Towards a Simple Relationship to Estimate the Capacity of Static and Mobile Wireless Networks

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***Abstract*—Extensive research has been done on studying the capacity of wireless multi-hop networks. These efforts have led to many sophisticated and customized analytical studies on the capacity of particular networks. While most of the analyses are intellectually challenging, they lack universal properties that can be extended to study the capacity of a different network. In this paper, we sift through various capacity-impacting parameters and present a simple relationship that can be used to estimate the capacity of both static and mobile networks. Specifically, we show that the network capacity is determined by the average number of simultaneous transmissions, the link capacity and the average number of transmissions required to deliver a packet to its destination. Our result is valid for both finite networks and asymptotically infinite networks. We then use this result to explain and better understand the insights of some existing results on the capacity of static networks, mobile networks and hybrid networks and the multicast capacity. The capacity analysis using the aforementioned relationship often becomes simpler. The relationship can be used as a powerful tool to estimate the capacity of different networks. Our work makes important contributions towards developing a generic methodology for network capacity analysis that is applicable to a variety of different scenarios.**

***Index Terms*—Capacity, mobile networks, wireless networks**

I. Introduction

WIRELESS multi-hop networks, in various forms, e.g. wireless sensor networks, underwater networks, vehicular networks, mesh networks and unmanned aerial vehicle formations, and under various names, e.g. ad-hoc networks, hybrid networks, delay tolerant networks and intermittently connected networks, are being increasingly used in military and civilian applications.

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Studying the capacity of these networks is an important problem. Since the seminal work of Gupta and Kumar [1], extensive research has been done in the area. Particularly, in [1] Gupta and Kumar considered an ad-hoc network with a total of n nodes uniformly and *i.i.d.* on an area of unit size. Furthermore, each node is capable of transmitting at W bit/s and using a fixed and identical transmission range. They showed that the transport capacity and the achievable per-node throughput, when each node randomly and independently chooses another node in the network as its destination, are $\Theta\left(W\sqrt{\frac{n}{\log n}}\right)$ and $\Theta\left(\frac{W}{\sqrt{n\log n}}\right)$ respectively[1]. When the nodes are optimally and deterministically placed to maximize throughput, the transport capacity and the achievable per-node throughput become $\Theta\left(W\sqrt{n}\right)$ and $\Theta\left(\frac{W}{\sqrt{n}}\right)$ respectively. In [2], Franceschetti *et al.* considered essentially the same random network as that in [1] except that nodes in the network are allowed to use two different transmission ranges. The link capacity between a pair of directly connected nodes is determined by their SINR through the Shannon–Hartley theorem. They showed that by having each source-destination pair transmitting via the so-called "highway system", formed by nodes using the smaller transmission range, the transport capacity and the per-node throughput can also reach $\Theta\left(\sqrt{n}\right)$ and $\Theta\left(\frac{1}{\sqrt{n}}\right)$ respectively even when nodes are randomly deployed. The existence of such highways was established using the percolation theory [3]. In [4] Grossglauser and Tse showed that in mobile networks, by leveraging on the nodes' mobility, a per-node throughput of $\Theta\left(1\right)$ can be

[1]The following notations are used throughout the paper. For two positive functions $f\left(x\right)$ and $h\left(x\right)$:

- $f\left(x\right)=o\left(h\left(x\right)\right)$ iff (if and only if) $\lim_{x\to\infty}\frac{f(x)}{h(x)}=0$;
- $f\left(x\right)=\omega\left(h\left(x\right)\right)$ iff $h\left(x\right)=o\left(f\left(x\right)\right)$;
- $f\left(x\right)=\Theta\left(h\left(x\right)\right)$ iff there exist a sufficiently large x_0 and two positive constants c_1 and c_2 such that for any $x>x_0$, $c_1h\left(x\right)\geq f\left(x\right)\geq c_2h\left(x\right)$;
- $f\left(x\right)\sim h\left(x\right)$ iff $\lim_{x\to\infty}\frac{f(x)}{h(x)}=1$;
- $f\left(x\right)=O\left(h\left(x\right)\right)$ iff there exist a sufficiently large x_0 and a positive constant c such that for any $x>x_0$, $f\left(x\right)\leq ch\left(x\right)$;
- $f\left(x\right)=\Omega\left(h\left(x\right)\right)$ iff $h\left(x\right)=O\left(f\left(x\right)\right)$;
- An event ξ is said to occur almost surely if its probability equals one;
- An event ξ_x depending on x is said to occur asymptotically almost surely (a.a.s.) if its probability tends to one as $x\to\infty$.

The above definition applies whether the argument x is continuous or discrete, e.g. assuming integer values.

achieved at the expense of unbounded delay. Their work [4] has sparked huge interest in studying the capacity-delay tradeoffs in mobile networks assuming various mobility models and the obtained results often vary greatly with the different mobility models being considered, see [5]–[10] and references therein for examples. In [11], Chen et al. studied the capacity of wireless networks under a different traffic distribution. In particular, they considered a set of n randomly deployed nodes transmitting to single sink or multiple sinks where the sinks can be either regularly deployed or randomly deployed. They showed that with single sink, the transport capacity is given by $\Theta(W)$; with k sinks, the transport capacity is increased to $\Theta(kW)$ when $k = O(n \log n)$ or $\Theta(n \log nW)$ when $k = \Omega(n \log n)$. Furthermore, there is also significant amount of work studying the impact of infrastructure nodes [12] and multiple-access protocols [13] on the capacity and the multicast capacity [14]. We refer readers to [15] for a comprehensive review of related work.

The above efforts have led to many sophisticated and customized analytical studies on the capacity of particular networks. The obtained results often vary greatly with even a slight change in the scenario being investigated. While most of the analyses are intellectually challenging, they lack universal properties that can be extended to study the capacity of a different network. In this paper, we sift through these capacity-impacting parameters, e.g. mobility, traffic distribution, spatial node distribution, the capability of nodes to adjust their transmission power, the presence of infrastructure nodes, multiple-access protocols and scheduling algorithms, and present a simple relationship that can be used to estimate the capacity of wireless multi-hop networks. In addition to capacity, delay is also an important performance metric that has been extensively investigated. In this paper we focus on the study of capacity. We refer readers to [5]–[9] for relevant work on delay.

The main contribution of this paper is the development of a simple relationship for estimating the capacity of wireless multi-hop networks applicable to various different scenarios. The following is a detailed summary of our contributions:

- Considering an arbitrary network, we show that the network capacity is determined by the link capacity, the average number of simultaneous transmissions, and the average number of transmissions required to deliver a packet to its destination;
- We extend the above relationship for arbitrary networks to random networks;
- We apply our new result to determine the asymptotic capacity of several typical random networks considered in the literature [1], [2], [4], [10], [12]–[14]. The capacity analysis using the aforementioned relationship often becomes simpler;
- Based on the intuitive understanding gained from our result, we point out limitations of some existing results and suggest further improvements;
- Furthermore, using our result, the capacity analysis for different networks can be transformed into the analysis of the three key parameters, i.e. the link capacity, the average number of simultaneous transmissions, and the average number of transmissions required to deliver a packet to its destination. Therefore our work makes important contributions towards developing a generic methodology for network capacity analysis that is applicable to a variety of different scenarios.

The rest of the paper is organized as follows: Section II gives a formal definition of the network models and notations considered in the paper. Section III gives the main results in this paper on the capacity of arbitrary networks and random networks. In Section IV, we demonstrate wide applications of our result by using it to analyze the asymptotic capacity of various random networks considered in the literature [1], [2], [4], [10], [12]–[14]. Finally Section V concludes this paper.

II. Network Models

We consider two classes of networks in this paper: *arbitrary networks* and *random networks*.

A. Arbitrary networks

We use the term arbitrary network to refer to a network with a total of n nodes arbitrarily and deterministically (i.e. not randomly) placed in a bounded area A initially. These nodes may be either stationary or moving following arbitrary and fixed (i.e. not random) trajectories. A node may choose an arbitrary and fixed number of other nodes as its destination(s). In the case that a source node has multiple destination nodes, the source node may transmit the same packets to its destinations, viz. multicast, or transmit different portions of its packets to different destinations, viz. unicast. Packets are transmitted from a source to its destination(s) via multiple intermediate relay nodes. Each node can be either a source, a relay, a destination or a mixture. It is assumed that there are always packets waiting at the source nodes to be transmitted, viz. a so-called saturated traffic scenario is considered.

Let V_n be the node set. Let E be the set of links. The establishment of a link between a pair of nodes may follow either the protocol model or the physical model [1]. Our analysis does not depend on the particular way a link is established. When nodes are mobile, the link between a pair of nodes may only exist temporarily and the link set at a particular time instant t may be more appropriately denoted by E_t to emphasize its temporal dependence. In this paper, we drop the subscript t for convenience. It is assumed that there is a spatial and temporal path between every source and destination pair.

Without loss of generality [1], [4], [10], [12]–[14], we further assume that each node can transmit at a fixed and known data rate of W bits per second over a common wireless channel. Following the same analytical approach as that in [1], it can be shown that it is immaterial to our result if the channel is broken into several subchannels of capacity $W_1, W_2, \cdots, W_M$ bits per second, as long as $\sum_{m=1}^{M} W_m = W$. This assumption allows us to ignore some physical layer details and focus on the topological aspects of the network that determine the capacity. Our result however can be readily extended to incorporate the situation that each link has a different and known capacity. We do not consider the impact of erroneous transmissions in our analysis. Transmission errors will cause a decrease in the effective link capacity and its impact can be captured in the parameter W, which is assumed to be known.

Denote the above network by $G\left(V_n, E\right)$ and in this paper, we study the capacity of $G\left(V_n, E\right)$.

