Road Traffic Density Estimation in Vehicular Networks

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***Abstract*—Road traffic density estimation provides important information for road planning, intelligent road routing, road traffic control, vehicular network traffic scheduling, routing and dissemination. The ever increasing number of vehicles equipped with wireless communication capabilities provide new means to estimate the road traffic density more accurately and in real time than traditionally used techniques. In this paper, we consider the problem of road traffic density estimation where each vehicle estimates its local road traffic density using some simple measurements only, i.e. the number of neighboring vehicles. A maximum likelihood estimator of the traffic density is obtained based on a rigorous analysis of the joint distribution of the number of vehicles in each hop. Analysis is also performed on the accuracy of the estimation and the amount of neighborhood information required for an accurate road traffic density estimation. Simulations are performed which validate the accuracy and the robustness of the proposed density estimation algorithm.**

***Index Terms*—Intelligent transportation systems, vehicle density estimation, vehicle-to-vehicle communication.**

I. Introduction

Vehicular networks (VANETs) or intelligent transport systems have been attracting increasing research interest. This is evidenced by recent developments from both automobile industry and the wireless communication community that make the vehicles more and more intelligent. Intelligent transport systems projects have been launched in the US, Japan and Europe. Several well-known projects include IntelliDrive(sm)/VII, V2V communication for safety, SAFESPOT and NoW [1]. VANETs or intelligent transport systems have been promising in improving the safety of drivers and passengers, improving the efficiency of road transportation and making the transportation experience more enjoyable by providing a number of services from collision warning, lane change assistance, speed limit notification, intelligent navigation and road traffic control, and multimedia content deliveries.

Road traffic density estimation provides important information in VANET and intelligent transport systems for road planning, intelligent road routing, road traffic control, network traffic scheduling, routing and dissemination. Traditionally road traffic density has been estimated using a number of techniques including roadside magnetic loop detectors, surveillance cameras, wireless vehicle sensors, and speed guns. They require the detection devices to be pre-installed, often at locations like cross the road, at traffic lights and in highway toll stations. The ever increasing number of vehicles equipped with wireless communication capabilities provide new opportunities to estimate the road traffic density more accurately and in real time than these traditional techniques.

In this paper, we investigate the problem of road traffic density estimation where each vehicle estimates its local road traffic density in a road segment using some simple measurements only, i.e. the number of neighbors.

The main contributions of this paper are:

- For a randomly chosen vehicle, a rigorous analysis of the joint distribution of the number of neighboring vehicles in each hop is provided.
- A maximum likelihood estimator of the road traffic density is obtained based on the above analysis, which uses a simple count of the number of neighboring vehicles in each hop only for estimation.
- The relationship between the amount of neighborhood information, measured in the number of hops, used in the estimation and the accuracy of the estimation is analyzed. The results can help to determine the amount of neighborhood information required in order to meet a designated accuracy requirement on density estimation.
- Simulations are performed which are indicative of good performance of the proposed algorithm.

The rest of this paper is organized as follows. Section II reviews related work. System model and problem formulation are described in Section III. The proposed road traffic density estimation algorithm is presented in Section IV. Section V analyzes the accuracy of the estimation. Simulation studies are presented in Section VI. Finally Section VII concludes this paper and suggests future work directions.

II. Related Work

Traditionally road traffic density has been estimated using a number of techniques including roadside magnetic loop detectors, surveillance cameras, wireless vehicle sensors, and speed guns [2], [3].

Magnetic loop detectors are often costly to install and maintain. The physical work involved in laying magnetic loops also prevent the method to be widely used on a large scale and in real-time applications.

In [4], Buch et al. provided an excellent review of the use of computer vision techniques for urban traffic monitoring. Video cameras are often installed at high poles to collect vehicle counts information. In [5], three vision sensors were used to detect the number of vehicles. In [6], cumulative road acoustics were used in estimating road traffic density and the impact of noise on the estimation was analyzed.

