Robust Localization Using Time Difference of Arrivals

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***Abstract*—We investigate a localization problem using time difference of arrival (TDOA) measurements with unknown and bounded measurement errors. Different from most existing algorithms, we consider the minimization of the worst-case position estimation error to improve the robustness of the algorithm. The localization problem is formulated as a non-convex optimization problem. We adopt semidefinite relaxation (SDR) to relax the original problem into a convex optimization problem, which can be solved using semidefinite programming (SDP). Simulation results show that our proposed algorithm has lower worst-case position estimation error than other existing algorithms.**

***Index Terms*—Time difference of arrival, worst-case estimation error, Chebyshev Center, semidefinite program**

I. INTRODUCTION

DUE to the significance of location information in many applications, localization has become an important topic in the research of wireless sensor networks (WSNs). Generally speaking, a target can be localized by fusing location related measurements, e.g., received signal strength (RSS) [1], [2], angle of arrival (AOA) [1], [2], time of arrival (TOA) [2], [3], time difference of arrival (TDOA) [1], [2], [4], [5], etc. Each kind of measurement method has its strength and weakness in terms of measurement acquisition, measurement accuracy, robustness to environment, etc. In this paper, we consider the use of TDOA measurements for target localization, which has been extensively used.

Many localization algorithms have been developed using TDOA measurements in recent years. One classic algorithm is the linear least squares (LS) method [2], [6]. Through preprocessing of TDOA measurements, a set of linear equations can be obtained. Apart from the unknown target's position, these equations also include an unknown distance between the target and the reference node, which is related to the target's position. A closed-form solution can be obtained by the standard linear LS estimator without considering the relationship between the target's position and this reference distance. To incorporate the constraint on the relationship between the target's position and the reference distance, the localization problem is usually formulated as an optimization problem that, however, is not convex in general. Doğançay and Hashemi-Sakhtsari [7] proposed an iterative Gauss-Newton algorithm based on a constrained weighted LS criterion, which may create local minima. To deal with the non-convexity of the problem, a common technique is semidefinite relaxation (SDR). Xu et al. [8] proposed two robust algorithms to minimize the maximum error between the squares of propagation times. These two algorithms transform the original nonconvex problem into two semidefinite programs (SDP) by SDR. Yang et al. [5] also proposed an SDP-based localization algorithm to minimize the sum of the squared error between the TDOA measurements and the TDOA estimates obtained from estimated target position. Their algorithm utilizes all pairwise TDOA measurements, i.e., $N(N-1)$ pairs of TDOA measurements for N sensors. It removes the constraint between the target's position and the reference distance at the expense of higher computational complexity.

In some applications, a lower worst-case estimation error is preferred. For instance, in vehicular applications, a lower worst-case estimation error can help to decrease collisions between vehicles [9]. To the best of our knowledge, there is no work in the literature considering the minimization of the worst-case position estimation error. Motivated by the above observation, we will deign a localization algorithm using TDOA measurements to minimize the worst-case position estimation error. The main contributions of this paper are summarized as follows:

1) We formulate the localization problem as an optimization problem to minimize the worst-case position estimation error using TDOA measurements with unknown and bounded measurement errors.
2) Through SDR, we transform the original nonconvex optimization problem into a solvable SDP problem.
3) Extensive simulations show that the maximum position estimation error of the proposed algorithm is smaller

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than other existing methods, especially when the measurement error is large.

The remainder of this paper is organized as follows. Section II gives the problem formulation. Section III intrduces the proposed localization algorithm. Simulation results are presented in Section IV. Conclusions are drawn in Section V.