In the following paragraphs, we give a formal definition of the capacity of $G\left(V_n, E\right)$. Let $v_i \in V$ be a source node and let $b_{i,j}$ be the j^{th} bit transmitted from v_i to its destination. Let $d\left(v_i, j\right)$ be the destination of $b_{i,j}$. For unicast transmission, $d\left(v_i, j\right)$ represents single destination; for multicast transmission, $d\left(v_i, j\right)$ represents the set of all destinations of $b_{i,j}$. Let $N_{i,T}^{\chi}$ be the number of bits transmitted by v_i *and* which reached, i.e. successfully received by, their respective destinations during a time interval $[0, T]$, with T being an arbitrarily large number. The superscript $\chi \in \Phi$ denotes the spatial and temporal scheduling algorithm used in the network and Φ denotes the set of all scheduling algorithms. If the same bit is transmitted from a source to multiple destinations, e.g. in the case of multicast, it is counted as one bit in the calculation of $N_{i,T}^{\chi}$.

It is assumed that the network is *stable* $\forall \chi \in \Phi$. A network is called *stable* if and only if for any fixed n, assuming that each node has an infinite queue, the queue length in any intermediate relay node storing packets in transit does not grow towards infinity as $T \to \infty$, or equivalently the long-term incoming traffic rate into the network equals the long-term outgoing traffic rate. It is further assumed that there is no traffic loss due to queue overflow.

The transport capacity when using the spatial and temporal scheduling algorithm χ, denoted by $\eta^{\chi}\left(n\right)$, is defined as:

$$\eta^{\chi}\left(n\right) \triangleq \lim_{T \to \infty} \frac{\sum_{i=1}^{n} N_{i,T}^{\chi}}{T} \tag{1}$$

and the transport capacity of the network is defined as

$$\eta\left(n\right) \triangleq \max_{\chi \in \Phi} \eta^{\chi}\left(n\right) \tag{2}$$

Obviously $\eta\left(n\right) \geq \eta^{\chi}\left(n\right), \forall \chi \in \Phi$.

An important special case occurs when the scheduling algorithm divides the transport capacity equally among all source-destination pairs asymptotically over time. Denote by $\Phi^f \subseteq \Phi$ the set of *fair* scheduling algorithms that divide the transport capacity equally among all m source-destination pairs asymptotically over time. The throughput per source-destination pair is defined as

$$\lambda_m \triangleq \max_{\chi \in \Phi^f} \frac{\eta^{\chi}\left(n\right)}{m} \tag{3}$$

The above definitions of the transport capacity and throughput capacity are valid for both finite n and asymptotically infinite n.

B. Random networks

In addition to arbitrary networks, random networks have also been extensively studied in the literature, particularly the asymptotic properties of random networks as the number of nodes n approaches infinity [1], [2], [4], [10], [12]–[14]. By a random network, we mean a network with a total of n nodes and each node is i.i.d. in a bounded area A initially following a known distribution. If these nodes are mobile, their trajectories may also be random and i.i.d. A link between a pair of nodes in a random network may be established following either the protocol model or the physical model [1]. Denote the above random network by G_n to distinguish it from the arbitrary network considered in the previous subsection.

Given the randomness involved in the problem statement, the above definitions of throughput capacity for arbitrary networks need to be modified to account for "vanishingly small probabilities" [1]. Particularly, for asymptotic random networks whose number of nodes n is sufficiently large, we say that under the spatial and temporal scheduling algorithm χ, the transport capacity of G_n is $\eta^{\chi}\left(n\right)$ if and only if $\eta^{\chi}\left(n\right)$ is the *maximum* transport capacity that can be achieved *asymptotically almost surely (a.a.s.)* as $n \to \infty$ under χ. Given the above modification on $\eta^{\chi}\left(n\right)$, the transport capacity of an arbitrary network defined in (2) can still be used for random networks.

The most extensively studied traffic distribution in random networks involves each node choosing another node independently as its destination and the transport capacity being divided equally among all source-destination pairs. In that case, the total number of source-destination pairs equals n and the capacity of the network is often studied using the metric known as the *per-node throughput* or the *throughput capacity*. Denote by $\Phi^f \subseteq \Phi$ the set of *fair* scheduling algorithms that divide the transport capacity equally among all n source-destination pairs asymptotically over time. The per-node throughput (or throughput capacity) is defined as

$$\lambda\left(n\right) \triangleq \max_{\chi \in \Phi^f} \frac{\eta^{\chi}\left(n\right)}{n} \tag{4}$$

Intuitively, a scheduling algorithm χ is fair if it divides the transport capacity equally among all source-destination pairs asymptotically over time and also distributes traffic evenly across A such that there is no traffic hot spot. For nodes uniformly and i.i.d. on A and each node choosing another node independently as its destination, which is the scenario studied in Sections III-B and IV, the technique is well known to establish the (asymptotic) fairness of a scheduling algorithm χ, or to construct a (asymptotically) fair scheduling algorithm. It typically involves partitioning A into a set of equal-size sub-areas, allocating transmission opportunities equally among all sub-areas and then demonstrating that using χ, the number of source-destination pairs crossing each sub-area varies by at most a constant factor. The conclusion readily follows that the throughput obtainable by each source-destination pair varies by at most a constant factor and each source-destination pair has access to throughput of the same order asymptotically, see [2], [16] for examples. The set of scheduling algorithms analyzed in Section IV are known to be fair in the sense that *a.a.s.*, each source-destination pair can achieve a throughput of the same order.

Note that the above definitions of transport capacity and throughput capacity for random networks are consistent with those in [1], [2], [4], [10], [12]–[14]. Particularly in [1], a throughput capacity of $\lambda(n)$ bits per second is called *feasible* if there is a spatial and temporal scheme for scheduling transmissions such that every node can send $\lambda(n)$ bits per second on average to its chosen destination [1]. The throughput capacity of random networks with n node is of order $\Theta(f(n))$ bits per second if there are deterministic constants $c > 0$ and $c' < +\infty$ such that

$$\lim_{n\to\infty} \Pr(\lambda(n) = cf(n) \text{ is feasible}) = 1$$

$$\liminf_{n\to\infty} \Pr(\lambda(n) = c'f(n) \text{ is feasible}) < 1$$

III. Capacity of Arbitrary and Random Networks

In this section, we analyze the capacity of arbitrary networks and the capacity of random networks respectively.

A. Capacity of Arbitrary Networks

The following theorem on the capacity of arbitrary networks summarizes a major result of the paper:

Theorem 1. *Consider an arbitrary network $G(V_n, E)$. Let χ be the spatial and temporal scheduling algorithm used in $G(V_n, E)$. Let $k^\chi(n)$ be the average number of transmissions required to deliver a randomly chosen bit to its destination. Let $Y^\chi(n)$ be the average number of simultaneous transmissions in $G(V_n, E)$, the transport capacity $\eta^\chi(n)$ satisfies:*

$$\eta^\chi(n) = \frac{Y^\chi(n)\, W}{k^\chi(n)} \tag{5}$$

Proof: Recall from Section II that $v_i \in V_n$ represents a source node, $b_{i,j}$ represents the j^{th} bit transmitted from v_i to its destination(s), $d(v_i, j)$, and $N_{i,T}^\chi$ is the number of bits successfully transmitted by v_i during a time interval $[0, T]$.

Let $h_{i,j}^\chi$ be the number of transmissions required to deliver $b_{i,j}$ to its destination (or all destination nodes in $d(v_i, j)$ in the case of multicast) when the spatial and temporal scheduling algorithm $\chi \in \Phi$ is used. Let $Y_t^\chi(n)$ be the number of simultaneous transmissions in the network $G(V_n, E)$ at time t. It follows from the definitions of $k^\chi(n)$ and $Y^\chi(n)$ that

$$k^\chi(n) = \lim_{T\to\infty} \frac{\sum_{i=1}^{n}\sum_{j=1}^{N_{i,T}^\chi} h_{i,j}^\chi}{\sum_{i=1}^{n} N_{i,T}^\chi} \tag{6}$$

and

$$Y^\chi(n) = \lim_{T\to\infty} \frac{\int_0^T Y_t^\chi(n)\, dt}{T} \tag{7}$$

Let $\tau_{i,j,l}$, $1 \le l \le h_{i,j}$ be the time required to transmit $b_{i,j}$ in the l^{th} transmission and assume that the transmitting node is active during the entire $\tau_{i,j,l}$ interval. As each node transmits at the same data rate W, $\tau_{i,j,l} = \frac{1}{W}$.

Given the above definitions, we are now ready to prove the theorem.

Remark 2. The technique used in the proof is based on first considering the *total transmission time*, viz. the amount of traffic transmitted, measured in bits, multiplied by the time required to transmit each bit, in the network on the individual node level by aggregating the transmissions at different nodes, viz. $\sum_{i=1}^{n}\sum_{j=1}^{N_{i,T}^\chi}\sum_{l=1}^{h_{i,j}^\chi} \tau_{i,j,l}$ shown in the latter equations, and then evaluating the total transmission time in the network on the network level by considering the number of simultaneous transmissions in the entire network, viz. $\int_0^T Y_t^\chi(n)\, dt$ shown in the latter equations. Obviously, the two values must be equal. On the basis of this observation, the theorem can be established.

At time T, the total transmission time during $[0, T]$ is given by

$$\sum_{i=1}^{n}\sum_{j=1}^{N_{i,T}^\chi}\sum_{l=1}^{h_{i,j}^\chi} \tau_{i,j,l} + q_T^\chi = \frac{1}{W}\sum_{i=1}^{n}\sum_{j=1}^{N_{i,T}^\chi} h_{i,j}^\chi + q_T^\chi \tag{8}$$

where $\sum_{i=1}^{n}\sum_{j=1}^{N_{i,T}^\chi}\sum_{l=1}^{h_{i,j}^\chi} \tau_{i,j,l}$ accounts for the transmission time for traffic that has reached its destination and q_T^χ accounts for the transmission time for traffic still in transit at time T.

