MLE-based Localization and Performance Analysis in Probabilistic LOS/NLOS Environment

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Abstract

Non-line-of-sight (NLOS) propagation, which widely exists in wireless systems, will degrade the performance of wireless positioning system if it is not taken into consideration in the localization algorithm design. Different from existing approaches which treat NLOS measurements as outliers and only work when there is a large number of measurements, in this paper, we propose to use Maximum Likelihood Estimator (MLE) for localization, which utilizes all the available measurements and explicitly takes the probabilities of occurrences of LOS and NLOS propagations into account. Furthermore, to evaluate the accuracy of the proposed localization algorithm, the position error bound of the positioning system is derived using Cramer-Rao Lower Bound (CRLB). Through numerical analysis, the impact of NLOS propagation on the position error bound is evaluated. The performance of our proposed algorithm is verified by both simulations and real world experimental data.

Keywords: Non-line-of-sight, distance-related probability, Cramer-Rao Lower Bound, Position Error Bound, Maximum Likelihood Estimator

1. Introduction

The tremendous development of wireless communication technology in the recent decade has sparked a large number of novel applications [1–7]. Particularly, driven by the increasing demand from location-based services, wireless lo-

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calization systems have attracted significant research interest [7–10]. Numerous localization systems have been developed, e.g., Global Positioning System (GPS), Cellular-based Positioning System (CPS), Wireless Local Area Network (WLAN)-based positioning system, etc. These positioning systems can often deliver very good performance in ideal environment, i.e., line-of-sight (LOS) propagation environment. In more complicated environment however, especially, indoors or urban areas, wireless signals often suffer from non-line-of-sight (NLOS) propagation, which will significantly degrade the localization performance of the conventional localization techniques.

Many techniques have been developed to deal with localization problem in LOS/NLOS environment [11–16]. The correct utilization of the sight condition (LOS or NLOS) of each path is very important during the localization, therefore, *a priori* identification of the LOS/NLOS paths is necessary in many localization approaches [11–13]. By analyzing the channel features, e.g., mean and standard deviation [11], the NLOS paths can be identified by hypothesis testing. Usually, a reliable identification needs to collect a large number of measurements, which is not suitable for real time localization. Since the statistical property of NLOS propagation is much more complicated than that of LOS propagation, many algorithms propose to discard the NLOS measurements [17, 18]. Only the LOS measurements are utilized for localization. However, this kind of approach needs sufficient number of LOS measurements, which renders it not suitable for environment with only a small number of base stations (BSs).

Generally, when the distance between a receiver and a transmitter is small, LOS propagation is more likely to occur. When the distance between the receiver and the transmitter increases, the probability of LOS will decrease, and the signals will suffer from NLOS propagation with a higher probability. This phenomenon is also verified by LOS/NLOS models suggested in the 3rd Generation Partnership Project (3GPP) [5, 6], which is derived based on large number of field measurements. Note that, there are some existing algorithms considering the LOS/NLOS probabilities [12, 19]. They usually assume the LOS/NLOS probabilities are fixed and known *a priori*, which does not capture the fact that the occurrence of LOS and N-LOS propagations varies with the distance between the transmitter and the receiver, or the distance-related LOS/NLOS probabilities are inappropriately modeled [20]. To the best of our knowledge, there is little work that takes the distance-related LOS/NLOS probabilities as suggested in 3GPP standard [5, 6] into localization algorithm design and performance analysis.

Based on the aforementioned observations, in this paper, we will study the localization problem considering distance-related LOS/NLOS probabilities.[1] The main contributions of this paper are summarized as follows:

- We propose an MLE-based localization algorithm for a single target localization problem, which incorporates the distance-related LOS/NLOS probabilities. The proposed algorithm uses both LOS and NLOS measurements and does not require identification of LOS and NLOS paths before localization.
- Cramer-Rao Lower Bound (CRLB) is used to analyze the localization accuracy of the considered positioning system under a general NLOS bias model. Based on CRLB, the position error bound (PEB) is derived. Localization performances under four different and widely used distributions of NLOS bias are numerically analyzed and compared.
- The performance of the proposed localization algorithm is verified by simulations, which considers a cellular network, and experiments in a realistic indoor scenario using range measurements obtained by ultra-wide bandwidth (UWB) devices. Both simulation and experimental studies demonstrate that it is important to consider the distance-related probabilities in localization, which results in better localization accuracy.

The remainder of this paper is organized as follows. Section 2 gives the related work. Section 3 introduces the system model. Section 4 presents the MLE-based localization algorithm. CRLB-based localization accuracy analysis is conducted in section 5. Both simulation and experimental results are presented in section 6. Conclusions are drawn in section 7.

2. Related Work

Many techniques have been developed to tackle the localization problem in LOS/NLOS environment. Basically, the existing approaches can be divided into three categories depending on how much *a priori* knowledge about the NLOS propagations is available: 1) both the statistical property of NLOS measurement and the sight condition of each path are perfectly known; 2) the statistical property of NLOS measurement is known but the sight condition of each path is unknown; 3) both the statistical property of NLOS measurement and the sight condition of each path are unknown.

[1]Part of this work has been made and reported in [21]

When both the statistical property of NLOS measurement and the sight condition of each path are perfectly known, MLE is the most common method for localization since it is statistically optimal and can asymptotically achieve the CRLB [12, 18, 22]. Least squares (LS) based methods can also be applied to such case. Nguyen *et al.* analyzed the localization performances of LS, squared-range LS, and squared-range weighted LS solutions from the fundamental limit [16]. However, since it is very difficult to obtain the exact sight condition of each path in advance, this is an ideal case that rarely exists in practice.