In [7]–[9], the authors presented methods to estimate vehicle density on a road segment based on the traffic data collected at the two wireless vehicle sensors placed at both ends of the road segment. In [8], Singh considered density estimation on a road segment with multiple lanes and the density estimation problem was solved using an Markov model approach. In [7], Yuan et al. proposed an Lagrangian state estimator-based approach to estimate vehicle density. A modified extended Kalman filter-based approach for density estimation is presented in [9]. In [10], [11], vehicle density estimation problem is formulated and solved using an Expectation Maximization approach.

In [12], vehicle density is estimated by using the average fraction of stop time of a probe vehicle. A main purpose for density estimation in that paper was to distinguish between the free-ow and the congested traffic phases.

Reference [13] is possibly the most closely related work to ours. In [13], assuming that each vehicle knows its own location and the location of all other vehicles, a density estimation algorithm was designed that uses the neighborhood information of neighbors separated by up to 2-hop away and the location of these neighbors. In this paper, we design a density estimation algorithm that uses a simple count of number of neighbors in each hop. The proposed algorithm does not rely on the use of location information and can utilize neighborhood information of an arbitrary number of hops. Further, we provide results characterizing the relation between the amount of neighborhood information used in the estimation and the accuracy of the estimation.

III. System Model and Problem Formulation

In this paper, we consider a set of vehicles Poissonly distributed on a road segment with an unknown density ρ. The use of Poisson distribution for modeling the distribution of vehicles has been supported by a number of measurement studies (see [14] and references therein). Each vehicle is able to directly communicate with nearby vehicles separated by a maximum Euclidean distance R away. That is, following the same model as that used in [13], a unit disk communication model is used in this paper. Without loss of generality, we assume that the road segment is sufficiently long, $>> R$, such that the effects associated with vehicles located near both ends of the road segment, viz. the boundary effect, can be neglected.

For a randomly chosen vehicle, this paper designs techniques for the vehicle to estimate the vehicle density ρ using its local neighborhood information. A more detailed problem statement is given below:

Without loss of generality, designate the location of the above randomly chosen vehicle to be the origin. Choose a direction randomly along the road segment and designate that direction to be the $+x$ direction. Let k_1 be the (random) number of vehicles along the $+x$ direction that are directly connected to the chosen vehicle; let k_2 be the (random) number of vehicles along the $+x$ direction that are exactly two hops away from the chosen vehicle; ...; and let k_n be the (random) of vehicles along the $+x$ direction that are exactly n hops away from the chosen vehicles (Note that it is a trivial extension to include the number of neighbors along the $-x$ direction into the analysis). See Fig. 1 for an illustration. This paper provides answers to the following two problems:

- Given the neighborhood information specified in terms of number of neighbors in each hop, k_1, k_2, ..., k_n, how to obtain an estimate of the vehicle density ρ.
- Intuitively as more and more neighborhood information is taken into account, i.e. when n increases, the accuracy of the estimation will improve. What is the amount of neighborhood information required, specified in terms of n, in order to meet a designated level of estimation accuracy.

Because the collection of the local neighborhood information can typically be completed quickly (in the order of ms) compared with the mobility of vehicles, the mobility of vehicles has little impact on the problem during the short time interval being considered. That is, we are essentially using a snap shot of the road traffic taken at a particular time instant to estimate the road traffic density.

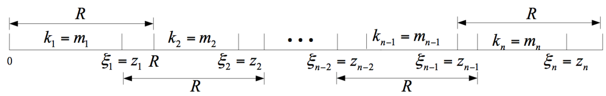

Figure 1. An illustration of the neighborhood of a randomly chosen vehicle at the origin. ξ_i is the furthest vehicle along the $+x$ direction that is i-hop away from the vehicle at the origin. k_i is the number of vehicles that are exactly i-hop away from the vehicle at the origin.

IV. Design of the Density Estimation Algorithm Using the Neighborhood Information

In this section, we first analyze the estimation of ρ when only k_1 is known. Then we gradually extend to the situations that (k_1, k_2) and (k_1, k_2, k_3) are known. Finally the more generic situation of the estimation of ρ using $(k_1, \ldots, k_n)$ is investigated.