Let p_{max}^χ be the maximum length, measured in the number of hops, of all routes in $G(V_n, E)$ under χ, obviously $p_{max}^\chi < n$. Furthermore, since the network is stable, there exists a positive constant C_1, independent of T, such that the total amount of traffic in transit is bounded by $C_1 n$. Therefore

$$q_T^\chi \le \frac{p_{max}^\chi C_1 n}{W} < \frac{C_1 n^2}{W} \tag{9}$$

On the other hand, the total transmission time during $[0, T]$ evaluated on the network level equals $\int_0^T Y_t^\chi(n)\, dt$. Obviously

$$\sum_{i=1}^{n}\sum_{j=1}^{N_{i,T}^\chi}\sum_{l=1}^{h_{i,j}^\chi} \tau_{i,j,l} + q_T^\chi = \int_0^T Y_t^\chi(n)\, dt$$

When T is sufficiently large and the network is *stable*, using (9), the amount of traffic in transit is negligibly small compared with the amount of traffic that has already reached its destination. Therefore, the following relationship can be established:

$$\lim_{T\to\infty} \frac{\sum_{i=1}^{n}\sum_{j=1}^{N_{i,T}^\chi}\sum_{l=1}^{h_{i,j}^\chi} \tau_{i,j,l}}{\int_0^T Y_t^\chi(n)\, dt} = 1 \tag{10}$$

Noting that $\tau_{i,j,l} = \frac{1}{W}$, Equation (5) follows readily by combing (1), (6), (7) and (10). ∎

Remark 3. Equation (5) can also be obtained using Little's formula [17]. Intuitively, defining the *system* as consisting of the set of all wireless channels in $G(V_n, E)$, the long-term average effective arrival rate into the system equals $k^\chi(n)\,\eta^\chi(n)$, the long-term average amount of traffic in the system equals $Y^\chi(n)$ and the average time in the system equals $\frac{1}{W}$. Equation (5) then readily follows using Little's formula.

Equation (5) is obtained under a very generic setting and is applicable to networks of any size. It reveals that the network capacity can be readily determined by evaluating the average number of simultaneous transmissions $Y^\chi(n)$, the average number of transmissions required for reaching the destinations $k^\chi(n)$ and the link capacity W. The two parameters $Y^\chi(n)$ and $k^\chi(n)$ are often related. For example, in a network where each node transmits using a fixed transmission range $r(n)$, reducing $r(n)$ (while keeping the network connected) will cause increases in both $Y^\chi(n)$ and $k^\chi(n)$ and the converse. On the other hand, $Y^\chi(n)$ and $k^\chi(n)$ also have their independent significance, and can be optimized and studied independently of each other. For example, an optimally designed routing algorithm can distribute traffic evenly and avoid creating bottlenecks which helps to significantly increase $Y^\chi(n)$ at the expense of slightly increased $k^\chi(n)$ only, compared with the shortest-path routing.

The following corollary is an easy consequence of Theorem 1:

Corollary 4. *Under the same setting as that in Theorem 1,*

$$\eta(n) = \max_{\chi\in\Phi} \frac{Y^\chi(n)\,W}{k^\chi(n)} \le \frac{\max_{\chi\in\Phi} Y^\chi(n)\,W}{\min_{\chi\in\Phi} k^\chi(n)}$$

Corollary 4 allows the two key parameters that determining the capacity of $G(V_n, E)$, viz. $Y^\chi(n)$ and $k^\chi(n)$ to be studied separately. Parameter $\max_{\chi\in\Phi} Y^\chi(n)\,W$ is determined by the maximum number of transmissions that can be accommodated in the network area. Assuming that each node transmits using a fixed transmission range $r(n)$, each transmission will then "consume" a disk area of radius at least $\frac{C_2 r(n)}{2}$ in the sense that two simultaneous active transmitters must be separated by an Euclidean distance of at least $C_2 r(n)$, where $C_2 > 1$ is a constant determined by the interference model [1]. The problem of finding the maximum number of simultaneous transmissions, viz. $\max_{\chi\in\Phi} Y^\chi(n)$, can be converted into one that finds the maximum number of non-overlapping equal-radius circles that can be packed into A and then studied as a densest circle packing problem (see [18] for an example). Parameter $Y^\chi(n)$ can also be studied as the transmission capacity of networks [19]. For unicast transmission, $k^\chi(n)$ becomes the average number of hops between two randomly chosen source-destination pairs and has been studied extensively [20]. As will also be shown in Section IV, $Y^\chi(n)$ and $k^\chi(n)$ can be optimized separately to maximize the network capacity.

B. Capacity of Random Networks

We now consider the capacity of random networks. Note the connection between random networks and arbitrary networks that an instance of a random network forms an arbitrary network. The following result on the capacity of an arbitrary network can be obtained from Theorem 1.

Corollary 5. *Consider a random network G_n. Let $\chi \in \Phi^f$ be the spatial and temporal scheduling algorithm used in G_n. Let $k^\chi(n)$ be the average number of transmissions required to deliver a randomly chosen bit to its destination in an instance of G_n. Let $Y^\chi(n)$ be the average number of simultaneous transmissions in an instance of G_n. Both $k^\chi(n)$ and $Y^\chi(n)$ are random numbers associated with a particular (random) instance of G_n. If there exist two positive functions $f(n)$ and $g(n)$ such that*

$$\Pr\left(\lim_{n\to\infty} \frac{k^\chi(n)}{f(n)} = 1\right) = 1$$

and

$$\Pr\left(\lim_{n\to\infty} \frac{Y^\chi(n)}{g(n)} = 1\right) = 1$$

the throughput capacity $\lambda^\chi(n)$ satisfies:

$$\Pr\left(\lim_{n\to\infty} \frac{\lambda^\chi(n)}{\frac{g(n)W}{nf(n)}} = 1\right) = 1 \tag{11}$$

Proof: Using the union bound,

$$1 - \Pr\left(\frac{\lambda^\chi(n)}{\frac{g(n)W}{nf(n)}} = 1\right)$$
$$\le \left(1 - \Pr\left(\frac{k^\chi(n)}{f(n)} = 1\right)\right) + \left(1 - \Pr\left(\frac{Y^\chi(n)}{g(n)} = 1\right)\right)$$

The result in the corollary readily follows from Theorem 1. ∎

In reality, such two functions $f(n)$ and $g(n)$ required by Corollary 5 do not necessarily exist or are very difficult to find. Therefore asymptotic capacity of random networks is more commonly studied by investigating its upper and lower bounds. The following two corollaries give respectively an upper and a lower bound on the asymptotic capacity of random networks. These two corollaries are used in Section IV to examine the asymptotic capacity of random networks.

Corollary 6. *Consider a random network G_n. Let $\chi \in \Phi^f$ be the spatial and temporal scheduling algorithm used in G_n. Let $f(n)$ and $g(n)$ be two positive functions such that*

$$\lim_{n\to\infty} \Pr\left(\min_{\chi\in\Phi^f} k^\chi(n) \ge f(n)\right) = 1$$

and let $g(n)$ be a function of n such that

$$\lim_{n\to\infty} \Pr\left(\max_{\chi\in\Phi^f} Y^\chi(n) \le g(n)\right) = 1$$

the throughput capacity of G_n satisfies:

$$\lim_{n\to\infty} \Pr\left(\lambda\left(n\right) \le \frac{g\left(n\right)W}{nf\left(n\right)}\right) = 1 \tag{12}$$

Corollary 7. *Consider a random network G_n. Let $\chi \in \Phi^f$ be the spatial and temporal scheduling algorithm used in G_n. Let $f(n)$ and $g(n)$ be two positive functions such that*

$$\lim_{n\to\infty} \Pr\left(k^{\chi}\left(n\right) \le f\left(n\right)\right) = 1$$

and

$$\lim_{n\to\infty} \Pr\left(Y^{\chi}\left(n\right) \ge g\left(n\right)\right) = 1$$

the throughput capacity of G_n satisfies:

$$\lim_{n\to\infty} \Pr\left(\lambda\left(n\right) \ge \frac{g\left(n\right)W}{nf\left(n\right)}\right) = 1,\ \ \forall \chi \in \Phi^f \tag{13}$$

As implied in Corollaries 4 and 6, finding the throughput capacity upper bound of G_n is achieved by analyzing the upper bound of $Y^{\chi}(n)$, $\forall\chi \in \Phi^f$, viz. $\max_{\chi\in\Phi^f} Y^{\chi}(n)$, and then the lower bound of $k^{\chi}(n)$, $\forall\chi \in \Phi^f$, viz. $\min_{\chi\in\Phi^f} k^{\chi}(n)$, separately. An upper bound of $\max_{\chi\in\Phi^f} Y^{\chi}(n)$ can usually be found by analyzing the maximum number of simultaneous transmissions that can be accommodated in A, which is in turn determined by such parameters like SINR threshold or the transmission range, independent of χ. A lower bound of $\min_{\chi\in\Phi^f} k^{\chi}(n)$ can often be found by analyzing the average number of hops between a randomly chosen source-destination pair along the shortest path, which is mainly determined by the network topology and node distribution, and is independent of χ. Finding the throughput capacity lower bound of G_n often involves using a constructive technique, i.e. constructing a particular scheduling algorithm $\chi \in \Phi^f$ and analyzing the throughput capacity $\lambda^{\chi}(n)$ under χ by analyzing the associated parameters $k^{\chi}(n)$ and $Y^{\chi}(n)$.

IV. Applications of the Relationship to Determine the Capacity of Random Networks

In this section, to demonstrate the usage and applicability of our results developed in Section III, we use these results to re-derive some well-known results in the literature obtained for different networks and through the use of some intellectually challenging and customized techniques [1], [2], [4], [10], [12]–[14]. Due to the large amount of existing work in the area, it is not possible for us to include all of them. Therefore the random networks considered [1], [2], [4], [10], [12]–[14] are chosen as typical examples only. We show that the use of our result often lead to simpler analysis. Furthermore, through the intuitive understanding revealed in our result on the interactions of these capacity-impacting parameters, we point out limitations in some existing results and suggest further improvement.