When the statistical property of NLOS measurement is known but the sight condition of each path is unknown, a large part of the existing methods need to conduct identification of the sight condition of each path beforehand. By analyzing the channel features, e.g., mean and standard deviation [11], Kurtosis [13], Rician K factor [13], etc., the LOS/NLOS paths can be identified by hypothesis testing. After the identification, Chan *et al.* proposed to estimate the target's position by only taking advantage of the LOS measurements [17]. In such kind of method, sufficient number of LOS measurements are necessary for accurate localization, which makes this method not suitable for a system with limited number of BSs. Some other kinds of methods do not discard the NLOS measurements, all the measurements are utilized for localization [15, 16, 23]. By assigning different weights to the measurements, the impact of NLOS measurements on the localization accuracy will be mitigated. All of the above methods are based on the identification results of the sight conditions, which will directly affect the localization accuracy. There are also some methods that do not need to identify LOS/NLOS conditions beforehand. By assuming *a priori* probability of LOS propagation, Cong *et al.* in [12] and Yin *et al.* in [24] proposed MLE-based localization algorithms, and Lui *et al.* in [19] proposed two Maximum *A Posteriori* (MAP) localization algorithms. The performance of the proposed algorithms will be affected by the values of the fixed LOS/NLOS probabilities. Moreover, how to set the values of LOS/NLOS probabilities is still unclear.

When both the statistical property of NLOS measurement and the sight condition of each path are unknown, the algorithms that rely on the statistical knowledge of measurement errors will not be applicable. Many algorithms that do not need the statistical knowledge of measurement errors are proposed. Based on some empirical knowledge, for instance, in time-of-arrival (TOA) based localization system, the NLOS bias is positive and much larger than the measurement noise, thus several robust algorithms are proposed. Wang *et al.* proposed to take all the measurements as NLOS measurements and formulated the localization problem as a semi-definite programming (SDP) to mitigate the effect of NLOS errors on the localization performance, which only considers the constraints on the upper bound of the NLOS errors [25]. Ekambaram *et al.* also formulated the localization problem of a set of points in Euclidean space as an SDP, which considers the nonnegativity of the N-LOS errors [26]. The robust localization algorithms can be applied to all the above three cases. However, the obtained localization accuracy might be lower than that of the algorithms which make use of the statistical property of measurements.

In this paper, we consider the case when the statistical property of NLOS measurement is known but the sight condition of each path is unknown, and design a localization algorithm that takes into account the distance-related LOS/NLOS probabilities and does not need *a priori* identification of LOS/NLOS paths.

Table 1: Notations Summary

Symbol	Definition
$p^L(\varepsilon)$	PDF of ε under LOS propagation
$p^{NL}(\varepsilon)$	PDF of ε under NLOS propagation
$p_G^{NL}(\varepsilon)$	PDF of ε under NLOS propagation when NLOS bias follows Gaussian distribution
$p_E^{NL}(\varepsilon)$	PDF of ε under NLOS propagation when NLOS bias follows exponential distribution
$p_U^{NL}(\varepsilon)$	PDF of ε under NLOS propagation when NLOS bias follows uniform distribution
$p_C^{NL}(\varepsilon)$	PDF of ε under NLOS propagation when NLOS bias is a constant
$\mathcal{N}(\mu, \sigma^2)$	Gaussian distribution with mean μ and variance σ^2
$Exp(\lambda)$	Exponential distribution with mean $\frac{1}{\lambda}$
$U(lb, ub)$	Uniform distribution with lower bound lb and upper bound up
$\mathrm{erfc}(\cdot)$	Complementary error function
$\Phi(\cdot)$	Cumulative distribution function (CDF) of the standard normal distribution
$p(LOS; d)$	Probability of LOS propagation when the MS-BS distance is d
$p(NLOS; d)$	Probability of NLOS propagation when the MS-BS distance is d
$p(z; \mathbf{x})$	PDF of measurement z when the target is located at $\mathbf{x}$
$p^L(z; \mathbf{x})$	PDF of measurement z when the target is located at $\mathbf{x}$ under LOS propagation
$p^{NL}(z; \mathbf{x})$	PDF of measurement z when the target is located at $\mathbf{x}$ under NLOS propagation
$\mathbf{J}(\mathbf{x})$	Fisher information matrix when the target is located at $\mathbf{x}$
$\mathbf{CRLB}(\mathbf{x})$	Cramer-Rao Lower Bound when the target is located at $\mathbf{x}$
$\mathrm{PEB}(\mathbf{x})$	Position error bound when the target is located at $\mathbf{x}$

3. System Model

We consider a 2-D localization system with N BSs, whose positions are known as $\mathbf{x}_i = [x_i, y_i], i = 1, \cdots, N$. The position of the mobile station (MS), i.e., the target, denoted by **x** $= [x, y]$, is unknown and needs to be estimated. It is assumed that the noisy range measurements between the MS and the BSs can be obtained, which are expressed as

$$z_i = d_i + \varepsilon_i, \quad i = 1, \cdots, N \tag{1}$$

where $d_i = \sqrt{(x_i - x)^2 + (y_i - y)^2}$ is the true distance between the MS and the i-th BS, and ε_i is measurement error. The measurement error is usually modeled as zero-mean white Gaussian noise under LOS propagation [16, 27, 28]. In practice, because of the complicated propagation environment, not all the paths are LOS paths. If path i is an NLOS path, ε_i also includes an NLOS bias in addition to the additive Gaussian noise because the occurrence of NLOS path means that the signal needs to travel a longer distance to reach the receiver. Consequently, the measurement error can be modeled as follows

$$\varepsilon_i = \begin{cases} \upsilon_i, & LOS\ path \\ b_i + \upsilon_i, & NLOS\ path \end{cases} \tag{2}$$

where $\upsilon_i \sim \mathcal{N}(0, \sigma_i^2)$ is the zero-mean white Gaussian noise, and b_i is the bias attributable to NLOS propagation. In the literature the bias term has been modeled as a random variable following Gaussian distribution [16], exponential distribution [27], uniform distribution [28], or just being a constant [16]. In this paper, we assume the distributions of υ_i and b_i are known *a priori*, which can be obtained from field measurements and *a priori* calibration. The measurement errors in different paths are assumed to be independent. Moreover, υ_i and b_i in the same NLOS path are also assumed to be independent [16, 27, 28]. Consequently, the probability density function (PDF) of the measurement error ε_i under LOS propagation is the PDF of the Gaussian variable υ_i,

$$p^L(\varepsilon_i) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left(-\frac{\varepsilon_i^2}{2\sigma_i^2}\right) \tag{3}$$