A. One-hop neighbor scenario

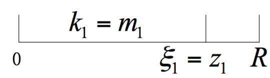

Figure 2. An illustration of the one-hop neighbor scenario

We first consider the one-hop scenario, i.e. only k_1 is known. This is illustrated in Figure 2. Let m_i be a non-negative integer. As an easy consequence of the Poisson distribution of vehicles, the probability that $k_1 = m_1$ is given by:

$$\Pr(k_1 = m_1) = \frac{(\rho R)^{m_1}}{m_1!} \times \exp(-\rho R) \quad (1)$$

Using the maximum likelihood estimation,

$$\hat{\rho} = \arg\max_{\rho} \Pr(k_1 = m_1) = \frac{m_1}{R} \quad (2)$$

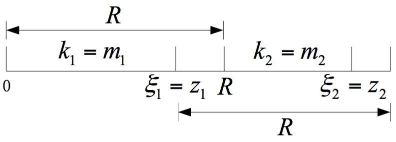

Figure 3. An illustration of two-hop neighbor scenario

B. Two-hop neighbor scenario

Now we move on to study the two-hop neighbor scenario, i.e. when both k_1 and k_2 are known. This is illustrated in Figure 3.

Let z_i be a positive real number. To avoid triviality, we require that $z_i > z_{i-1}$. Conditioned on $k_1 = m_1$, the probability that the distance between the furthest one-hop neighbor, denoted by ξ_1, and the vehicle at the origin is less than or equal to z_1 is given by:

$$\Pr(\xi_1 \leq z_1 | k_1 = m_1) = \left(\frac{z_1}{R}\right)^{m_1}$$

Combining the above equation with (1),

$$\Pr(\xi_1 \leq z_1, k_1 = m_1) = \frac{(\rho z_1)^{m_1}}{m_1!} \times \exp(-\rho R)$$

It follows that:

$$\begin{aligned}\Pr(\xi_1 = z_1, k_1 = m_1) &= \frac{\partial}{\partial z_1}\Pr(\xi_1 \leq z_1, k_1 = m_1)\\ &= \frac{\rho^{m_1} z_1^{m_1-1}}{(m_1-1)!} \times \exp(-\rho R) \end{aligned} \tag{3}$$

Note that in the above equation, it is implicitly assumed that $k_1 \geq 1$ (hence $m_1 \geq 1$). This is required for (3) to be meaningful. In the special case of $k_1 = 0$, conclusion readily follows that $k_i = 0$, $i > 1$, the density estimation can be quickly solved using (2). Therefore in the following analysis, we assume that $k_i > 0$ to avoid triviality.

Note that $\xi_1 = z_1$ implies that there is no vehicle within $(z_1, R]$ (otherwise that vehicle becomes the furthest one-hop neighbor which violates the condition that $\xi_1 = z_1$). Further, any two hop neighbors must be located within $(R, R + z_1]$, which has a length of z_1 (see Figure 3 for an illustration.). Therefore

$$\begin{aligned} &\Pr(k_2 = m_2 | k_1 = m_1, \xi_1 = z_1)\\ =\ &\Pr(k_2 = m_2 | \xi_1 = z_1)\\ =\ &\frac{(\rho z_1)^{m_2}}{m_2!} \times \exp(-\rho z_1) \end{aligned} \tag{4}$$

where in the first step of the above equation, the property of Poisson distribution that the (random) number of vehicles in two non-overlapping intervals are independent is used. Combining the two equations (3) and (4):

$$\begin{aligned} &\Pr(k_1 = m_1, k_2 = m_2, \xi_1 = z_1)\\ =&\frac{\rho^{m_1+m_2} z_1^{m_1+m_2-1}}{(m_1-1)!m_2!}\exp[-\rho(z_1+R)] \end{aligned} \tag{5}$$

Using the total probability theorem,

$$\begin{aligned} &\Pr(k_1 = m_1, k_2 = m_2)\\ =&\frac{\rho^{m_1+m_2} \times \exp(-\rho R)}{(m_1-1)!m_2!}\int_0^R z_1^{m_1+m_2-1} \times \exp(-\rho z_1) dz_1 \end{aligned} \tag{6}$$

Using the maximum likelihood estimation,

$$\hat{\rho} = \arg\max_{\rho} \Pr(k_1 = m_1, k_2 = m_2) \tag{7}$$

A closed-form solution for $\hat{\rho}$ becomes difficult but the above equation can be readily solved numerically.