A. Capacity of static ad-hoc networks with uniform transmission capability

In [1], Gupta and Kumar first considered a random network with n nodes uniformly and *i.i.d.* on a unit square A and each node is capable of transmitting at a fixed rate of W bit/s using a common channel. Every node chooses its destination randomly and independently of other nodes and transmits using a fixed and identical transmission range $r(n)$. Both the protocol model and the physical model are considered for modeling the interference. As shown in [1], results obtained assuming the protocol model can be readily extended to those assuming the physical model. Therefore, in this paper, we focus on the protocol model only.

In the protocol model, a direct transmission from a transmitter v_i located at X_i to a receiver v_j located at X_j is successful if the Euclidean distance between v_i and v_j is smaller than or equal to $r(n)$ *and* for every other node v_k simultaneously transmitting over the same channel, $\|X_k - X_j\| \ge (1+\triangle)\, r(n)$ where the parameter $\triangle > 0$ defines a guard zone which prevents a nearby node from transmitting on the same channel at the same time and $\|\bullet\|$ denotes the Euclidean norm.

Given the above setting, it is straightforward to show that each transmitter defines a disk with a radius equal to $\frac{1}{2}\triangle r(n)$ and centered at itself such that for the set of concurrent transmitters, their respective associated disks do not overlap. Therefore, each transmitter located in A "consumes" a disk of area at least $\frac{1}{4}\pi\left(\frac{1}{2}\triangle r(n)\right)^2 = \frac{\pi}{16}\triangle^2 r^2(n)$ in A (The worst case happens for a transmitter located at the corners of A where only one quarter of the disk falls in A.). It follows that

$$\max_{\chi\in\Phi} Y^{\chi}\left(n\right) \le \frac{1}{\frac{\pi}{16}\triangle^2 r^2\left(n\right)} \tag{14}$$

We now establish a lower bound of $\min_{\chi\in\Phi^f} k^{\chi}(n)$. Let A_1 be a $\frac{1}{4}\times\frac{1}{4}$ square located at the lower left corner of A and let A_2 be a $\frac{1}{4}\times\frac{1}{4}$ square located at the upper right corner of A. Using the property that nodes are uniformly and i.i.d. on A, it can be shown that *a.a.s.* the expected fraction of source-destination pairs with the source located in A_1 (or A_2) and the destination located in A_2 (or A_1) equals $2\times\frac{1}{16}\times\frac{1}{16} = \frac{1}{128}$. The minimum Euclidean distance between these source-destination pairs is $\frac{\sqrt{2}}{2}$ and thus the minimum number of hops between these source-destination pairs is $\frac{\sqrt{2}}{2r(n)}$. It is then follows that

$$\lim_{n\to\infty} \Pr\left(\min_{\chi\in\Phi^f} k^{\chi}\left(n\right) \ge \frac{\sqrt{2}}{256}\times\frac{1}{r\left(n\right)}\right) = 1 \tag{15}$$

Note that $\Phi^f \subseteq \Phi$, the following lemma can be obtained as an easy consequence of Corollary 6, (14) and (15).

Lemma 8. *In the random network considered by Gupta and Kumar [1] and assuming the protocol model, the per-node throughput satisfies*

$$\lim_{n\to\infty} \Pr\left(\lambda\left(n\right) \le \frac{2048\sqrt{2}}{\pi\triangle^2} W \frac{1}{nr\left(n\right)}\right) = 1$$

In Lemma 8, the upper bound of $\lambda(n)$ is expressed as a function of the transmission range $r(n)$ and an increase in $r(n)$ will reduce the upper bound. As the minimum transmission range required for the network to be *a.a.s.* connected is well known to be $r(n) = \sqrt{\frac{\log n + f(n)}{\pi n}}$ where $f(n) = o(\log n)$ and $f(n) \to \infty$ as $n \to \infty$ [21], the conclusion readily follows that $\lim_{n\to\infty} \Pr\left(\lambda(n) \le \frac{2048\sqrt{2}}{\triangle^2} W \frac{1}{\sqrt{\pi n \log n}}\right) = 1$.

We now proceed to obtaining a lower bound of $\lambda(n)$. The lower bound is obtained constructively. Specifically, using the scheduling algorithm $\chi \in \Phi^f$ presented in [1], we will analyze the associated $k^\chi(n)$ and $Y^\chi(n)$ and then obtain a lower bound of $\lambda^\chi(n)$. The lower bound obtained under a particular scheduling algorithm is of course also a lower bound of $\lambda(n)$.

We first recall the scheduling algorithm used in [1]. In [1], the network area A is partitioned into a set of Voronoi cells such that every Voronoi cell contains a disk of radius $\rho(n) = \sqrt{\frac{100 \log n}{\pi n}}$ and is contained in a disk of radius $2\rho(n)$. Packets are relayed sequentially from a node in a Voronoi cell to another node in an adjacent Voronoi cell along the Voronoi cells intersecting the direct line connecting the source and the destination. Denote the above scheduling scheme by χ.

The following result on a lower bound of $Y^\chi(n)$ is required for obtaining the lower bound of $\lambda^\chi(n)$:

Lemma 9. *In the random network considered by Gupta and Kumar [1] and assuming the protocol model, there exists a small positive constant c_1 such that the average number of simultaneous transmissions using χ satisfies*

$$\lim_{n\to\infty} \Pr\left(Y^\chi(n) \ge c_1 \frac{n}{\log n}\right) = 1$$

Note that each Voronoi cell has an area of at most $\frac{400 \log n}{n}$. Therefore the total number of Voronoi cells in A is at least $\frac{n}{400 \log n}$. The result in Lemma 9 follows readily from [1, Lemma 4.4][2] which states that *a.a.s.* there exists a positive *constant* c_2 such that every $(1 + c_2)$ slots, each cell gets at least one slot in which to transmit.

In addition to Lemma 9 , we also need the following lemma that provides an upper bound of $k^\chi(n)$.

Lemma 10. *Under the same setting as that in Lemma 9, there exists a positive constant c_3 such that*

$$\lim_{n\to\infty} \Pr\left(k^\chi(n) \le c_3 \sqrt{\frac{n}{\log n}}\right) = 1$$

Proof: In [1, Lemma 4.4], it was shown that for every line connecting an arbitrary source-destination pair, denoted by L, and every Voronoi cell $V \in \Gamma_n$ where Γ_n denotes the set of Voronoi cells, there exists a positive constant c_4 such that $\Pr(L \text{ intersect } V) \le c_4 \sqrt{\frac{\log n}{n}}$. Since each Voronoi cell has an area of at least $\frac{100 \log n}{n}$, the maximum number of Voronoi cells is bounded by $\frac{n}{100 \log n}$. Denoting by $N(L)$ the expected number of cells intersected by a randomly chosen source-destination line and using the union bound, it follows from the above results that $N(L) \le \frac{c_4}{100} \sqrt{\frac{n}{\log n}}$. This result, together with the result in [1, Lemma 4.8], which shows that there exists a sequence $\delta(n) \to 0$ as $n \to \infty$ such that $\Pr(\text{Every cell } V \in \Gamma_n \text{ contains at least one node}) \ge 1 - \delta(n)$, allow us to conclude that there exists a positive constant $c_3 = \frac{c_4}{100}$ such that

$$\lim_{n\to\infty} \Pr\left(k^\chi(n) \le c_3 \sqrt{\frac{n}{\log n}}\right) = 1$$

■

Combing the results in Lemmas 9 and 10, and also using Corollary 7, the following result can be shown:

Lemma 11. *In the random network considered by Gupta and Kumar [1] and assuming the protocol model, there exists a positive constant c_5 such that the per-node throughput satisfies*

$$\lim_{n\to\infty} \Pr\left(\lambda(n) \ge c_5 W \sqrt{\frac{1}{n \log n}}\right) = 1$$

Combing Lemmas 8 and 11, conclusion readily follows that *a.a.s.* $\lambda(n) = \Theta\left(W\sqrt{\frac{1}{n \log n}}\right)$.

In [1], Gupta and Kumar also investigated the capacity of arbitrary networks and showed that by placing nodes optimally and deterministically to maximize the capacity, e.g. on grid points, $\lambda(n) = \Theta\left(W\sqrt{\frac{1}{n}}\right)$. Realizing that when nodes are optimally placed, a reduced transmission range of $r(n) = \Theta\left(\sqrt{\frac{1}{n}}\right)$ is required for the network to be connected. Following a similar analysis leading to Lemma 8 and using Theorem 1 and (4), result readily follows that $\lambda(n) \le \frac{2048\sqrt{2}}{\pi(1+\triangle)^2} W \frac{1}{nr(n)}$ and hence $\lambda(n) = O\left(W\sqrt{\frac{1}{n}}\right)$. To obtain a lower bound of $\lambda(n)$, first it can be shown that when $r(n) = \Theta\left(\sqrt{\frac{1}{n}}\right)$, a scheduling algorithm χ can be easily constructed such that $Y^\chi(n) = \Theta\left(\sqrt{\frac{1}{n}}\right)$ and $k^\chi(n) = \Theta\left(\sqrt{\frac{1}{n}}\right)$ (for example an algorithm that first routes packets along a horizontal line to a node on the same vertical height as the destination node and then routes packets along a vertical line to the destination). Conclusion then follows that $\lambda^\chi(n) = \Theta\left(W\sqrt{\frac{1}{n}}\right)$ and $\lambda(n) = \Omega\left(W\sqrt{\frac{1}{n}}\right)$. Combing the lower and the upper bound, results follows using Theorem 1 that for an arbitrary network with optimally placed nodes, $\lambda(n) = \Theta\left(W\sqrt{\frac{1}{n}}\right)$.