The PDF of ε_i under NLOS propagation is the PDF of the sum of two independent random variables, i.e., $\upsilon_i + b_i$, which can be derived through convolution

$$p^{NL}(\varepsilon_i) = \int_{-\infty}^{\infty} p^L(\varepsilon_i - \tau) p_{b_i}(\tau) d\tau \tag{4}$$

where p_{b_i} stands for the PDF of b_i. The analytical form of $p^{NL}(\varepsilon_i)$ with b_i assuming four different distributions as mentioned previously can be derived as follows:

- **Gaussian distributed NLOS bias**: $b_i \sim \mathcal{N}(\mu_{bi}, \sigma_{bi}^2)$:

$$p_G^{NL}(\varepsilon_i) = \frac{1}{\sqrt{2\pi\sigma_{\varepsilon i}^2}} \exp\left(-\frac{(\varepsilon_i - \mu_{\varepsilon i})^2}{2\sigma_{\varepsilon i}^2}\right)$$

  where $\mu_{\varepsilon i} = \mu_{bi}$ and $\sigma_{\varepsilon i}^2 = \sigma_{bi}^2 + \sigma_i^2$.

- **Exponentially distributed NLOS bias**: $b_i \sim Exp(\lambda)$:

$$p_E^{NL}(\varepsilon_i) = \frac{\lambda}{2} \exp\left(-\lambda(\varepsilon_i - \frac{\lambda\sigma_i^2}{2})\right) \operatorname{erfc}\left(\frac{\lambda\sigma_i^2 - \varepsilon_i}{\sqrt{2}\sigma_i}\right)$$

- **Uniformly distributed NLOS bias**: $b_i \sim U(lb_i, ub_i)$:

$$p_U^{NL}(\varepsilon_i) = \frac{1}{(ub_i - lb_i)} \left(\Phi\left(\frac{ub_i - \varepsilon_i}{\sigma_i}\right) - \Phi\left(\frac{lb_i - \varepsilon_i}{\sigma_i}\right)\right)$$

- **Constant NLOS bias**: $b_i = b$:

$$p_C^{NL}(\varepsilon_i) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left(-\frac{(\varepsilon_i - b)^2}{2\sigma_i^2}\right)$$

According to 3GPP [5], the occurrence of LOS path depends on the MS-BS distance d_i, whose probability is denoted as $p(LOS; d_i)$ and NLOS probability is $p(NLOS; d_i) = 1 - p(LOS; d_i)$. The LOS probability is modeled as different forms under different environmental conditions, including indoor hotspots, urban micro, urban macro and rural macro cellular networks [5]. For example, as shown in [5], the LOS probability in indoor environment is

$$p(LOS; d_i) = \begin{cases} 1, & 0 < d_i \leq 18 \\ \exp(-\frac{d_i - 18}{27}), & 18 < d_i < 37 \\ 0.5, & d_i \geq 37 \end{cases} \tag{5}$$

We can see from (5) that, with the increase of d_i, the LOS probability will decrease. It has been shown that it is important to take into account the distance-related LOS/NLOS probabilities in system design and performance analysis [4, 29].

4. MLE-based Localization

In this work, we assume the distributions of measurement noise and NLOS bias are known, along with the distance-related LOS/NLOS probabilities, which makes MLE the preferred option in estimating the MS's position, since MLE is maximizing the joint likelihood function of the MS's position.

Incorporating the LOS and NLOS probabilities, the PDF of measurement z_i is expressed as

$$p(z_i; \mathbf{x}) = p^L(z_i; \mathbf{x})p(LOS; d_i) + p^{NL}(z_i; \mathbf{x})p(NLOS; d_i) \tag{6}$$

where $p^L(z_i; \mathbf{x})$ is the PDF of z_i assuming LOS propagation, which can be easily obtained from (3) as

$$p^L(z_i; \mathbf{x}) = p^L(z_i - d_i) \tag{7}$$

and $p^{NL}(z_i; \mathbf{x})$ is the PDF of z_i assuming NLOS propagation, which can be obtained from (4) as

$$p^{NL}(z_i; \mathbf{x}) = p^{NL}(z_i - d_i) \tag{8}$$

The expression $p^{NL}(z_i - d_i)$ varies with different distributions of NLOS bias, as discussed in Sec. 3. Since the measurement errors in different paths are mutually independent, the joint PDF of the whole localization system is

$$p(\mathbf{z}; \mathbf{x}) = \prod_{i=1}^{N} p(z_i; \mathbf{x}) \tag{9}$$

where $\mathbf{z} = [z_1, \cdots, z_N]^T$ is the measurement vector. The estimate of $\mathbf{x}$ by MLE can be obtained as

$$\hat{\mathbf{x}}_{MLE} = \arg\max_{\mathbf{x}} \ln p(\mathbf{z}; \mathbf{x}) = \arg\max_{\mathbf{x}} \sum_{i=1}^{N} \ln p(z_i; \mathbf{x}) \tag{10}$$

This MLE does not need to identify the NLOS paths beforehand and takes full advantage of all the available measurements, including both the LOS measurements and the NLOS measurements. It will achieve the optimum result assuming that the distribution of measurement error is known. Starting from a good initial estimate, the solution can usually be obtained by many exiting algorithms, e.g., Newton-Raphson method [30].

5. CRLB-based Performance Analysis

In this section, we will analyze the fundamental limits of localization accuracy in probabilistic LOS/NLOS environment using CRLB, which is often utilized as a benchmark to evaluate the estimation accuracy [16, 18, 27, 30].

5.1. Position Error Bound

For a localization problem in $\mathbb{R}^2$, CRLB is a 2×2 matrix indicating the lower bound of the covariance matrix of estimation error. To simplify the analysis, the position error bound (PEB) is usually taken as a scalar metric for localization accuracy, which is defined as

$$\mathrm{PEB}(\mathbf{x}) = \sqrt{\mathrm{Tr}(\mathbf{CRLB}(\mathbf{x}))} = \sqrt{\mathrm{Tr}(\mathbf{J}(\mathbf{x})^{-1})} \tag{11}$$

where $\mathrm{Tr}(\cdot)$ denotes the trace operation of one matrix and $\mathbf{J}(\mathbf{x})$ is the Fisher Information Matrix (FIM) at $\mathbf{x}$. For a localization system, regardless of the localization algorithm, the root mean square error (RMSE) of the position estimate satisfies [10, 16]

$$\mathrm{RMSE} \geq \mathrm{PEB}(\mathbf{x})$$

About the PEB of our considered localization system in probabilistic LOS/NLOS environment, we have the following theorem.