C. Three-hop neighbor scenario

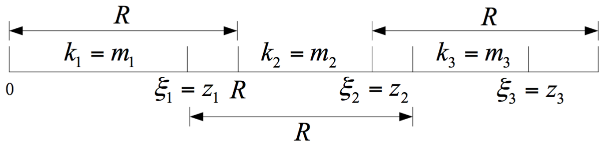

Figure 4. Three-hop Neighbor Scenario

Now we continue to investigate the three-hop neighbor scenario, i.e. when k_1, k_2 and k_3 are all known. This is illustrated in Figure 4. The investigation of the three-hop neighbor scenario will enable us to obtain recursively the solution for the generic situation when $(k_1, \ldots, k_n)$ are known.

We first obtain the distribution of ξ_2, i.e. the distance between the furthest two-hop neighbor and the vehicle at the origin, and then use it as a tool to obtain the final result.

Using the same analysis as that leads to (4), it can be obtained that

$$\begin{aligned} &\Pr(R \leq \xi_2 \leq z_2 | k_2 = m_2, k_1 = m_1, \xi_1 = z_1)\\ =&\left(\frac{z_2 - R}{z_1}\right)^{m_2} \end{aligned} \tag{8}$$

Obviously in the above equation z_2 has to be greater than R in order for the result to be meaningful, i.e. a two-hop neighbor cannot be within distance R of the vehicle at the origin.

As an easy consequence of the above equation and (5), it can be obtained that

$$\begin{aligned} &\Pr(k_2 = m_2, R \leq \xi_2 \leq z_2, k_1 = m_1, \xi_1 = z_1)\\ =&\frac{\rho^{m_1+m_2} z_1^{m_1-1}(z_2-R)^{m_2}}{(m_1-1)!m_2!} \times \exp[-\rho(z_1+R)] \end{aligned}$$

By taking the derivative of $\Pr(k_2 = m_2, R \leq \xi_2 \leq z_2, k_1 = m_1, \xi_1 = z_1)$ with respect to z_2, it can be shown that

$$\begin{aligned} &\Pr(k_2 = m_2, \xi_2 = z_2, k_1 = m_1, \xi_1 = z_1)\\ =&\frac{\rho^{m_1+m_2} z_1^{m_1-1}(z_2-R)^{m_2-1}}{(m_1-1)!(m_2-1)!} \times \exp[-\rho(z_1+R)] \end{aligned} \tag{9}$$

Note that $\xi_1 = z_1$ implies that a) there is no three-hop neighbor within $(z_2, z_1 + R]$; and $\xi_2 = z_2$ implies that b) any three-hop neighbor must be located in within $(z_2, z_2 + R]$.

The combination of the above two conditions means when $\xi_1 = z_1$ and $\xi_2 = z_2$, the three-hop neighbor can only be within $(z_1 + R, z_2 + R]$. Using the property that vehicles are Poissonly distributed, it follows that

$$
\begin{aligned}
&\Pr(k_3 = m_3 \mid k_2 = m_2, \xi_2 = z_2, k_1 = m_1, \xi_1 = z_1)\\
=&\Pr(k_3 = m_3 \mid \xi_2 = z_2, \xi_1 = z_1)\\
=&\frac{[\rho(z_2 - z_1)]^{m_3}}{m_3!} \times \exp[-\rho(z_2 - z_1)]
\end{aligned} \tag{10}
$$

Remark 1. Note that the expression for $\Pr(k_3 = m_3 \mid k_2 = m_2, \xi_2 = z_2, k_1 = m_1, \xi_1 = z_1)$ has a quite different form from the expression of $\Pr(k_2 = m_2 | k_1 = m_1, \xi_1 = z_1)$. This explains why we need to investigate the three-hop scenario in order to extend to the more generic situation.