The above results on the throughput capacity of arbitrary networks and random networks unsurprisingly are consistent with those in [1]. In addition to the above rigorous analysis, we also offer the following intuitive explanation on the capacity results in [1] using the relationship revealed in Section III. In the network considered by Gupta and Kumar, each node transmits using a fixed and identical transmission range $r(n)$. Therefore each transmission consumes a disk area of radius $\Theta(r(n))$ and $Y(n) = O\left(\frac{1}{r^2(n)}\right)$. Here we dropped the superscript χ when we discuss $k(n)$ and $Y(n)$ generally and the result does not depend on a particular scheduling algo-

[2]Strictly speaking, the result in [1, Lemma 4.4] was derived for nodes on the surface of a sphere. However the result can be readily modified for a planar area with due consideration to the boundary effect. Thus we ignore the difference and use the result directly.

rithm being used. Furthermore, a scheduling algorithm can be readily constructed that distributes the transmissions evenly across A such that $Y(n) = \Theta\left(\frac{1}{r^2(n)}\right)$. Given that the average Euclidean distance between a randomly chosen pair of source-destination nodes equals a constant, independent of n [22], it can be shown that $k(n) = \Theta\left(\frac{1}{r(n)}\right)$. Thus result follows that the throughput capacity $\lambda(n) = \Theta\left(\frac{W}{nr(n)}\right)$, viz. a smaller transmission range will result in a larger throughput. The minimum transmission range required for a random network to be *a.a.s.* connected is known to be $r(n) = \Theta\left(\sqrt{\frac{\log n}{n}}\right)$ while the minimum transmission range required for a network with optimally and deterministically deployed nodes is known to be $r(n) = \Theta\left(\sqrt{\frac{1}{n}}\right)$. Accordingly, the throughput capacity of random networks and arbitrary networks with optimally placed nodes are $\Theta\left(\frac{W}{\sqrt{n\log n}}\right)$ and $\Theta\left(\frac{W}{\sqrt{n}}\right)$ respectively. Therefore the $\frac{1}{\sqrt{\log n}}$ factor is the price in reduction of network capacity to pay for placing nodes randomly, instead of optimally.

B. Capacity of static networks with non-uniform transmission capability

In [2], Franceschetti *et al.* considered a network with n nodes uniformly and *i.i.d.* on a square of $\sqrt{n}\times\sqrt{n}$. A node v_i can transmit to another node v_j directly at a rate of

$$R(v_i, v_j) = \log\left(1 + \frac{Pl(X_i, X_j)}{N_0 + \sum_{k\in\Gamma_i} Pl(X_k, X_j)}\right)$$

where Γ_i denotes the set of indices of nodes that are simultaneously active as v_i, $l(X_i, X_j)$ denotes the power attenuation function and $l(X_i, X_j) = \min\left\{1, e^{-\gamma\|X_i - X_j\|}/\|X_i - X_j\|^{\alpha}\right\}$ with $\gamma > 0$ or $\gamma = 0$ and $\alpha > 2$, and N_0 represents the background noise. It is assumed that all nodes transmit at the same power level P. Each node chooses its destination randomly and independently of other nodes.

Remark 12. Strictly speaking, the results derived in Section III can only be used when the link capacity W is fixed. However it is straightforward to extend these results to study the capacity of the network considered in [2] where the link capacity depends on its SINR and is a variable. More specifically, given the two functions $g(n)$ and $f(n)$ defined in Corollary 6, if a third function $h(n)$ can be found such that $W = O(h(n))$, it can be readily shown using Corollary 6 that $\lambda(n) = O\left(\frac{g(n)h(n)}{nf(n)}\right)$. Similarly, given the two functions $g(n)$ and $f(n)$ defined in Corollary 7, if a third function $h(n)$ can be found such that $W = \Omega(h(n))$, then $\lambda(n) = \Omega\left(\frac{g(n)h(n)}{nf(n)}\right)$.

We first introduce the scheduling algorithm used in [2]. The network area is partitioned into non-overlapping squares of size c^2, called cells hereinafter. These cells are grouped into l^2 non-overlapping sets of cells where $l = 2(d+1)$ and within each set, adjacent cells are separated by an Euclidean distance of $(l-1)c$, see Fig. 4 of [2] for an illustration. Parameter d is a positive integer to be specified later. The time is also divided into l^2 time slots, which are equally distributed among the l^2 sets of cells. Within each time slot, at most one node in a cell can transmit. Furthermore, nodes located in cells belonging to the same set can transmit at the same time and nodes located in cells of different sets should use different time slots to transmit. The following result was established in [2] on the transmission rate between a pair of directly connected transmitter and receiver, which will be used in the later analysis:

Lemma 13. *Using the above scheduling algorithm, for any integer $d > 0$, there exists an $W(d) > 0$ such that a.a.s., when a node is scheduled to transmit, the node can transmit directly to any other node located within an Euclidean distance of $\sqrt{2}c(d+1)$ at rate $W(d)$. Furthermore, as d tends to infinity, we have*

$$W(d) = \Omega\left(d^{-\alpha}e^{-\gamma\sqrt{2}cd}\right)$$

Lemma 13 is essentially the same as Theorem 3 in [2] except that in [2, Theorem 3], it was considered that $W(d)$ is further multiplied by the fraction of time a cell is scheduled to be active, i.e. $1/l^2$, and the data rate is given in terms of rate per cell whereas in Lemma 13, $W(d)$ corresponds to the link rate, i.e. W in Theorem 1 and Corollaries 4, 5, 6 and 7.

In addition to the above result, capacity analysis in [2] also relies on the use of the percolation theory. More specifically, the $\sqrt{n}\times\sqrt{n}$ square is partitioned into $L = \left\lceil\frac{\sqrt{n}}{\kappa\log(\sqrt{n})}\right\rceil$ non-overlapping horizontal slabs where κ is a positive constant and each slab is of size $\frac{\sqrt{n}}{L}\times\sqrt{n}$. By symmetry, the $\sqrt{n}\times\sqrt{n}$ square can also be partitioned into $L = \left\lceil\frac{\sqrt{n}}{\kappa\log(\sqrt{n})}\right\rceil$ non-overlapping vertical slabs and each slab is of size $\sqrt{n}\times\frac{\sqrt{n}}{L}$. Using the percolation theory, it was shown that there exists positive constants c_1 and c_2 such that by directly connecting nodes separated by an Euclidean distance of at most c_1 only, *a.a.s.* there are at least $c_2\log(\sqrt{n})$ *disjoint* left-to-right (top-to-bottom) crossing paths within every horizontal (vertical) slab as $n\to\infty$ [2, Theorem 5]. These crossing paths are termed "highway" in [2]. Furthermore it was shown that for nodes not part of the highway, they can access their respective nearest highway node in single hops of length at most proportional to $\log(\sqrt{n})$, i.e. the Euclidean distance between non-highway nodes and their respective nearest highway nodes is $O(\log(\sqrt{n}))$.

On the basis of the above results, the following scheduling algorithm was used in [2] to deliver a packet from its source to its destination. The algorithm uses four separate phases, and in each phase time is divided into $l^2 = 4(d+1)^2$ slots where the value of d varies in each phase. The first phase is used by source nodes to access their nearest highway nodes; in the second phase, information is transported on the horizontal highways; in the third phase information is transported on vertical highways to highway nodes nearest their respective destinations; and in the fourth phase information is delivered to the respective destinations. The first and fourth phases use direct transmissions to deliver information from the source nodes to the respective highway nodes within Euclidean distance $O\left(\log\left(\sqrt{n}\right)\right)$ away; while the second and third phases use multiple hops to deliver information hop-by-hop along the highway and each hop is separated by a maximum Euclidean distance of c_1. Denote the above scheduling algorithm by ξ. The following result on the throughput capacity can be established:

Lemma 14. *Using the scheduling algorithm ξ, the throughput capacity in the random networks considered in [2] satisfies $\lambda^{\xi}(n) = \Omega\left(\frac{1}{\sqrt{n}}\right)$.*

Proof: Denote the per-node throughput in the four different phases by $\lambda_1^{\xi}(n)$, $\lambda_2^{\xi}(n)$, $\lambda_3^{\xi}(n)$ and $\lambda_4^{\xi}(n)$ respectively. We analyze $\lambda_1^{\xi}(n)$, $\lambda_2^{\xi}(n)$, $\lambda_3^{\xi}(n)$ and $\lambda_4^{\xi}(n)$ separately in the following paragraphs to obtain $\lambda^{\xi}(n)$ where $\lambda^{\xi}(n) = \min\left\{\lambda_1^{\xi}(n), \lambda_2^{\xi}(n), \lambda_3^{\xi}(n), \lambda_4^{\xi}(n)\right\}$.

We first analyze the link capacity in phase 1. From the earlier result that the Euclidean distance between non-highway nodes and their respective nearest highway nodes is $O\left(\log\left(\sqrt{n}\right)\right)$, there exists a positive constant c_3 such that *a.a.s.* the Euclidean distance between non-highway nodes and their respective nearest highway nodes is smaller than or equal to $c_3 \log n$. Choosing the value of d such that d is the smallest integer satisfying $\sqrt{2}c(d+1) \geq c_3 \log n$ and using Lemma 13, it follows that each non-highway node can transmit to its nearest highway node at a rate of $\Omega\left(d^{-\alpha}e^{-\gamma\sqrt{2}cd}\right) = \Omega\left((\log n)^{-\alpha} n^{-c\gamma\frac{\sqrt{2}}{2}}\right)$ *a.a.s.* using ξ.