II. Problem Formulation

We consider a target localization problem using TDOA measurements in 2-dimensional space. The position of the target, denoted by $\boldsymbol{x} = [x, y]^T$, is unknown. There are N ($N \geq 4$) non-collinear sensors, whose positions are known as $\boldsymbol{s}_i = [x_i, y_i]^T, \ i = 1, \cdots, N$. We assume the sensors are synchronized to a common clock. Each sensor can measure the TOA of the signal transmitted from the target. The TOA measurement at sensor i is

$$z_i = \frac{1}{c} \parallel \boldsymbol{x} - \boldsymbol{s}_i \parallel + t_0 + \upsilon_i \tag{1}$$

where c is the signal propagation speed, $\parallel \cdot \parallel$ denotes the Euclidean norm, t_0 is the local time at the sensors when the signal leaves the target, and $|\upsilon_i| \leq \gamma_t$ is the unknown and bounded measurement error. We assume the error bound γ_t is known *a priori*. The assumption of bounded measurement error has been widely applied in many areas [10]–[13]. In TDOA-based localization, it is common to select one receiver as a reference node and the TDOA measurements are taken with respect to this reference node. Here, we take sensor 1 as the reference node; then the TDOA measurement between sensor i and 1 is

$$z_{i1} = z_i - z_1 = \frac{1}{c} \parallel \boldsymbol{x} - \boldsymbol{s}_i \parallel - \frac{1}{c} \parallel \boldsymbol{x} - \boldsymbol{s}_1 \parallel + \upsilon_{i1} \tag{2}$$

where $\upsilon_{i1} = \upsilon_i - \upsilon_1 \in [-2\gamma_t, 2\gamma_t]$.

From the constraints on the TDOA measurements, we can say that the target lies in the following closed feasible set

$$\begin{aligned} \mathcal{C}_x = \{\boldsymbol{x} : \ & \underline{d_{i1}} \leq \parallel \boldsymbol{x} - \boldsymbol{s}_i \parallel - \parallel \boldsymbol{x} - \boldsymbol{s}_1 \parallel \leq \overline{d_{i1}}, \\ & i = 2, \cdots, N\} \end{aligned} \tag{3}$$

where $\underline{d_{i1}} = c(z_{i1} - 2\gamma_t)$ and $\overline{d_{i1}} = c(z_{i1} + 2\gamma_t)$.

Note that in TDOA measurements, the measurement error bound will be enlarged by subtracting two TOA measurements, which causes the constraint set $\mathcal{C}_x$ to be larger than that constructed by TOA measurements. Nevertheless, the TDOA measurement formulation can avoid estimating t_0, which is the reason why we use TDOA measurements but not TOA measurements.

Since the true position of the target is unknown, we cannot minimize the position estimation error directly; therefore, we suggest to minimize the worst-case estimation error over all the feasible positions. Specifically, let $\boldsymbol{y}$ denote one feasible point for the target, which satisfies the TDOA measurement constraints in (3); our problem can be formulated as

$$\min_{\hat{\boldsymbol{x}}} \left\{ \max_{\boldsymbol{y} \in \mathcal{C}} \parallel \boldsymbol{y} - \hat{\boldsymbol{x}} \parallel \right\} \tag{4}$$

$$\begin{aligned} \mathcal{C} = \{\boldsymbol{y} : & \underline{d_{i1}} \leq \parallel \boldsymbol{y} - \boldsymbol{s}_i \parallel - \parallel \boldsymbol{y} - \boldsymbol{s}_1 \parallel \leq \overline{d_{i1}}, \\ & i = 2, \cdots, N\} \end{aligned} \tag{5}$$

where $\hat{\boldsymbol{x}}$ denotes the position estimate. Such formulation can guarantee that the localization error is upper bounded by the minimum worst-case estimation error.