Theorem 1. *For a localization system in probabilistic LOS/NLOS environment, its PEB is*

$$PEB(\boldsymbol{x}) = \sqrt{\frac{\sum\limits_{i=1}^{N} f_i(\boldsymbol{x})}{\sum\limits_{i=1}^{N}\sum\limits_{j>i}^{N} f_i(\boldsymbol{x}) f_j(\boldsymbol{x}) sin^2(\theta_i - \theta_j)}} \tag{12}$$

where

$$f_i(\boldsymbol{x}) = \mathbb{E}\left[\left(\frac{\partial \ln p(z_i;\boldsymbol{x})}{\partial d_i}\right)^2\right] \tag{13}$$

and $\cos\theta_i = \frac{x-x_i}{d_i}$, $\sin\theta_i = \frac{y-y_i}{d_i}$.

PROOF. We first compute $\mathbf{CRLB}(\mathbf{x})$. Generally, CRLB is obtained from its inverse matrix, i.e., FIM, which is defined as

$$\mathbf{J}(\mathbf{x}) = \mathbb{E}\begin{bmatrix} \left(\frac{\partial \ln p(\mathbf{z};\mathbf{x})}{\partial x}\right)^2 & \frac{\partial \ln p(\mathbf{z};\mathbf{x})}{\partial x}\frac{\partial \ln p(\mathbf{z};\mathbf{x})}{\partial y} \\ \frac{\partial \ln p(\mathbf{z};\mathbf{x})}{\partial y}\frac{\partial \ln p(\mathbf{z};\mathbf{x})}{\partial x} & \left(\frac{\partial \ln p(\mathbf{z};\mathbf{x})}{\partial y}\right)^2 \end{bmatrix}$$

The entry of $\mathbf{J}(\mathbf{x})$ at the first row and the first column is computed as follows

$$\begin{aligned}
J_{11}(\mathbf{x}) &= \mathbb{E}\left[\left(\sum_{i=1}^{N} \frac{\partial \ln p(z_i;\mathbf{x})}{\partial x}\right)^2\right] \\
&= \mathbb{E}\left[\sum_{i=1}^{N}\left(\frac{\partial \ln p(z_i;\mathbf{x})}{\partial x}\right)^2\right] \\
&\quad + 2\mathbb{E}\left[\sum_{i=1}^{\mathrm{N}}\sum_{j>i}^{N} \frac{\partial \ln p(z_i;\mathbf{x})}{\partial x}\frac{\partial \ln p(z_j;\mathbf{x})}{\partial x}\right] \\
&= g_1(\mathbf{x}) + 2g_2(\mathbf{x})
\end{aligned}$$

where

$$\begin{aligned}
g_1(\mathbf{x}) &= \sum_{i=1}^{N}\mathbb{E}\left[\left(\frac{\partial \ln p(z_i;\mathbf{x})}{\partial x}\right)^2\right] \\
&= \sum_{i=1}^{N}\mathbb{E}\left[\left(\frac{\partial \ln p(z_i;\mathbf{x})}{\partial d_i}\right)^2\left(\frac{\partial d_i}{\partial x}\right)^2\right]
\end{aligned}$$

and

$$\begin{aligned}
g_2(\mathbf{x}) &= \mathbb{E}\left[\sum_{i=1}^{N}\sum_{j>i}^{N} \frac{\partial \ln p(z_i;\mathbf{x})}{\partial x}\frac{\partial \ln p(z_j;\mathbf{x})}{\partial x}\right] \\
&= \sum_{i=1}^{N}\sum_{j>i}^{N}\mathbb{E}\left[\frac{\partial \ln p(z_i;\mathbf{x})}{\partial d_i}\frac{\partial \ln p(z_j;\mathbf{x})}{\partial d_j}\right]\frac{\partial d_i}{\partial x}\frac{\partial d_j}{\partial x}
\end{aligned}$$

Since $\frac{\partial d_i}{\partial x} = \frac{x-x_i}{d_i} = \cos\theta_i$ and $\frac{\partial d_i}{\partial y} = \frac{y-y_i}{d_i} = \sin\theta_i$, $g_1(\mathbf{x})$ can be rewritten as

$$g_1(\mathbf{x}) = \sum_{i=1}^{N}\mathbb{E}\left[\left(\frac{\partial \ln p(z_i;\mathbf{x})}{\partial d_i}\right)^2\right]\cos^2\theta_i$$

Note that measurement errors in different paths are independent, it follows that

$$\mathbb{E}\left[\frac{\partial \ln p(z_i;\mathbf{x})}{\partial d_i}\frac{\partial \ln p(z_j;\mathbf{x})}{\partial d_j}\right] = \mathbb{E}\left[\frac{\partial \ln p(z_i;\mathbf{x})}{\partial d_i}\right]\mathbb{E}\left[\frac{\partial \ln p(z_j;\mathbf{x})}{\partial d_j}\right]$$

Moreover, from the regulation condition of CRLB in [30],

$$\mathbb{E}\left[\frac{\partial \ln p(z_i;\mathbf{x})}{\partial d_i}\right] = 0$$

Thus, $g_2(\mathbf{x}) = 0$. Consequently,

$$J_{11}(\mathbf{x}) = g_1(\mathbf{x}) = \sum_{i=1}^{N} \mathbb{E}\left[\left(\frac{\partial \ln p(z_i;\mathbf{x})}{\partial d_i}\right)^2\right] \cos^2\theta_i$$

Accordingly, the other entries of $\mathbf{J}(\mathbf{x})$ are obtained as

$$J_{12}(\mathbf{x}) = J_{21}(\mathbf{x}) = \sum_{i=1}^{N} \mathbb{E}\left[\left(\frac{\partial \ln p(z_i;\mathbf{x})}{\partial d_i}\right)^2\right] \cos\theta_i \sin\theta_i$$

$$J_{22}(\mathbf{x}) = \sum_{i=1}^{N} \mathbb{E}\left[\left(\frac{\partial \ln p(z_i;\mathbf{x})}{\partial d_i}\right)^2\right] \sin^2\theta_i$$