Now we analyze the number of simultaneous transmissions in phase 1. Note that each highway node is separated from its nearest highway node by at most an Euclidean distance c_1. Therefore if a node has no other node located within an Euclidean distance of c_1 from itself, that node must be a non-highway node. Let N_h be the number of cells where each cell has at least one non-highway node, let N_o be the number of cells where each cell has exactly one non-highway node and let N_{iso} be the number of cells where each cell has exactly one node *and* that node has no other node located within an Euclidean distance of c_1 from itself. It follows from the above observation that

$$N_h \geq N_o \geq N_{iso} \tag{16}$$

Now we further analyze the asymptotic property of N_{iso}. Let Γ denote the set of all cells. Let I_i be an indicator random variable such that if the i^{th} cell, denoted by C_i, has exactly one node *and* that node has no other node located within an Euclidean distance of c_1 from itself, $I_i = 1$; otherwise $I_i = 0$. It follows from the definition of N_{iso} that $N_{iso} = \sum_{C_i \in \Gamma} I_i$. Using the property that nodes are uniform and i.i.d., it can be shown that $\lim_{n\to\infty} E(I_i) = p = c^2 e^{-c^2} e^{-\pi c_1^2}$ where $c^2 e^{-c^2}$ is the probability that C_i has exactly one node and $e^{-\pi c_1^2}$ is the probability that the node has no other node located within an Euclidean distance of c_1 from itself. Furthermore $Var(I_i) = E\left(I_i^2\right) - E^2(I_i) = E(I_i) - E^2(I_i)$ and $\lim_{n\to\infty} Var(I_i) = p - p^2$. Note that I_i and I_j are asymptotically independent as $n \to \infty$ if the associated cells C_i and C_j are separated by an Euclidean distance greater than or equal to $2c_1$. Denote by Γ_{ind} a maximal set of cells where adjacent cells are separated by an Euclidean distance $\mu = \left\lceil \frac{2c_1}{c} \right\rceil c$. It can be readily shown that $|\Gamma_{ind}| \geq \left(\frac{\sqrt{n}}{\mu+c}\right)^2$, where $|\Gamma_{ind}|$ denotes the cardinality of Γ_{ind}. Therefore using the central limit theorem,

$$\lim_{n\to\infty} \Pr\left(\sum_{C_i \in \Gamma_{ind}} I_i \geq \frac{n}{(\mu+c)^2} - h(n)\right) = 1$$

where $h(n)$ is an arbitrary positive function satisfying $h(n) = o(n)$ and $\lim_{n\to\infty} h(n) = \infty$. Noting that $N_{iso} = \sum_{C_i\in\Gamma} I_i \geq \sum_{C_i\in\Gamma_{ind}} I_i$ and using inequality (16) and the above equation, *a.a.s.* $N_h = \Omega(n)$ as $n \to \infty$. Using ξ, every $l^2 = 4(d+1)^2$ time slots, each cell gets one time slot to transmit. Therefore *a.a.s.* the average number of simultaneous transmissions in phase 1 equals $\Omega\left(\frac{n}{4(d+1)^2}\right)$.

Note that in phase 1, only direct transmission is allowed. It then follows from Corollary 7 that in the first phase, each node can have access a per-node throughput of $\lambda_1^{\xi}(n)$ where

$$\lambda_1^{\xi}(n) = \Omega\left(\frac{n}{4(d+1)^2}\right) \times \frac{\Omega\left((\log n)^{-\alpha} n^{-c\gamma\frac{\sqrt{2}}{2}}\right)}{n}$$

or equivalently $\lambda_1^{\xi}(n) = \Omega\left((\log n)^{-\alpha-2} n^{-c\gamma\frac{\sqrt{2}}{2}}\right)$.

Using a similar analysis, it can be shown that $\lambda_4^{\xi}(n) = \Omega\left((\log n)^{-\alpha-2} n^{-c\gamma\frac{\sqrt{2}}{2}}\right)$.

Now we analyze the throughput capacity in phases 2 and 3. We consider phase 2 first. In phase 2, d is chosen such that d is the smallest integer satisfying $\sqrt{2}c(d+1) \geq c_1$. It follows from Lemma 13, *a.a.s.* there exists a positive constant c_4 such that each highway node can transmit at a rate of at least c_4 bits per second, i.e. $W > c_4$ in phase 2.

As introduced earlier, *a.a.s.* each horizontal slab of size $\frac{\sqrt{n}}{L} \times \sqrt{n}$ has at least $c_2 \log\left(\sqrt{n}\right)$ *disjoint* highways where $L = \left\lceil \frac{\sqrt{n}}{\kappa \log\left(\sqrt{n}\right)} \right\rceil$. Two nodes belonging to two disjoint highways are separated by an Euclidean distance of at least c_1. Therefore the number of disjoint highways that can cross a cell is at most $\left\lceil \frac{c^2}{\frac{1}{4}\pi c_1^2} \right\rceil$. Each horizontal slab has $\frac{\sqrt{n}}{L} \times \frac{\sqrt{n}}{c^2}$ cells. Thus each horizontal highway crosses at most $\frac{\sqrt{n}}{L} \times \frac{\sqrt{n}}{c^2} \times \left\lceil \frac{c^2}{\frac{1}{4}\pi c_1^2} \right\rceil / \left(c_2 \log\left(\sqrt{n}\right)\right) = O\left(\sqrt{n}\right)$ cells. A packet moves by at least one cell in each hop. Therefore the average number of hops traversed by a packet in phase 2 is $O\left(\sqrt{n}\right)$.

Furthermore, *a.a.s.* the total number of disjoint horizontal highways is at least $c_2 L \log\left(\sqrt{n}\right) > \frac{c_2}{\kappa}\sqrt{n}$ and each horizontal highway crosses at least $\frac{\sqrt{n}}{c}$ cells where $\sqrt{n}$ is the minimum length of a left-to-right line in A. The number of disjoint highways that can cross a cell is at most $\left\lceil \frac{c^2}{\frac{1}{4}\pi c_1^2} \right\rceil$. Therefore, *a.a.s.* the number of cells where each cell contains at least one high-way node is at least $\frac{c_2}{\kappa}\sqrt{n} \times \frac{\sqrt{n}}{c} / \left\lceil \frac{c^2}{\frac{1}{4}\pi c_1^2} \right\rceil$. Using ξ, every $l^2 = 4\left(d+1\right)^2$ time slots, each cell gets one time slot to transmit. It follows that *a.a.s.* the average number of simultaneous transmissions in phase 2 is greater than or equal to $\frac{c_2}{\kappa}\sqrt{n} \times \frac{\sqrt{n}}{c} \times \frac{1}{l^2} / \left\lceil \frac{c^2}{\frac{1}{4}\pi c_1^2} \right\rceil = c_5 n$, where $c_5 = \frac{c_2}{\kappa} \times \frac{1}{c} \times \frac{1}{l^2} / \left\lceil \frac{c^2}{\frac{1}{4}\pi c_1^2} \right\rceil$ is a positive constant independent of n.

It follows from the above analysis and Corollary 7 that

$$\lambda_2^{\xi}\left(n\right) = \Omega\left(\frac{1}{\sqrt{n}}\right)$$

By symmetry, $\lambda_3^{\xi}\left(n\right) = \Omega\left(\frac{1}{\sqrt{n}}\right)$. By choosing the value of c such that $c\gamma\frac{\sqrt{2}}{2} < \frac{1}{2}$, the conclusion in the lemma readily follows. ■

Lemma 14 allows us to conclude that the throughput capacity in the random network considered by Franceschetti et al. satisfies $\lambda\left(n\right) = \Omega\left(\frac{1}{\sqrt{n}}\right)$, which is consistent with the result in [2].

In [2], essentially nodes are allowed to use two transmission ranges, viz. a smaller transmission range of $\Theta\left(1\right)$ for nodes forming the highways and a larger transmission range of $O\left(\log\left(\sqrt{n}\right)\right)$ for non-highway nodes to access their respective nearest highway nodes. Most transmissions are through the highway using the smaller transmission range while the larger transmission range is only used for the last mile in phases 1 and 4. It can be shown that phases 1 and 4 do not become the bottleneck in determining the throughput capacity. Therefore both $Y\left(n\right)$ and $k\left(n\right)$ are dominated by the smaller transmission range and accordingly $Y\left(n\right) = \Theta\left(n\right)$, $k\left(n\right) = \Theta\left(\sqrt{n}\right)$. Furthermore, as a consequence of Lemma 13, $W = \Omega\left(1\right)$. It then readily follows that $\lambda\left(n\right) = \Omega\left(\frac{1}{\sqrt{n}}\right)$. This higher throughput capacity, compared with that in [1], is achieved by allowing nodes to adjust their transmission capabilities as required.

In [13], Chau, Chen and Liew showed that the higher throughput capacity of $\lambda\left(n\right) = \Omega\left(\frac{1}{\sqrt{n}}\right)$ can also be achieved in large-scale CSMA wireless networks if wireless nodes performing CSMA operations are allowed to use two different carrier-sensing ranges. The capacity analysis in [13] is based on two findings: a) by adjusting the count-down rate, a tunable parameter in CSMA protocols, of each node, a distributed and randomized CSMA scheme can achieve the same capacity as a centralized deterministic scheduling scheme [23]; b) by using the highway system defined in [2], a higher throughput capacity of $\lambda\left(n\right) = \Omega\left(\frac{1}{\sqrt{n}}\right)$ can be achieved using a centralized deterministic scheduling algorithm. Using [13, Lemma 9], which states that in CSMA schemes, there exists a set of count-down rates such that the throughput of each and every link is not smaller than that can be achieved with a centralized deterministic scheduling scheme, and a similar analysis above for analyzing the capacity of networks in [2], the result in [13] can also be obtained using the relationship established in this paper. Except for some analysis on particular details of CSMA networks, i.e. hidden node problem and distributedness of CSMA protocols, the analysis is similar as the analysis earlier in the section and hence is omitted in the paper.