Geometrically, our objective is to find the Chebyshev center of the feasible set, i.e., the center of the minimum circle enclosing $\mathcal{C}$ [13]. For example, in Fig.1, $\boldsymbol{s}_1$, $\boldsymbol{s}_2$, $\boldsymbol{s}_3$ and $\boldsymbol{s}_4$ are sensors with known positions, $\boldsymbol{s}_1$ is the reference node. From each TDOA measurement, we can obtain two hyperbolas, from which one feasible set for the target can be determined. Since the sensors are non-collinear, the intersection of all the feasible sets, i.e., the region surrounded by the bold black curve, is a bounded set. The minimum circle that encloses this bounded feasible set is the blue circle. Its center (the red dot) is the Chebyshev center $\boldsymbol{x}_{Cheby}$, which satisfies

$$\boldsymbol{x}_{Cheby} = \arg\min_{\hat{\boldsymbol{x}}} \left\{ \max_{\boldsymbol{y} \in \mathcal{C}} \parallel \boldsymbol{y} - \hat{\boldsymbol{x}} \parallel \right\} \tag{6}$$

Finding the Chebyshev center is an NP-hard problem except in some special cases [14]. In our problem, because the feasible set is not convex, it is certainly difficult to obtain $\boldsymbol{x}_{Cheby}$. We will find a relaxed estimate of the Chebyshev center in next section.

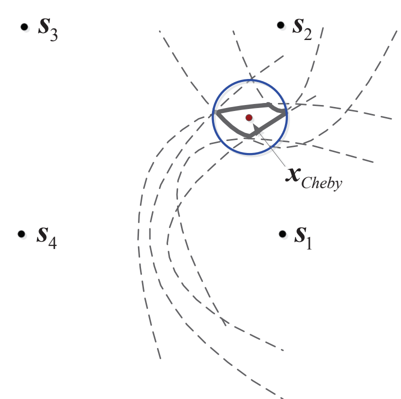


Fig. 1. Chebyshev center of the feasible set for the target.

III. The Relaxed Estimation

In this section, we explain how to find a relaxed estimate of the Chebyshev center. We first relax the non-convex set into a convex set, then transform our problem into an SDP problem.

In our 2-dimensional localization problem, a circle that encloses the feasible set $\mathcal{C}$ can be expressed as

$$C_2 = \{\boldsymbol{x}_c + \boldsymbol{v} | \parallel \boldsymbol{v} \parallel = \tau\} \tag{7}$$

where $\boldsymbol{v}$ is a 2-dimensional vector, $\boldsymbol{x}_c$ and τ are the center and the radius of this circle, respectively.

Since set $\mathcal{C}$ is inside this circle, clearly, the feasible point $\boldsymbol{y}$ aforementioned is inside this circle, which can be expressed as follows:

$$\boldsymbol{y} = \boldsymbol{x}_c + \boldsymbol{u} \tag{8}$$

where $\boldsymbol{u}$ is a 2-dimensional vector, which satisfies

$$\parallel \boldsymbol{u} \parallel \leq \tau \tag{9}$$

We are seeking the minimum circle that encloses $\mathcal{C}$, therefore, the equivalent problem to (4) is

$$\begin{aligned} &\min_{\boldsymbol{x}_c,\boldsymbol{y}} \tau && (10)\\ &s.t.\quad \boldsymbol{y}=\boldsymbol{x}_c+\boldsymbol{u},\ \parallel \boldsymbol{u} \parallel \leq \tau \\ &\qquad \underline{d_{i1}} \leq \parallel \boldsymbol{y}-\boldsymbol{s}_i \parallel - \parallel \boldsymbol{y}-\boldsymbol{s}_1 \parallel \leq \overline{d_{i1}} \\ &\qquad i=2,\cdots,N && (11) \end{aligned}$$

From (11), we can directly obtain

$$\underline{d_{i1}}+ \parallel \boldsymbol{y}-\boldsymbol{s}_1 \parallel \leq \parallel \boldsymbol{y}-\boldsymbol{s}_i \parallel \leq \overline{d_{i1}}+ \parallel \boldsymbol{y}-\boldsymbol{s}_1 \parallel \quad (12)$$