Let $f_i(\mathbf{x}) = \mathbb{E}\left[\left(\frac{\partial \ln p(z_i;\mathbf{x})}{\partial d_i}\right)^2\right]$, then $\mathbf{J}(\mathbf{x})$ is expressed as

$$\mathbf{J}(\mathbf{x}) = \sum_{i=1}^{N} f_i(\mathbf{x}) \begin{bmatrix} \cos^2\theta_i & \cos\theta_i \sin\theta_i \\ \cos\theta_i \sin\theta_i & \sin^2\theta_i \end{bmatrix} \tag{14}$$

According to the relationship between CRLB and FIM, we have

$$\mathrm{Tr}(\mathbf{CRLB}(\mathbf{x})) = \frac{\mathrm{Tr}(\mathbf{J}(\mathbf{x}))}{\det(\mathbf{J}(\mathbf{x}))}$$

where

$$\mathrm{Tr}(\mathbf{J}(\mathbf{x})) = \sum_{i=1}^{N} f_i(\mathbf{x}) \tag{15}$$

and by referring to the derivation in [31], we have

$$\det(\mathbf{J}(\mathbf{x})) = \sum_{i=1}^{N} \sum_{j>i}^{N} f_i(\mathbf{x}) f_j(\mathbf{x}) \sin^2(\theta_i - \theta_j) \tag{16}$$

It is straightforward that (12) holds.

From (12), we can find that the value of $f_i(\mathbf{x})$ will affect the value of PEB. The relationship between $f_i(\mathbf{x})$ and $\mathrm{PEB}(\mathbf{x})$ is introduced in the following property.

Property 1. *PEB($\boldsymbol{x}$) monotonically decreases with the value of $f_i(\boldsymbol{x})$.*

PROOF. Let $g_{ij} = \frac{f_i(\mathbf{x}) f_j(\mathbf{x})}{\sum\limits_{i=1}^{N} f_i(\mathbf{x})}$, then

$$\text{PEB}(\mathbf{x}) = \sqrt{\frac{1}{\sum\limits_{i=1}^{N}\sum\limits_{j>i}^{N} g_{ij} sin^2(\theta_i - \theta_j)}} \tag{17}$$

Obviously, PEB$(\mathbf{x})$ is a monotonically decreasing function of g_{ij}. Also,

$$\frac{\partial g_{ij}}{\partial f_i(\mathbf{x})} = \frac{f_j(\mathbf{x})(\sum\limits_{i=1}^{N} f_i(\mathbf{x}) - f_i(\mathbf{x}))}{(\sum\limits_{i=1}^{N} f_i(\mathbf{x}))^2} > 0 \tag{18}$$

Then, g_{ij} is a monotonically increasing function of $f_i(\mathbf{x})$. Therefore, a larger $f_i(\mathbf{x})$ implies a lower PEB$(\mathbf{x})$.

5.2. Numerical Analysis

In this subsection, we analyze the impact of NLOS propagations on the localization accuracy. The NLOS bias and LOS/NLOS probabilities can only affect the value of $f_i(\mathbf{x})$, therefore, we just need to analyze the influence of NLOS propagation on $f_i(\mathbf{x})$, which can be further computed as

$$f_i(\mathbf{x}) = \mathbb{E}\left[\left(\frac{\partial \ln p(z_i;\mathbf{x})}{\partial d_i}\right)^2\right] = \int_{-\infty}^{+\infty} \frac{\left(\frac{\partial p(z_i;\mathbf{x})}{\partial d_i}\right)^2}{p(z_i;\mathbf{x})} dz_i \tag{19}$$

Because of the complicated analytical form of $p(z_i;\mathbf{x})$ and its partial derivative over d_i, the integral in (19) is difficult to express analytically. We resort to using numerical method to approximate the integral in (19). As is common, we use Monte Carlo integration to get the numerical solution [32, 33], i.e.,

$$f_i(\mathbf{x}) \approx \frac{1}{N_{MC}} \sum_{n=1}^{N_{MC}} \left(\frac{\partial \ln p(z_n^{MC};\mathbf{x})}{\partial d_i}\right)^2 \tag{20}$$

where $z_n^{MC},\ n = 1, \cdots, N_{MC}$ are samples independently generated from $p(z_i;\mathbf{x})$ and N_{MC} is the number of samples.

To investigate the influence of LOS/NLOS probabilities and NLOS biases on the localization performance, in the numerical analysis, we adopt the LOS probability model in (5) for indoor environment [5]. The parameters in the measurement error model are set to be the same as those in [16], i.e., the additive Gaussian noise $v_i \sim \mathcal{N}(0, 1)$, the mean and standard deviation of NLOS bias are respectively $\mu = 2$ and $\sigma = 2$. The function of $f_i(\mathbf{x})$ is computed as introduced before with b_i assuming Gaussian distribution $b_i \sim \mathcal{N}(\mu, \sigma^2)$, uniform distribution $b_i \sim U(\mu - \sqrt{3}\sigma, \mu + \sqrt{3}\sigma)$, exponential distribution $b_i \sim Exp(1/\mu)$ and constant $b_i = \mu$, respectively.

Fig.1 shows the impact of NLOS bias on $f_i(\mathbf{x})$. In this figure, when $d_i \leq 18$, no NLOS propagation occurs. When $18 < d_i < 37$, with the increase of d_i, $p(NLOS; d_i)$ increases, and the impact of NLOS bias on $f_i(\mathbf{x})$ increases. When $d_i \geq 37$, $p(NLOS; d_i)$ keeps unchanged, and the impact of NLOS bias on $f_i(\mathbf{x})$ becomes stable. We can see NLOS biases with different distributions have different impacts on $f_i(\mathbf{x})$. Since a lower $f_i(\mathbf{x})$ indicates a higher PEB, we can say that the PEB associated with a Gaussian distributed bias is larger than the PEB with a uniformly distributed bias and again larger than the PEB with an exponentially distributed bias. Constant bias results in the smallest PEB.