Combing (10) and (9) using Bayes' formula:

$$
\begin{aligned}
&\Pr(k_3 = m_3, k_2 = m_2, \xi_2 = z_2, k_1 = m_1, \xi_1 = z_1)\\
=&\frac{\rho^{m_1+m_2+m_3} z_1^{m_1-1}(z_2 - R)^{m_2-1}(z_2 - z_1)^{m_3}}{(m_1 - 1)!(m_2 - 1)!m_3!}\\
&\times \exp[-\rho(z_2 + R)]
\end{aligned}
$$

Using the total probability theorem,

$$
\begin{aligned}
&\Pr(k_3 = m_3, k_2 = m_2, k_1 = m_1)\\
=&\frac{\rho^{m_1+m_2+m_3} \times \exp(-\rho R)}{(m_1 - 1)!(m_2 - 1)!m_3!} \times \int_0^R \int_R^{R+z_1} z_1^{m_1-1}(z_2 - R)^{m_2-1}\\
&\times (z_2 - z_1)^{m_3} \exp(-\rho z_2) dz_2 dz_1
\end{aligned} \tag{11}
$$

Finally it can be obtained that

$$
\hat{\rho} = \arg\max_{\rho} \Pr(k_3 = m_3, k_2 = m_2, k_1 = m_1) \tag{12}
$$

D. Vehicle density estimation for the more generic scenario

Now we are ready to study the more generic scenario of vehicle density estimation when $(k_1, \ldots, k_n)$ are known. Note that (10) reveals that the distribution of k_3 depends only on the distribution of ξ_2 and ξ_1, i.e. the distribution of the distances of furthest vehicles in the previous two hops. This relationship can be generalized to that the distribution of k_i depends only on the distribution of ξ_{i-1} and ξ_{i-2} (see Fig. (5) for an illustration). The above observation forms the basis of the following analysis on density estimation for the generic scenario that $(k_1, \ldots, k_n)$ are known.

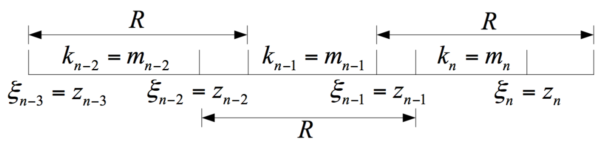

Figure 5. An Illustration of the N-hop Neighbor Scenario

The following theorem forms a major contribution of this paper:

Theorem 2. *The joint distribution of k_n, k_{n-1},...,k_1, where $n \geq 3$ is given by*

$$
\begin{aligned}
&\Pr(k_n = m_n, k_{n-1} = m_{n-1}, \ldots, k_2 = m_2, k_1 = m_1)\\
=&\frac{\rho^{\sum_{i=1}^{n} m_n} \times \exp(-\rho R)}{\prod_{i=1}^{n-1}(m_i - 1)!m_n!} \int_0^R \int_R^{R+z_1} \int_{z_1+R}^{z_2+R} \int_{z_2+R}^{z_3+R} \cdots\\
&\int_{z_{n-3}+R}^{z_{n-2}+R} z_1^{m_1-1}(z_2 - R)^{m_2-1} \prod_{i=3}^{n-1}(z_i - z_{i-2} - R)^{m_i-1}\\
&\times (z_{n-1} - z_{n-2})^{m_n} \exp(-\rho z_{n-1}) dz_{n-1} dz_{n-2} \ldots dz_2 dz_1
\end{aligned} \tag{13}
$$

Proof: In the following, we shall prove the theorem by recursion. Particularly, assuming that (13) is valid for n (Note that the correctness of (13) has been demonstrated in the earlier subsection for the three-hop scenario), we shall prove that it is also correct for $n + 1$.

First, using the same procedure as that results in (8) (see also Fig. 5), it can be obtained that

$$
\begin{aligned}
&\Pr(z_{n-2} + R \leq \xi_n \leq z_n | k_n = m_n, k_{n-1} = m_{n-1}, \xi_{n-1}\\
&= z_{n-1}, \ldots, k_1 = m_1, \xi_1 = z_1) = (\frac{z_n - z_{n-2} - R}{z_{n-1} - z_{n-2}})^{m_n}
\end{aligned}
$$