It is straightforward that

$$(\underline{d_{i1}}+ \parallel \boldsymbol{y}-\boldsymbol{s}_1 \parallel)^2 \leq \parallel \boldsymbol{y}-\boldsymbol{s}_i \parallel^2 \leq (\overline{d_{i1}}+ \parallel \boldsymbol{y}-\boldsymbol{s}_1 \parallel)^2 \quad (13)$$

Remark 1: Since $\boldsymbol{y}$ represents any feasible value of $\boldsymbol{x}$, we have

$$\begin{aligned} &\underline{d_{i1}}+ \parallel \boldsymbol{y}-\boldsymbol{s}_1 \parallel \\ &= c(z_{i1}-2\gamma_t)+ \parallel \boldsymbol{y}-\boldsymbol{s}_1 \parallel \\ &= \parallel \boldsymbol{x}-\boldsymbol{s}_i \parallel - \parallel \boldsymbol{x}-\boldsymbol{s}_1 \parallel + cv_{i1} - 2c\gamma_t + \parallel \boldsymbol{x}-\boldsymbol{s}_1 \parallel \\ &= \parallel \boldsymbol{x}-\boldsymbol{s}_i \parallel + cv_{i1} - 2c\gamma_t \qquad (14) \end{aligned}$$

It is possible that $\underline{d_{i1}}+ \parallel \boldsymbol{y}-\boldsymbol{s}_1 \parallel < 0$. It only occurs when $\parallel \boldsymbol{x}-\boldsymbol{s}_i \parallel$ is very small or γ_t is very large. In practice, the sensor-target distance is usually larger than the measurement error bound, therefore, the probability of occurrence of $\underline{d_{i1}}+ \parallel \boldsymbol{y}-\boldsymbol{s}_1 \parallel < 0$ is small and negligible.

Let $d_1 = \parallel \boldsymbol{y}-\boldsymbol{s}_1 \parallel$, $\Delta = \boldsymbol{y}^T\boldsymbol{y}$ and $d_s = d_1^2$; then the inequality constraint (13) can be written as

$$\Delta - 2\boldsymbol{s}_i^T\boldsymbol{y} - d_s - 2\underline{d_{i1}}d_1 + \boldsymbol{s}_i^T\boldsymbol{s}_i - \underline{d_{i1}}^2 \geq 0 \quad (15)$$

$$\Delta - 2\boldsymbol{s}_i^T\boldsymbol{y} - d_s - 2\overline{d_{i1}}d_1 + \boldsymbol{s}_i^T\boldsymbol{s}_i - \overline{d_{i1}}^2 \leq 0 \quad (16)$$

Then the problem (10) can be written as

$$\begin{aligned} &\min_{\boldsymbol{x}_c,\boldsymbol{y},\Delta,d_1,d_s} \tau \\ &s.t.\quad \boldsymbol{y}=\boldsymbol{x}_c+\boldsymbol{u},\ \parallel \boldsymbol{u} \parallel \leq \tau \\ &\qquad \Delta - 2\boldsymbol{s}_i^T\boldsymbol{y} - d_s - 2\underline{d_{i1}}d_1 + \boldsymbol{s}_i^T\boldsymbol{s}_i - \underline{d_{i1}}^2 \geq 0 \\ &\qquad \Delta - 2\boldsymbol{s}_i^T\boldsymbol{y} - d_s - 2\overline{d_{i1}}d_1 + \boldsymbol{s}_i^T\boldsymbol{s}_i - \overline{d_{i1}}^2 \leq 0 \\ &\qquad i=2,\cdots,N \\ &\qquad d_s = \Delta - 2\boldsymbol{s}_1^T\boldsymbol{y} + \boldsymbol{s}_1^T\boldsymbol{s}_1 \\ &\qquad \Delta = \boldsymbol{y}^T\boldsymbol{y},\ d_s = d_1^2 \qquad (17) \end{aligned}$$