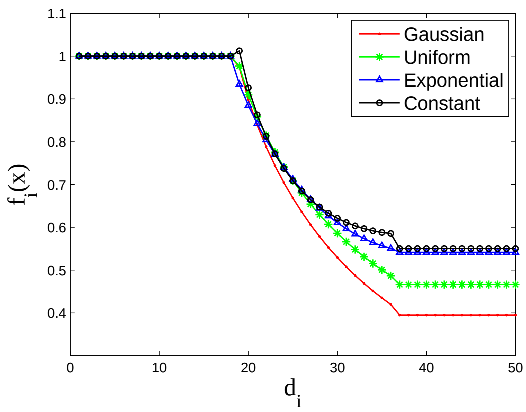


Figure 1: $f_i(\mathbf{x})$ with NLOS biases under four different distributions with the common mean and standard deviation, i.e., $\mu = 2$, $\sigma = 2$

6. Performance Evaluation

In this section, both simulations and experimental data are utilized for performance evaluation of the proposed localization algorithm.

6.1. Simulation Results

In the simulations, the same as [33, 34], we consider a cellular network in a Germany city center, where the size of the test area is approximately $3km \times 3km$.

The location of the MS is $[250m, 250m]$. There are 7 BSs, whose locations are respectively known as $[-0.75km, 0.75km]$, $[-0.25km, 1.5km]$, $[0.75km, 1.75km]$, $[0.5km, -0.75km]$, $[1.5km, 0km]$, $[2km, 1.9km]$ and $[-0.75km, -0.6km]$. We adopt the LOS probability model proposed in [5] for the Urban Micro cellular network, as follows

$$p(LOS; d) = \min(\frac{18}{d}, 1) \cdot (1 - \exp(-\frac{d}{36})) + \exp(-\frac{d}{36}) \tag{21}$$

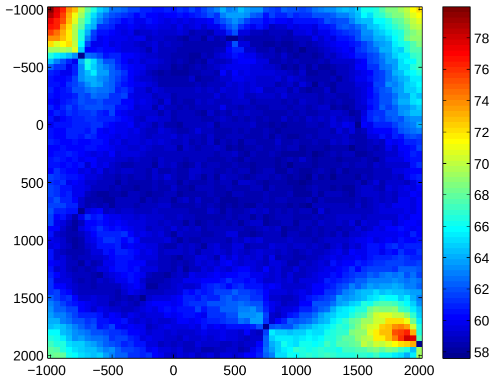
Both the measurement error under LOS condition and the measurement error under NLOS condition are Gaussian distributed: $\varepsilon_L \sim \mathcal{N}(0, 55m)$ and $\varepsilon_{NL} \sim \mathcal{N}(380m, 120m)$. Therefore, both the measurement noise and NLOS bias are Gaussian distributed, i.e., $v_i \sim \mathcal{N}(0, 55m)$ and $b_i \sim \mathcal{N}(380m, 106.65m)$. Fig. 2 shows the PEBs when the MS is located at different places in the area and all the BSs are utilized for localization. Fig. 2(a) and Fig. 2(b) respectively illustrate the PEBs when the LOS probability is fixed to be 0.5 and the LOS probability varies with the true distance following (21). It can be easily seen that, different assumptions on the LOS probability will result in different PEBs. Compared with the assumption of the fixed LOS probability, considering the distance-related LOS/NLOS probability will provide a tighter lower bound for the position estimation error.

Fig. 3 and Fig. 4 compare the localization performance of different localization algorithms. In these two figures, MLE-PC is the MLE based on distance-related LOS/NLOS probabilities, which is proposed in Sec. 4; MLE-TC is the MLE assuming the accurate knowledge of LOS and NLOS paths is known *a priori*; MLE-FP is the MLE assuming the LOS/NLOS probabilities are fixed, for which, two cases are considered, i.e., $p(LOS) = 0.5$ and $p(LOS) = 0.2$; PEB is the one considering distance-related LOS/NLOS probabilities.

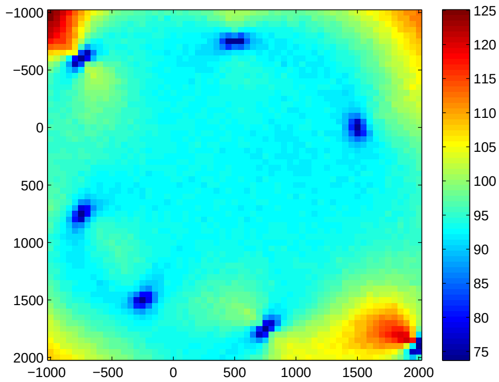
Fig. 3 compares the RMSEs of different localization algorithms by considering a varying number of BSs. The statistical distributions of the LOS and NLOS measurement errors are the same as those in Fig. 2. From this figure, we can find, with the increase of the number of utilized BSs, the RMSEs will be lower, which indicates higher localization accuracies. The RMSEs of the proposed MLE-PC is very close to the lower bounds PEB. Since MLE-TC utilizes and relies on the prior knowledge of the true sight condition of each path, its RMSEs are slightly lower than PEB. RMSEs of MLE-FP are larger than others, which again shows the necessity of considering the distance-related LOS/NLOS probabilities.

Fig. 4 compares the RMSEs of different localization algorithms when the NLOS bias is Gaussian distributed with different standard deviations. The statistical distribution of LOS measurement errors is the same as that in Fig. 2 and all the 7 BSs are utilized for localization. It can be seen that, with the increase of σ_b, the

RMSEs will be higher, which indicates lower localization accuracies. By comparing the performance of different algorithms, again, we can find the RMSEs of the proposed algorithm can approach PEB and are smaller than MLE-FP.



(a) $p(LOS) = 0.5$



(b) $p(LOS; d)$

Figure 2: PEBs in the test area. Both the LOS and NLOS measurement errors follow Gaussian distributions.