By taking the derivative of the above equation with regards to z_n, it follows that

$$
\begin{aligned}
&\Pr(k_n = m_n, \xi_n = z_n, \ldots, k_1 = m_1, \xi_1 = z_1)\\
=&\frac{\rho^{\sum_{i=1}^{n} m_n} z_1^{m_1-1}(z_2 - R)^{m_2-1} \prod_{i=3}^{n}(z_i - z_{i-2} - R)^{m_i-1}}{\prod_{i=1}^{n}(m_i - 1)!}\\
&\times \exp[-\rho(z_{n-1} + R)]
\end{aligned} \tag{14}
$$

As mentioned in the beginning of this section, the distribution of k_{n+1} only depends on the distribution of ξ_n and ξ_{n-1}. Therefore

$$
\begin{aligned}
&\Pr(k_{n+1} = m_{n+1} \mid k_n = m_n, \xi_n = z_n, \ldots, k_1 = m_1, \xi_1 = z_1)\\
=&\Pr(k_{n+1} = m_{n+1} \mid \xi_n = z_n, \xi_{n-1} = z_{n-1})\\
=&\frac{[\rho(z_n - z_{n-1})]^{m_{n+1}}}{m_{n+1}!} \times \exp[-\rho(z_n - z_{n-1})]
\end{aligned} \tag{15}
$$

As an easy consequence of (14) and (15):

$$
\begin{aligned}
&\Pr(k_{n+1} = m_{n+1}, k_n = m_n, \xi_n = z_n, \ldots, k_1 = m_1, \xi_1 = z_1)\\
=&\frac{\rho^{\sum_{i=1}^{n+1} m_n} z_1^{m_1-1}(z_2 - R)^{m_2-1} \prod_{i=3}^{n}(z_i - z_{i-2} - R)^{m_i-1}}{\prod_{i=1}^{n}(m_i - 1)!m_{n+1}!}\\
&\times (z_n - z_{n-1})^{m_{n+1}} \exp[-\rho(z_n + R)]
\end{aligned}
$$

Then using the total probability theorem,

$$
\begin{aligned}
&\Pr(k_{n+1} = m_{n+1}, k_n = m_n, \ldots, k_1 = m_1)\\
=&\frac{\rho^{\sum_{i=1}^{n+1} m_n} \times \exp(-\rho R)}{\prod_{i=1}^{n}(m_i - 1)!m_{n+1}!} \int_0^R \int_R^{R+z_1} \cdots \int_{z_{n-2}+R}^{z_{n-1}+R} z_1^{m_1-1}\\
&(z_2 - R)^{m_2-1} \prod_{i=3}^{n}(z_i - z_{i-2} - R)^{m_i-1}(z_n - z_{n-1})^{m_{n+1}}\\
&\times \exp(-\rho z_n) dz_n dz_{n-1} \ldots dz_2 dz_1
\end{aligned} \tag{16}
$$

■

Using Theorem (2), a maximum likelihood estimator of ρ given the knowledge of $(k_1, \ldots, k_n)$ is

$$\hat{\rho} = \arg\max_{\rho} \Pr(k_n = m_n, \ldots, k_1 = m_1) \qquad (17)$$

where $\Pr(k_n = m_n, \ldots, k_1 = m_1)$ is given in (13).

V. An Analysis of the Accuracy of the Estimation

In the earlier sections, we have given a maximum likelihood estimator of ρ given an observation of $(k_1, \ldots, k_n)$. It is intuitively true that when more and more neighborhood information is taken into account, i.e. when n increases, a more accurate estimate of ρ can be obtained. In this section, we shall quantitatively characterization the relation between $\hat{\rho}$ and the amount of neighborhood information used in the observation, which is quantitatively represented by n.