The equality constraints $\Delta = \boldsymbol{y}^T\boldsymbol{y}$ and $d_s = d_1^2$ are not affine, which means the constraint set is a non-convex set. The problem (17) cannot therefore be directly solved by convex optimization methods. We make the following relaxation:

$$\Delta \geq \boldsymbol{y}^T\boldsymbol{y},\ d_s \geq d_1^2 \quad (18)$$

To distinguish the relaxed Δ and d_s from the original ones, we use Δ_r and d_{sr} to denote the relaxed Δ and d_s. As is well-known, the above constraints can be written into the following linear forms

$$\begin{bmatrix} \boldsymbol{I}_2 & \boldsymbol{y} \\ \boldsymbol{y}^T & \Delta_r \end{bmatrix} \succeq \boldsymbol{0},\ \begin{bmatrix} 1 & d_1 \\ d_1 & d_{sr} \end{bmatrix} \succeq \boldsymbol{0} \quad (19)$$

where $\boldsymbol{I}_2$ denotes a 2×2 identity matrix. The relaxed problem becomes

$$\begin{aligned} &\min_{\boldsymbol{x}_c,\boldsymbol{y},\Delta_r,d_1,d_{sr}} \tau \\ &s.t.\quad \boldsymbol{y}=\boldsymbol{x}_c+\boldsymbol{u},\ \parallel \boldsymbol{u} \parallel \leq \tau \\ &\qquad \Delta_r - 2\boldsymbol{s}_i^T\boldsymbol{y} - d_{sr} - 2\underline{d_{i1}}d_1 + \boldsymbol{s}_i^T\boldsymbol{s}_i - \underline{d_{i1}}^2 \geq 0 \\ &\qquad \Delta_r - 2\boldsymbol{s}_i^T\boldsymbol{y} - d_{sr} - 2\overline{d_{i1}}d_1 + \boldsymbol{s}_i^T\boldsymbol{s}_i - \overline{d_{i1}}^2 \leq 0 \\ &\qquad i=2,\cdots,N \\ &\qquad d_{sr} = \Delta_r - 2\boldsymbol{s}_1^T\boldsymbol{y} + \boldsymbol{s}_1^T\boldsymbol{s}_1 \\ &\qquad \begin{bmatrix} \boldsymbol{I}_2 & \boldsymbol{y} \\ \boldsymbol{y}^T & \Delta_r \end{bmatrix} \succeq \boldsymbol{0},\ \begin{bmatrix} 1 & d_1 \\ d_1 & d_{sr} \end{bmatrix} \succeq \boldsymbol{0} \qquad (20) \end{aligned}$$

Furthermore, the constraint $\parallel \boldsymbol{u} \parallel \leq \tau$ can be expressed in a linear form

$$\begin{bmatrix} \tau\boldsymbol{I}_2 & \boldsymbol{u} \\ \boldsymbol{u}^T & \tau \end{bmatrix} \succeq \boldsymbol{0},\ \ \tau \geq 0 \quad (21)$$

Our problem becomes a standard SDP:

$$\begin{aligned} &\min_{\boldsymbol{x}_c,\boldsymbol{y},\Delta_r,d_1,d_{sr}} \tau \\ &s.t.\quad \boldsymbol{y}=\boldsymbol{x}_c+\boldsymbol{u} \\ &\qquad \begin{bmatrix} \tau\boldsymbol{I}_2 & \boldsymbol{u} \\ \boldsymbol{u}^T & \tau \end{bmatrix} \succeq \boldsymbol{0},\ \tau \geq 0 \\ &\qquad \Delta_r - 2\boldsymbol{s}_i^T\boldsymbol{y} - d_{sr} - 2\underline{d_{i1}}d_1 + \boldsymbol{s}_i^T\boldsymbol{s}_i - \underline{d_{i1}}^2 \geq 0 \\ &\qquad \Delta_r - 2\boldsymbol{s}_i^T\boldsymbol{y} - d_{sr} - 2\overline{d_{i1}}d_1 + \boldsymbol{s}_i^T\boldsymbol{s}_i - \overline{d_{i1}}^2 \leq 0 \\ &\qquad i=2,\cdots,N \\ &\qquad d_{sr} = \Delta_r - 2\boldsymbol{s}_1^T\boldsymbol{y} + \boldsymbol{s}_1^T\boldsymbol{s}_1 \\ &\qquad \begin{bmatrix} \boldsymbol{I}_2 & \boldsymbol{y} \\ \boldsymbol{y}^T & \Delta_r \end{bmatrix} \succeq \boldsymbol{0},\ \begin{bmatrix} 1 & d_1 \\ d_1 & d_{sr} \end{bmatrix} \succeq \boldsymbol{0} \qquad (22) \end{aligned}$$

