments. There exists a scheduling algorithm with

$$t_j = \frac{1}{P_j} \min_{1 \leq i \leq k-2} \frac{1}{\frac{1}{P_i} + \frac{1}{P_{i+1}} + \frac{1}{P_{i+2}}}, 1 \leq j \leq k \tag{32}$$

that allows the k-hop path to achieve its maximum throughput while satisfying the constraints in (2)-(4). ■

VII. Conclusion

In this paper, we investigated the maximum throughput of a single chain multi-hop wireless path analytically.

We showed that in an ideal error free radio environment, a wireless multi-hop path, in which all nodes are optimally placed and all links has an identical normalized capacity of 1, has a maximum throughput of $\frac{1}{3}$. This maximum throughput can only be achieved when all links of the path are separated into three maximal independent sets and each set of links become active alternatively for the same proportion of time.

In an erroneous radio environment, we showed that the maximum throughput of the k-hop path is determined by its bottleneck consecutive three-hop segment. A scheduling algorithm achieving this maximum throughput is also presented. The research outcomes in this paper will help to design guidelines for optimum scheduling algorithms.

In this paper we have ignored the correlation among link activities by assuming each link has an independent successful transmission probability. In a real radio environment, one would expect the link activities to be correlated due to interference. It is an objective of our future work to investigate into the effect of this correlation.

This paper suggests that in order to achieve the maximum throughput, it is necessary to synchronize the transmission of every three links and designing the scheduling algorithm using the maximal independent sets as the basic unit. It remains to be investigated as to how these design principles can be realized in a distributed scheduling algorithm.

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