Observing that in a large network, a much smaller transmission range is required to connect most nodes in the network (i.e. forming a giant component) whereas the larger transmission range of $\Theta\left(\sqrt{\frac{\log n}{n}}\right)$ is only required to connect the few hard-to-reach nodes [24], a routing scheme can be designed, which achieves a per-node throughput of $\lambda\left(n\right) = \Theta\left(\frac{1}{\sqrt{n}}\right)$ and does not have to use the highway system, such that a node uses the smaller transmission ranges for most communications and only uses the larger transmission if the next-hop node cannot be reached when using the smaller transmission ranges.

C. Capacity of mobile ad-hoc networks

In [4], Grossglauser and Tse considered mobile ad hoc networks consisting of n nodes uniformly and i.i.d. on a unit square A initially. Nodes are mobile and the spatial distribution of nodes is stationary and ergodic with stationary distribution uniform on A. The trajectories of nodes are i.i.d. Each node chooses its destination randomly and independently of other nodes. At time t, a node v_i can transmit directly to another node v_j at rate W if the SINR at v_j is above a prescribed threshold β:

$$\frac{P_i\left(t\right)\gamma_{ij}\left(t\right)}{N_0 + \frac{1}{L}\sum_{k\in\Gamma_i(t)} P_k\left(t\right)\gamma_{kj}\left(t\right)} > \beta$$

where N_0 is the background noise power, L is the processing gain, $\Gamma_i\left(t\right)$ is the set of nodes, not including v_i itself, simultaneously transmitting with v_i at time t and $P_i\left(t\right)$ is the transmitting power of v_i at time t. The transmitting power $P_i\left(t\right)$ is determined by the scheduling algorithm and is chosen to be a constant independent of n. For a narrowband system $L = 1$. Parameter $\gamma_{ij}\left(t\right)$ is the channel gain and is given by $\gamma_{ij}\left(t\right) = \left\|X_i\left(t\right) - X_j\left(t\right)\right\|^{-\alpha}$ where $X_i\left(t\right)$ represents the location of v_i at time t and α is a parameter greater than 2.

A two-hop relaying strategy is adopted. In the first phase, a source transmits a packet to a nearby node (acting as a relay). As the source moves around, different packets are transmitted to different relay nodes. In the second phase, either the source or a relay transmits the packet to the destination when it is close to the destination and is scheduled to transmit to the destination. Within each time slot, the set of concurrent transmissions are scheduled randomly and independently of transmissions in the previous time slot. More specifically, a parameter $\theta \in (0, 0.5)$, called the transmitter density, is fixed first. $n_S = \theta n$ number of nodes are randomly designated as transmitters and the remaining nodes are designated as *potential receivers*. Denote the set of potential receivers by R_t. Each transmitter transmits its packets to its nearest neighbor among all nodes in R_t. Among all the n_S sender-receiver pairs, only those whose SINR is above β are retained. Denote the number of such pairs by N_t. Note that the set of transmitter-receiver pairs is random in each time slot (thus N_t is a random integer) and depends on the time varying locations of nodes. Denote the above scheduling algorithm by χ.

From the above description of the scheduling algorithm χ, obviously $1 \le k^{\chi}(n) \le 2$. Furthermore, it can be shown [4, Theorem III-4] that $Y^{\chi}(n) = E(N_t)$ and that there exists a positive constant c such that

$$\lim_{n\to\infty} \Pr\left(\frac{Y^{\chi}(n)}{n} \ge c\right) = 1 \tag{17}$$

The following result on the asymptotical throughput capacity of the random mobile ad hoc networks considered in [4] readily follows:

Lemma 15. *In the random mobile ad hoc network considered by Grossglauser and Tse [4], a.a.s.* $\lambda(n) = \Theta(1)$.

Proof: We first consider an upper bound of $\lambda(n)$. It can be easily shown that $\min_{\chi\in\Phi^f} k^{\chi}(n) = \Omega(1)$ and $\max_{\chi\in\Phi^f} Y^{\chi}(n) = O(n)$. It then follows using Corollary 6 that $\lambda(n) = O(1)$.

Now we consider the lower bound. Using the two-phase scheduling algorithm χ introduced above, $1 \le k^{\chi}(n) \le 2$. Using the above result, (17) and Corollary 7, conclusion readily follows that $\lim_{n\to\infty} \Pr\left(\lambda(n) \ge \frac{c}{2}W\right) = 1$ where W is a constant independent of n. ∎

The capacity result in [4] and the use of the two hop relaying strategy can be intuitively explained as follows. Obviously the two-hop relaying strategy helps to cap $k^{\chi}(n)$ at 2. Compared with a one-hop strategy where a source is only allowed to transmit when it is close to its destination, the two-hop relaying strategy also helps to spread the traffic stream between a source-destination pair to a large number of intermediate relay nodes such that in steady state, the packets of every source node will be distributed across all the nodes in the network. This arrangement ensures that every node in the network will have packets buffered for every other node. Therefore a node always has a packet to send when a transmission opportunity is available. Thus the role of the two-hop relaying strategy, compared with a one-hop strategy is to maximize $Y^{\chi}(n)$ such that $Y^{\chi}(n) = \Theta(n)$ [4] at the expense of a slightly increased $k^{\chi}(n)$. A lower bound on $\lambda(n)$ readily results using $Y^{\chi}(n) = \Theta(n)$, $k^{\chi}(n) \le 2$ and Corollary 7. An upper bound on $\lambda(n)$ can be easily obtained using Corollary 6. Therefore conclusions readily follows for $\lambda(n)$. Capacity of mobile ad-hoc networks assuming other mobility models and routing strategies [10] can also be obtained analogously.

Given the insight revealed in Theorem 1 and Corollaries 4, 5, 6 and 7, it can be readily shown that in a network with a different traffic model than that in [4], e.g. each node has an infinite stream of packets for every other node in the network, a one-hop strategy can also achieve a transport capacity of $\eta(n) = \Theta(n)$. Therefore the insight revealed in Theorem 1 and Corollaries 4, 5, 6 and 7 helps to design the optimum routing strategy for different scenarios of mobile ad-hoc networks.

D. Multicast capacity

In the previous three subsections, we have used Theorem 1 and Corollaries 4, 5, 6 and 7 established in Section III to analyze the capacity of the random static and mobile networks considered in [1], [2], [4]. An upper bound on the throughput capacity can often be readily obtained using Corollary 6. For the lower bound, the procedure generally involves using existing results and scheduling algorithms already established in [1], [2], [4] to obtain $k^{\chi}(n)$ and $Y^{\chi}(n)$, and then using Corollary 7 to obtain the throughput capacity lower bound. The use of Theorem 1 and Corollaries 4, 5, 6 and 7 often results in simpler analysis. Similar methods can also be used to obtain the multicast capacity and capacity of hybrid networks considered in this subsection and the next subsection. To avoid repetition and to focus on the main ideas, in this subsection and the next subsection, we choose to give an intuitive explanation of the results on the multicast capacity and capacity of hybrid networks only using Theorem 1 and Corollaries 4, 5, 6 and 7.

In [14], Li considered the multicast capacity of a network with n nodes uniformly and i.i.d. on a $a \times a$ square, denoted by A. It is assumed that all nodes have the same transmission range $r(n) = \Theta\left(\sqrt{\frac{\log n}{n}}\right)$ and are capable of transmitting at W bits per second over a common channel. Furthermore, a protocol interference model is assumed and two concurrent transmitters must be separated by an Euclidean distance of at least $(1+\triangle)\, r(n)$. A subset $S \subseteq V_n$ of $n_s = |S|$ nodes are randomly chosen to serve as the source nodes of n_s multicast sessions where n_s is assumed to be sufficiently large. Each node $v_i \in S$ chooses a set of $l-1$ points randomly and independently from A and multicast its data to the nearest node of each point. Denote by Φ^f the set of scheduling algorithm that allocate the transport capacity equally among all multicast sessions. Denote by $\eta^{\chi}(n)$ the maximum transport capacity that can be achieved *a.a.s.* using χ. The multicast capacity $\eta(n)$ is the maximum transport capacity that can be achieved *a.a.s.* for all $\chi \in \Phi^f$: $\eta(n) = \max_{\chi\in\Phi^f} \eta^{\chi}(n)$. Note that a bit multicast to $l-1$ destinations is counted as a single bit in the calculation of the multicast transport capacity. Therefore our definition of transport capacity in Section II is consistent with the definition of the multicast transport capacity in [14] and the results established in Section III can be used directly here.

We first consider the situation that $l = O\left(\frac{n}{\log n}\right)$. We will obtain an upper bound on the multicast transport capacity. It can be readily shown that $\max_{\chi\in\Phi^f} Y^{\chi}(n) = O\left(\frac{1}{r^2(n)}\right)$. Furthermore, it can be shown that *a.a.s.* any multicast tree spanning l nodes that are randomly placed in A has a total edge length of at least $ca\sqrt{l}$ [14, Lemma 9] where c is a positive constant. It follows that $\min_{\chi\in\Phi^f} k^{\chi}(n) = \Omega\left(\frac{ca\sqrt{l}}{r(n)}W\right)$. Therefore, as an easy consequence of Corollary 6, $\eta(n) = O\left(\frac{1}{r(n)\sqrt{l}}W\right) = O\left(\frac{W}{\sqrt{l}}\sqrt{\frac{n}{\log n}}\right)$.