The above SDP can be solved by many existing SDP solvers, e.g., SeDuMi [15], etc.

IV. Simulation Results

In the simulations, we compare the performance of our proposed algorithm, denoted by SDP-Cheby, with other algorithms: 1) the classic LS algorithm in [2]; 2) SDPO and SDPI in [8], which minimize the maximum matching error between the TDOA measurements by solving two SDP problems; 3) Algorithm in [5], denoted by SDP-AP, which combines all the pairwise TDOA measurements to minimize the error between TDOA measurements and TDOA estimates; SDP-AP also uses SDR to relax the localization problem into an SDP problem.

In the examples of our simulation, there are 8 sensors in a 2D area. The locations of the sensors are

$$\begin{aligned} &\boldsymbol{s}_1 = [40,40]^T,\ \boldsymbol{s}_2 = [40,-40]^T,\ \boldsymbol{s}_3 = [-40,40]^T, \\ &\boldsymbol{s}_4 = [-40,-40]^T, \boldsymbol{s}_5 = [40,0]^T,\ \boldsymbol{s}_6 = [0,40]^T, \\ &\boldsymbol{s}_7 = [-40,0]^T,\ \boldsymbol{s}_8 = [0,-40]^T \end{aligned}$$

We mainly compare the root mean square errors (RMSEs) and the maximum position errors (MPEs) of the localization algorithms. We use SeDuMi as the SDP solver in the implementation of the localization algorithms. In the following examples, for simplicity, the measurement error is converted

TABLE I
ROBUSTNESS TO THE SOURCE LOCATION

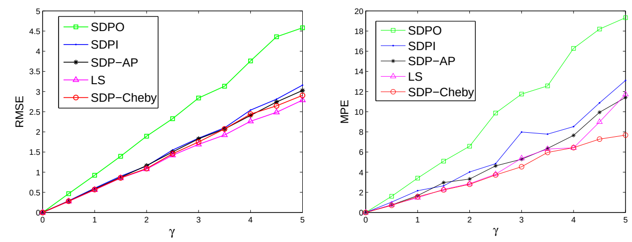
	SDPO	SDPI	SDP-AP	LS	SDP-Cheby
RMSE(Inside the convex hull)	2.5325	2.4159	1.8812	2.1300	1.6296
RMSE(Outside the convex hull)	16.3611	13.2851	10.3627	11.7446	18.9258
MPE(Inside the convex hull)	18.4129	13.5262	10.9544	12.1807	6.8291
MPE(Outside the convex hull)	178.8428	84.3838	71.9970	74.9970	64.8029

from time domain into distance domain, which means that, the measurement error bound in the simulations is the distance error bound, i.e. $\gamma = c\gamma_t$. The actual measurement error is determined in accordance with a uniform distribution or Gaussian distribution, but our localization algorithm runs under the assumption that the error is limited by a fixed known bound.