Note that the maximum likelihood estimate of ρ given in (17) (hence also in (12), (7) and (2)) can be more accurate written as $\hat{\rho}|\left(k_1 = m_1, \ldots, k_n = m_n\right)$. That is, it is an estimate of ρ when a particular (random) instance of $(k_1, \ldots, k_n)$ is observed. Because of the randomness of $(k_1, \ldots, k_n)$, $\hat{\rho}|\left(k_1 = m_1, \ldots, k_n = m_n\right)$ is also a random variable whose distribution is solely determined by the joint distribution of $(k_1, \ldots, k_n)$. Based on the above observation, it can be obtained that the bias of the estimation, denoted by $\delta\left(\hat{\rho}\right)$, is given by

$$\delta\left(\hat{\rho}\right) = E\left(\hat{\rho}\right) - \rho$$

where

$$\begin{aligned} E\left(\hat{\rho}\right) &= \sum_{m_n=0}^{\infty} \cdots \sum_{m_1=0}^{\infty} \{[\hat{\rho}|\left(k_1 = m_1, \ldots, k_n = m_n\right)] \\ &\quad \Pr\left(k_1 = m_1, \ldots, k_n = m_n\right)\} \end{aligned}$$

The variance of the estimation is given by

$$Var\left(\hat{\rho}\right) = E\left(\hat{\rho}^2\right) - \left(E\left(\hat{\rho}\right)\right)^2$$

where

$$\begin{aligned} E\left(\hat{\rho}^2\right) &= \sum_{m_n=0}^{\infty} \cdots \sum_{m_1=0}^{\infty} \{[\hat{\rho}|\left(k_1 = m_1, \ldots, k_n = m_n\right)]^2 \\ &\quad \Pr\left(k_1 = m_1, \ldots, k_n = m_n\right)\} \end{aligned}$$

Another commonly used metric to measure the accuracy of the estimation, the mean absolute error, denoted by $\epsilon\left(\hat{\rho}\right)$, is given by

$$\begin{aligned} \epsilon\left(\hat{\rho}\right) &= E\left(|\hat{\rho} - \rho|\right) \\ &= \sum_{m_n=0}^{\infty} \cdots \sum_{m_1=0}^{\infty} \{|\hat{\rho}|\left(k_1 = m_1, \ldots, k_n = m_n\right) - \rho| \\ &\quad \Pr\left(k_1 = m_1, \ldots, k_n = m_n\right)\} \end{aligned}$$

The above equations allow us to quantitatively characterize the relationship between the accuracy of the estimation and the amount of neighborhood information used in the estimation. It helps to determine the amount of neighborhood information that needs to be collected in order to meet a prescribed level of accuracy. In the next section, we shall further illustrate this relationship more intuitively using figures (more specifically in Fig. 7 and 6).

VI. Simulations

In this section, we used simulations to validate the performance of the proposed density estimation algorithm. The three metrics: bias, mean absolute error and variance, are used to measure the performance of the proposed algorithm.

Fig. 6 illustrates the mean absolute error (MAE) of the proposed density estimation algorithm for four different transmission ranges, i.e. $R = 75m$, $R = 100m$, $R = 150m$ and $R = 200m$. The MAE is plotted as a function of the number of hops used in the measurements, i.e. n. The true vehicle density used in the simulation is $\rho = 0.08$ vehicles/m. Each simulation is repeated 300 times. The MAE shown in the figure are the results from these 300 simulations.

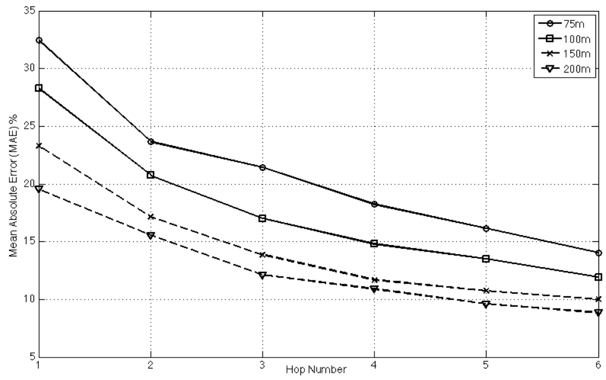

Figure 6. Variation of the Mean Absolute Error with the Number of Hops. The MAE is plotted as a percentage of the true value, where the true value $\rho = 0.08$ vehicles/m.

As expected, when the amount of neighborhood information used in the estimation, measured by the hop number n, increases, the MAE decreases. When $n = 4$, a further increase in the number of hops used in the estimation appears to have a reduced effect on the reduction of the MAE. Therefore, a small number of hops may be used to achieve an optimum tradeoff between the amount of neighborhood information used in the estimation and the accuracy of the estimation.