To obtain a lower bound on the multicast transport capacity, a scheduling algorithm χ is constructed (see [14] for a detailed description of the scheduling algorithm χ). More specifically, A is partitioned into non-overlapping squares and each square is of size $\frac{r(n)}{\sqrt{5}} \times \frac{r(n)}{\sqrt{5}}$. Calling these squares cells, the total number of cells equals $\frac{5a^2}{r^2(n)}$. Furthermore, nodes located in adjacent cells are directly connected, where two cells are *adjacent* if they have at least one point in common. Using the property that nodes are uniformly and i.i.d., *a.a.s. every* cell has at least one node [14, Lemma 18]. Dividing time into time slots of equal length, it can be shown that there exists a positive integer c_1, independent of n, such that every $\frac{1}{c_1}$ time slots, *every* cell gets at least one time slot to transit. Using the above results, *a.a.s.* $Y^{\chi}(n) \geq \frac{5a^2}{c_1 r^2(n)}$.

Choosing one node from each cell, it can be shown that these nodes form a connected component, termed *connected dominating set*. All other nodes are directly connected to at least one node in the connected dominating set. Multicast traffic is routed using the connected dominating set. Using the result that for an arbitrary cell, *a.a.s.*, the probability that a randomly chosen multicast flow is routed via the cell is at most $c_2\sqrt{l}r(n)/a$ [14, Lemma 20], *a.a.s.* the number of cells crossed by a randomly chosen multicast flow is at most $\frac{c_2\sqrt{l}r(n)}{a} \times \frac{5a^2}{r^2(n)} = 5c_2 a\frac{\sqrt{l}}{r(n)}$. Therefore *a.a.s.* $k^{\chi}(n) = O\left(\frac{\sqrt{l}}{r(n)}\right)$ and $\eta^{\chi}(n) = \Omega\left(\frac{1}{r(n)\sqrt{l}}W\right) = \Omega\left(\frac{W}{\sqrt{l}}\sqrt{\frac{n}{\log n}}\right)$.

Combing the upper and lower bounds on the transport capacity, conclusion can be obtained that when $l = O\left(\frac{n}{\log n}\right)$, *a.a.s.* $\eta(n) = \Theta\left(\frac{W}{\sqrt{l}}\sqrt{\frac{n}{\log n}}\right)$.

When $l = \Omega\left(\frac{n}{\log n}\right)$, the situation becomes slightly different. More specifically, the density of the multicast destination nodes becomes high enough such that the probability that a single transmission will deliver the packet to more than one multicast destination nodes becomes high. In fact, using the above connected dominating set, it can be shown that *a.a.s.* the number of transmissions required to deliver a packet to all nodes (hence the $l-1$ multicast destination nodes) is at most $\frac{5a^2}{r^2(n)}$, which is independent of l. Consequently $k(n) = \Theta\left(\frac{1}{r^2(n)}\right)$. Conclusion then readily follows that when $l = \Omega\left(\frac{n}{\log n}\right)$, $\eta(n) = \Theta(W)$.

E. Capacity of hybrid networks

Now we consider the impact of infrastructure nodes on network capacity. In addition to n ordinary nodes uniformly and i.i.d. on a unit square A, a set of M infrastructure nodes are regularly or randomly placed in the same area A where $M \leq n$. These infrastructure nodes act as relay nodes only and do not generate their own traffic. Following the same setting as that in [12], it is assumed that the infrastructure nodes have the same transmission range $r(n) = \Theta\left(\sqrt{\frac{\log n}{n}}\right)$ and link capacity W when they communicate with the ordinary nodes and these infrastructure nodes are inter-connected via a backbone network with much higher capacity. Furthermore a protocol interference model is adopted.

The routing algorithm used in the above network [12] has been optimized such that these infrastructure nodes do not become the bottleneck, which may be possibly caused by a poorly designed routing algorithm diverting excessive amount of traffic to the infrastructure nodes.

First consider the case when $M = o\left(\frac{1}{r^2(n)}\right) = o\left(\frac{n}{\log n}\right)$. In this situation, the number of transmissions involving an infrastructure node as a transmitter or receiver is small and has little impact on $Y(n)$, which has been shown in previous subsections to be $\Theta\left(\frac{1}{r^2(n)}\right)$. Furthermore, it can be shown that the average Euclidean distance between a randomly chosen pair of infrastructure nodes is $\Theta(1)$ [22]. That is, a packet transmitted between two infrastructure nodes moves by an Euclidean distance of $\Theta(1)$ whereas a packet transmitted by a pair of directly connected ordinary nodes moves by an Euclidean distance of $\Theta(r(n))$. Therefore a transmission between two infrastructure nodes *is equivalent to* $\Theta\left(\frac{1}{r(n)}\right)$ transmissions between ordinary nodes and the *equivalent* average number of simultaneous ordinary node transmissions equals $\Theta\left(\left(\frac{1}{r^2(n)} - M\right) + \frac{M}{r(n)}\right) = \Theta\left(\frac{1}{r^2(n)} + \frac{M}{r(n)}\right)$. It follows using a similar procedure outlined in Section IV-A that

$$\eta(n) = \Theta\left(\frac{\left(\frac{1}{r^2(n)} + \frac{M}{r(n)}\right)W}{\frac{1}{r(n)}}\right) = \Theta\left(\left(\sqrt{\frac{n}{\log n}} + M\right)W\right)$$

Therefore when $M = o\left(\sqrt{\frac{n}{\log n}}\right)$, the infrastructure nodes have little impact on the order of $\eta(n)$; when $M = \Omega\left(\sqrt{\frac{n}{\log n}}\right)$ (and $M = o\left(\frac{n}{\log n}\right)$), the infrastructure nodes start to have dominant impact on the network capacity and the above equation on the transport capacity reduces to $\eta(n) = \Theta(MW)$. Noting that the fundamental reason why infrastructure nodes improve capacity is that they help a pair of ordinary nodes separated by a large Euclidean distance to leapfrog some very long hops, thereby reducing $k(n)$. Therefore the same result in the above equation can also be obtained by analyzing the reduction in $k(n)$ directly. The analysis is albeit more complicated.

When $M = \Omega\left(\frac{n}{\log n}\right)$, assuming that the transmission range stays the same as when $M = o\left(\frac{n}{\log n}\right)$ at $r(n) = \Theta\left(\sqrt{\frac{\log n}{n}}\right)$, the number of simultaneous active infrastructure nodes becomes limited by the transmission range. More specifically, only $\Theta\left(\frac{1}{r^2(n)}\right) = \Theta\left(\frac{n}{\log n}\right)$ infrastructure nodes can be active simultaneously. Furthermore, *a.a.s.* each ordinary node can access its nearest infrastructure node in $\Theta(1)$ hops. Following a similar analysis as that in the last paragraph, it can be shown that $\eta(n) = \Theta\left(\frac{nW}{\log n}\right)$ when $M = \Omega\left(\frac{n}{\log n}\right)$.

The above results are consistent with the results in [12].

However we further note that when $M = \Omega\left(\frac{n}{\log n}\right)$, a smaller transmission range of $r(n) = \Theta\left(\frac{1}{\sqrt{M}}\right)$ is sufficient for an ordinary node to reach its nearest infrastructure node and hence achieving connectivity. A smaller transmission range helps to increase $Y(n)$ and it has been shown previously that $Y(n) = \Theta\left(\frac{1}{r^2(n)}\right)$, while $k(n) = \Theta(1)$. Therefore the achievable transport capacity using the smaller transmission range is $\eta(n) = \Theta(MW) = \Omega\left(\frac{nW}{\log n}\right)$, which is better than the result $\eta(n) = \Theta\left(\frac{nW}{\log n}\right)$ in [12]. Moreover, different from the conclusion in [12] suggesting that when $M = \Omega\left(\frac{n}{\log n}\right)$, further investment in infrastructure nodes will not lead to improvement in capacity, our result suggests that even when $M = \Omega\left(\frac{n}{\log n}\right)$, capacity still keeps increasing linearly with M. This capacity improvement is achieved by reducing the transmission range with the increase in M.

V. Conclusion and Further Work

In this paper, we show that the network capacity can be determined by estimating the three parameters, viz. the average number of simultaneous transmissions, the link capacity and the average number of transmissions required to deliver a packet to its destination. Our result is valid for both finite networks and asymptotically infinite networks. We have demonstrated the usage and the applicability of our result by using the result to analyze the capacity of a number of different networks studied in the literature. The use of our result often simplifies analysis. More importantly, we showed that the same methodology can be used to analyze the capacity of networks under different conditions. Therefore our work makes important contributions towards developing a generic methodology for network capacity analysis that is applicable to a variety of different scenarios. Furthermore, as illustrated in Section IV-E, the simple capacity-determing relationship revealed in the paper can be used as a powerful and convenient tool to quickly estimate the capacity of networks based on an intuitive understanding of the networks. However we readily acknowledge that the analysis of the three parameters: the average number of simultaneous transmissions, the link capacity and the average number of transmissions required to deliver a packet to its destination, may still need some customized analysis that takes into account details of a network different from other networks.

For asymptotically infinite random networks, the use of our result to estimate the capacity often involves estimating the capacity upper bound and the capacity lower bound separately. The capacity upper bound can be readily obtained by estimating the maximum number of simultaneously active transmissions satisfying the interference constraints that can be accommodated in the network area and the minimum number of transmissions required to deliver a packet. The capacity lower bound is more difficult to find. It usually involves constructing a spatial and temporal scheduling algorithm for the particular network and demonstrating that the network capacity is achievable using that algorithm. It remains to be investigated on whether a generic technique can be found such that the capacity lower bound can be obtained without resorting to designing customized algorithm for a particular network.

In this paper, we have ignored physical layer details by assuming that each node is capable of transmitting at a fixed and identical data rate. This assumption allows us to focus on the topological aspects of networks that determine capacity. It remains to be investigated on how to develop a generic methodology to incorporate the impact of physical layer techniques, e.g. coding and MIMO, on capacity. We refer readers to recent work by Jiang et al. [25], which suggests a possible direction to extend our result to incorporate physical layer details.

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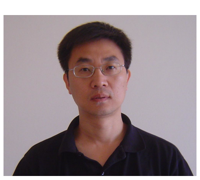

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