Example 1. In this example, the position of the target is $\boldsymbol{x} = [20, 10]^T$, which is inside the convex hull of the sensors. The measurement error is uniformly distributed. Fig.2(a) compares the RMSEs of the localization algorithms under different error bounds. Except for SDPO, RMSEs of all other four algorithms are very close. RMSEs of our proposed SDP-Cheby are slightly higher than LS, and lower than SDPO, SDPI and SDP-AP. By comparing the MPEs of different algorithms, as shown in Fig. 2(b), we can see that SDP-Cheby yields the lowest MPE.

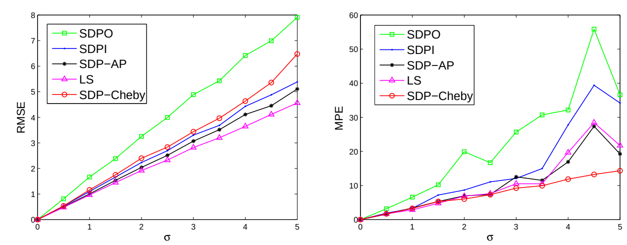
Example 2. In this example, let the target be randomly placed at 200 different locations in a square area of $[-100, 100]^2$, among which, 100 locations are inside the convex hull of the sensors, and the other 100 locations are outside the convex hull of the sensors. The measurement error is uniformly distributed with measurement error bound $\gamma = 3$. At each location, the target's location is estimated by 50 Monte Carlo trials. Table I shows the localization results, including both RMSEs and MPEs. The simulation results reveal, as expected, that when the target is located inside the convex hull of the sensors, the localization performance is much better than that when the target is located outside the convex hull of the sensors. By comparing RMSEs of different algorithms, we can find that RMSE of SDP-Cheby is the smallest when the target is inside the convex hull of the sensors; however, RMSE of SDP-Cheby becomes the largest one when the target is outside the convex hull of the sensors. On the other hand, by comparing MPEs of different algorithms, we can find that MPE of SDP-Cheby is smaller than that of other algorithms both inside and outside the convex hull.

Example 3. In this example, the position of the target is $\boldsymbol{x} = [20, 10]^T$, which is inside the convex hull of the sensors. The measurement error is Gaussian distributed with zero-mean. The error bound is set as $\gamma = 3\sigma$, where σ is the standard deviation of the Gaussian distributed measurement error. Fig.3(a) compares the RMSEs of the localization algorithms as σ varies. We can see the RMSEs of SDP-Cheby are higher than SDPI, SDP-AP and LS, and are lower than SDPO. On the other hand, by comparing the MPEs of different algorithms, as shown in Fig. 3(b), we can see that the MPEs of SDP-Cheby are lower than other algorithms. [1] [2]



(a) RMSE (b) Maximum position error (MPE)

Fig. 2. Localization performance comparison of SDP-Cheby, SDPO, SDPI, SDP-AP and LS under uniformly distributed measurement errors when the target is inside the convex hull of the sensors.



(a) RMSE (b) Maximum position error (MPE)

Fig. 3. Localization performance comparison of SDP-Cheby, SDPO, SDPI, SDP-AP and LS under Gaussian distributed measurement errors when the target is inside the convex hull of the sensors. The error bound is $\gamma = 3\sigma$.

V. CONCLUSION

In this paper, we investigate a single target localization problem using TDOA measurements with unknown and bounded measurement errors. We consider the minimization of the worst-case position estimation error, and formulate our problem as a non-convex optimization problem to find the Chebyshev center of the target's feasible set. Through SDR, we transform this non-convex optimization problem into a solvable SDP problem, and obtain a relaxed estimate of the Chebyshev center. Simulation results show that although the RMSEs of our proposed algorithm are higher than other algorithms in many cases, the MPEs of our proposed algorithm are lower than other algorithms. That is to say, our algorithm achieves the best worst-case location estimation error.

[1] One might consider use of CDFs to compare performance, but the differences are not as striking as with RMSE and MPE.

[2] Further simulations reveal that the proposed method is robust to inaccurate measurement error bounds.

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