Fig. (7) plots the variation of the MAE with the number of hops for different values of ρ.

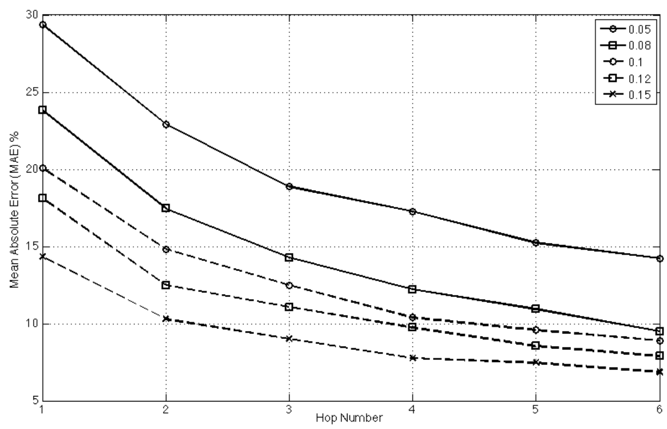

Figure 7. Variation of the Mean Absolute Error with the Number of Hops. The MAE is plotted as a percentage of the true value ρ, where the true value varies from 0.05 to 0.15. The transmission range is fixed at $R = 150m$.

The following table further shows the expected value of $\hat{\rho}$, the bias of the estimation and the variance where $\rho = 0.08$ and $R = 150m$. As shown in the table, the bias and the variance is generally small, compared with the true value $\rho = 0.08$. When $n = 2$, the bias increases compared with when only k_1 is used in the estimation, viz. $n = 1$. Comparing (2) and (7), it is easy to see that the maximum likelihood estimator is unbiased when only k_1 is used. When (k_1, k_2) are used, the estimator becomes non-linear. The non-linearity will introduce bias into the estimation however Table (I) shows that the bias is small. Note that variance shows a consistent trend of decreasing with an increase in n.

Table I
AN ILLUSTRATION OF THE EXPECTATION, BIAS AND VARIANCE

Hop Number	Expectation	Bias	Variance
1	0.079910	-4.51 $\times 10^{-5}$	3.96×10^{-4}
2	0.079662	-2.84×10^{-4}	2.13×10^{-4}
3	0.079655	-2.55 $\times 10^{-4}$	1.44×10^{-4}
4	0.079911	-1.94 $\times 10^{-4}$	1.09×10^{-4}

VII. CONCLUSION AND FUTURE WORK

In this paper, we proposed a vehicle density estimation algorithm that each vehicle only uses a simple count of the number of its neighboring vehicles in each hop to estimate the vehicle density. A maximum likelihood estimator of the vehicle density was obtained and its performance was validated using analytical studies and simulations. Further, we also analyzed the relationship between the amount of neighborhood information used in the estimation and the accuracy of the estimation, and obtained results can help to decide on the amount of neighborhood information required to meet a prescribed level of accuracy on the estimation.

In our analysis, it was assumed that vehicles communicate with each other following a unit disk model. The unit disk model has been widely used in the area and the results obtained using the unit disk model have been shown in numerous literature to be good indicator of the performance in a more realistic scenario. However the unit disk model often overly simplifies the real scenario. Therefore it is part of our future work plan to extend our work to more realistic scenario, e.g. vehicles communicate with each other following the log-normal communication model.

In this paper, we presented an algorithm that allows each vehicle to obtain a local estimate of vehicle density using its neighborhood information. An interesting problem arises when the above analysis is extended to a larger scale. More specifically, inside a road segment, there may exist many such vehicles and each vehicle has its own local estimate of the vehicle density. Assuming that these vehicles can report their local density estimates to a central entity or can exchange the density estimates, an interesting problem is how to form a more accurate "global" estimate using the local estimates. A main obstacle in solving the above problem is the so-called spatial correlation problem. That is, a vehicle may be neighbors of many other vehicles and therefore counted many times in the local density estimates of these vehicles. Consequently, these local density estimates become correlated. It is also part of our future work plan to tackle the above problem.

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