Fig. 5 compare the localization performance of MLE-PC when NLOS bias follows different distributions. In this figure, MLE-PC (Gaussian), MLE-PC (Uniform), MLE-PC (Exponential) and MLE-PC (Constant) denote MLE-PC when NLOS bias follows Gaussian distribution, uniform distribution, exponential distribution and being a constant, respectively. The mean and standard deviation of the

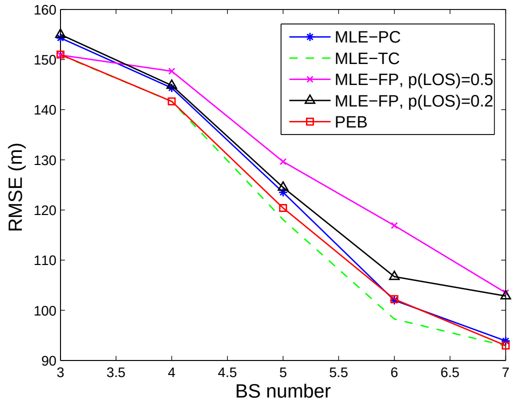


Figure 3: RMSEs of different algorithms versus the number of utilized BSs.

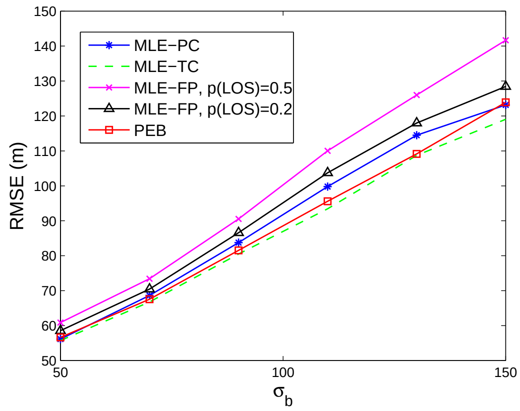


Figure 4: RMSEs of different algorithms versus the standard deviation of NLOS bias.

NLOS bias under four different distributions are the same as that those in Fig. 2. We can see the performance of MLE-PC (Constant) is the best; MLE-PC (Exponential) performs better than MLE-PC (Uniform) and MLE-PC (Gaussian). RMSEs of MLE-PC (Gaussian) are the highest among these four cases.

6.2. Experimental Results

In this section, we test the performance of the proposed localization algorithm using the measurement database WPR.B in [35]. This database includes extensive range measurements which are taken by 2 commercial UWB devices, PulsON220, in a typical indoor office environment, as shown in Fig. 6. In Fig. 6, a set of

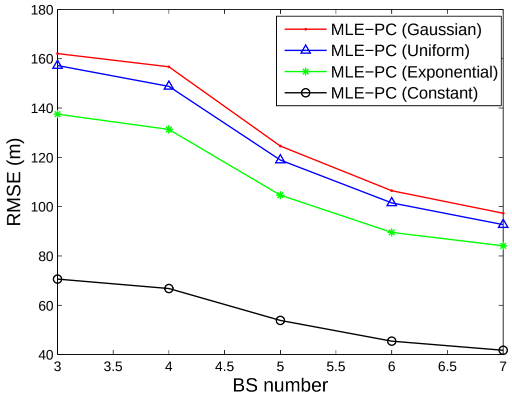


Figure 5: RMSEs of different algorithms versus the number of utilized BSs.

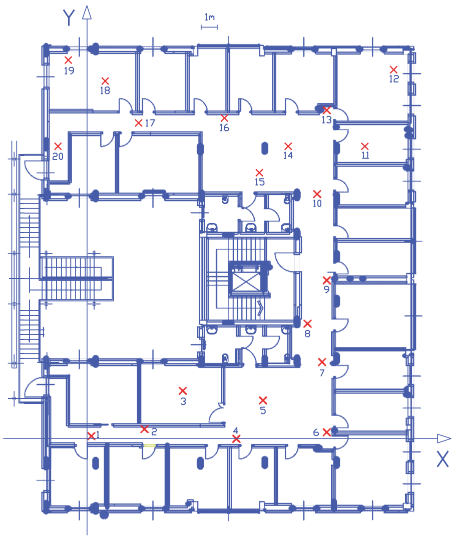


Figure 6: The experimental environment in WiLab, University of Bologna, Cesena Campus [35].

20 static positions, marked by red crosses, are used for the UWB measurements. By putting the UWB devices at two of the 20 positions, the TOA ranging measurements between these two points can be obtained and stored as a range measurement vector, which includes more than 200 range measurements. Because of the strong attenuations and limited radiated power of the UWB devices, not all UWB devices in the 20 static positions can communicate with each other. There exist only 69 links where the UWB devices can communicate with each other and conduct rang-

ing. In these 69 links, some of them are LOS, and the others are NLOS. We will first derive the model of LOS probability using this database. Then, we will localize the target at these 20 positions using the corresponding range measurements and the LOS/NLOS probabilities.

Ideally, we would want to use measurements obtained from cellular networks to validate the performance of the proposed localization algorithms. Limited by experimental conditions, here we use indoor UWB measurements instead. However this does not affect the main focus of this paper, i.e., it is important to consider both LOS and NLOS propagations in localization.

6.2.1. LOS probability fitting

About the MS-BS distance related LOS probability model, we mainly consider two models:

a) Linear model proposed in [6]:

$$p_l(LOS; d) = \alpha_l d + \beta_l \tag{22}$$

where α_l and β_l are the linear model parameters;

b) Exponential model proposed in [5]:

$$p_e(LOS; d) = \exp(\frac{-(d - \alpha_e)}{\beta_e}) \tag{23}$$

where α_e and β_e are the exponential model parameters. In our experimental environment, the LOS probability is always changing with the distance, that is why we dismiss the parts where the LOS probability keeps unchanged in (5).

In the database, there are 33 LOS links with true internode distances $d_i^{los}, i = 1, \cdots, 33$, and 36 NLOS links with true internode distances $d_j^{nlos}, j = 1, \cdots, 36$. We use MLE to estimate the LOS probability model parameters in (22) and (23). The PDF of α_l and β_l is computed as

$$\begin{aligned} P(\alpha_l, \beta_l) &= \prod_{i=1}^{33} p_l(LOS; d_i^{los}) \prod_{j=1}^{36} p_l(NLOS; d_j^{nlos}) \\ &= \prod_{i=1}^{33} \Big(\alpha_l d_i^{los} + \beta_l\Big) \prod_{j=1}^{36} \Big(1 - (\alpha_l d_j^{nlos} + \beta_l)\Big) \end{aligned}$$

Correspondingly, the PDF of α_e and β_e is computed as

$$
\begin{aligned}
P(\alpha_e, \beta_e) &= \prod_{i=1}^{33} p_e(LOS; d_i^{los}) \prod_{j=1}^{36} p_e(NLOS; d_j^{nlos}) \\
&= \prod_{i=1}^{33} \exp(\frac{-(d_i^{los} - \alpha_e)}{\beta_e}) \prod_{j=1}^{36} \left(1 - \exp(\frac{-(d_j^{nlos} - \alpha_e)}{\beta_e})\right)
\end{aligned}
$$

The estimates of α_l, β_l, α_e and β_e are

$$(\hat{\alpha_l}, \hat{\beta_l}) = \arg\max_{\alpha_l, \beta_l} \ln P(\alpha_l, \beta_l)$$

$$(\hat{\alpha_e}, \hat{\beta_e}) = \arg\max_{\alpha_e, \beta_e} \ln P(\alpha_e, \beta_e)$$

The estimated LOS probability models for this database are

$$p_l(LOS; d) = -0.018d + 0.6237 \tag{24}$$

$$p_e(LOS; d) = \exp(\frac{-(d + 4.7901)}{16.6700}) \tag{25}$$

The fitted results under different probability models are shown in Fig.7. The Mean Square Errors (MSE) of the fitted models (24) and (25) are almost equal. Considering that the linear model is simpler than the exponential model to analyze, we will apply the linear probability model (24) to the localization.

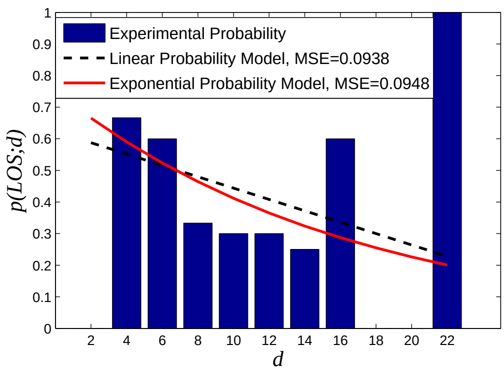


Figure 7: Fitted results of LOS probability.

6.2.2. Localization results

In the localization part, we use the range measurements provided by the W-PR.B database. Through numerical fitting using the range measurements in the whole database, the range measurement error under LOS condition follows a Gaussian distribution, i.e., $\varepsilon_L \sim \mathcal{N}(-0.0214, 0.1149)$, and the range measurement error under NLOS condition also follows a Gaussian distribution, i.e., $\varepsilon_{NL} \sim \mathcal{N}(0.7613, 0.9860)$. The NLOS bias is found to be Gaussian distributed. In the localization process, we assume the target is at one of the 20 static positions and the neighbors of this target are anchors, whose positions are assumed to be known. Then we use range measurements between this target and its neighbors to estimate the target's position by the MLE proposed in Sec. 4 combining the LOS probability model in (24). For example, if we want to localize the target at position 1, the anchors for the target are 2, 3, 4, 6, 7, the range measurements between 1 and 2, 3, 4, 6, 7 are utilized for localization.

Fig. 8 compares the RMSEs of different algorithms. The results shown in Fig. 8 are obtained by conducting 200 independent Monte Carlo trials, i.e., using 200 different range measurements for each pair of neighbors. In MLE-FP, the LOS probability is assumed to be 0.5. From this figure, we can see the RMSEs of MLE-FP is higher than those of MLE-PC. We can conclude that the performance of the MLE considering distance-related LOS probability is better than that of the MLE considering a fixed LOS probability. Except for some particular points, e.g., number 11 and 14, the RMSEs of MLE-PC can approach PEB. As for MLE-TC, its RMSEs at a few points are much larger than those of the other two algorithms, which might be counter-intuitive at the first sight. The reason behind this phenomenon is the statistical distributions of LOS and NLOS measurements are obtained by fitting technique, which incorporates all the measurements at the 20 positions. There exist many outliers in this database that may not fit well the obtained statistical model. For example, at position number 11, all the measurements are under NLOS condition, despite that the target at position 11 has small distances to its neighbors, whereas the actual errors in the range measurements obtained at position 11 are all very small, which make the associated measurements more like LOS measurements. In our MLE-PC algorithm, since the distances between 11 and its neighbors are small, those range measurements are taken as LOS measurements with a higher probability. And in the proposed algorithm, all the measurements are taken as NLOS measurements (which are the true conditions) with larger mean and larger standard deviation. As a result, some location estimates obtained by MLE-TC which has accurate knowledge of LOS and NLOS paths are worse than that obtained by the proposed algorithm.

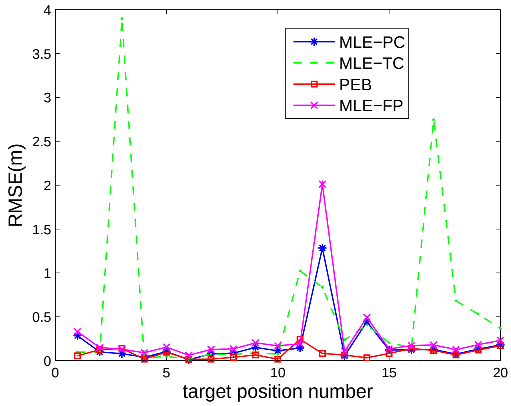


Figure 8: RMSE of different algorithms using experimental data at different positions.

7. Conclusion

In this paper, the localization problem in an environment including both LOS and NLOS paths is investigated. By combining the distance-related LOS/NLOS probabilities, an MLE-based localization algorithm is proposed for location estimation. The PEB for a general LOS/NLOS probability model and NLOS bias model is derived and its property is also analyzed. The numerical analysis shows that Gaussian NLOS bias leads to the worst localization performance, and constant NLOS bias leads to the best localization performance. Both the simulations results and experimental results using real UWB measurements show the necessity of considering the distance-related LOS/NLOS probabilities. The RMSEs of our proposed algorithm can approach the PEB and are lower than that of MLE assuming the LOS/NLOS probabilities to be fixed.

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