Evaluation of the Probability of K-hop Connection in Homogeneous Wireless Sensor Networks

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***Abstract*— Given a wireless sensor network (WSN) whose sensors are randomly and independently distributed in a bounded area following a homogeneous Poisson process with density ρ and each sensor has a uniform transmission radius of r_0, we investigate the probability that two random sensors separated by a known distance x are k-hop neighbors for some positive integer k in this paper. We give a closed-form equation for computing this probability for $k = 2$; and also give a recursive equation for evaluating this probability for $k > 2$ by using some approximations. The accuracy of the approximate analytical solution is validated by simulations. Furthermore, we present an empirical method to correct the discrepancies between the analytical results and the simulation results caused by the approximation. The result of this paper can be useful in a number of sensor network problems, e.g., estimating the transmission delay between two sensors and energy consumed in the transmission, and WSN routing problems.**

I. Introduction

Wireless sensor networks (WSNs) have been widely investigated and discussed in recent years. Generally, a typical wireless sensor network consists of a large number of small, inexpensive, low-power and multi-functional sensor nodes, deployed either randomly or according to some predefined statistical distribution, over a geographic region of interest [1]. Extensive research has been done in the area of WSNs, including routing protocols [2], localization algorithms [3], power control technique [4], and so on. Many of these problems can be studied in the framework of graph theory.

Consider a WSN whose sensors are randomly and independently distributed in a bounded region in $\Re^2$, according to a homogeneous Poisson process. Each sensor has a transmission radius of r_0. For many purposes, this sensor network can be modeled by a unit disk graph with a vertex in the graph representing every sensor and an edge in the graph representing every sensor pair for which the two sensors can directly communicate with each other. Any two sensors can directly communicate with each other iff (if and only if) their Euclidean distance is smaller than the given threshold r_0. The resulting graph $G = (V, E)$, where V is the vertex set and E is the edge set, is called the *underlying graph* of the network. Many interesting aspects of a WSN can be studied using its underlying graph, such as the network connectivity [5], [6], the probability that any two sensors in the network are k-hop neighbors for some positive integer k [7], [8] and the critical transmission radius required to achieve an asymptotically connected network [9].

In this paper we investigate the conditional probability $\Phi_k(x)$ that any two sensors separated by a known distance x are k-hop neighbors for some positive integer k. Two sensors being k-hop neighbors means that the length of the shortest path between the two sensors, measured in the number of hops, is k. A recursive analytical equation embodying an approximation is given for computing this probability. The results developed in the paper may lead to solution to problems such as estimation of the energy consumption for data transmission between two sensors and the associated transmission delay and WSN routing problems [10], [11]. Also given the probability density $p(x)$ for the distance x between any two sensors [12], our result can be used to compute Φ_k, i.e., the unconditional probability that any two random sensors are k-hop neighbors, which is useful for studying many WSN problems such as estimating the overall energy consumption and the lifetime of a WSN. Furthermore, once $\Phi_k(x)$ has been obtained, $\Phi_x(k)$ can then be easily found using Bayes' formula. Knowledge of $\Phi_x(k)$ may be used to estimate the geographical distance x between two sensors via maximum likelihood estimation if the hop number k is known, which may be used to improve existing localization algorithms [10].

In [7], Bettstetter *et al.* investigated the probability that two random sensors are k-hop neighbors for $k = 1$ and $k = 2$, where n sensors are uniformly distributed in a rectangular area. Their results are based on the distribution of the distance between two random sensors derived by Ghosh [13]. For $k > 2$, only simulation results were presented. In [8], Miller considered sensors distributed following a two dimensional Gaussian distribution, and derived an approximation for the probability that two random sensors are two-hop neighbors. In [14], Chandler analyzed the probability two random packet radio stations separated by a known distance can communicate in k or less hops where stations are uniformly distributed over flat earth.

In this paper, we provide a recursive equation for computing the probability that two random sensors are k-hop neighbors for some positive integer k. The technique used in deriving

[1]National ICT Australia is funded by the Australian Government Department of Communications, Information Technology and the Arts and the Australian Research Council through the *Backing Australia Ability initiative* and the ICT Centre of Excellence Program.

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the recursive equation is the same as that used in [14]. A contribution of this paper is we point out the reliance of the analysis on the *independence assumption*, which is defined later, and the performance impact of the *independence assumption*. In addition, we show that the probability that two random sensors are k-hop neighbors is only determined by two parameters, i.e., the normalized distance and the average vertex degree. We also provide simulation results as well as discrepancy analysis between the simulation results and the analytical results. Furthermore, we present an empirical method to correct the discrepancy caused by the *independence assumption*.

The rest of this paper is organized as follows. In Section II we derive a closed-form equation for the probability that any two sensors separated by a known distance x are two-hop neighbors. In Section III we develop (using an approximating assumption) a recursive analytical equation for evaluating the probability that any two random sensors separated by a known distance x are k-hop neighbors for $k > 2$. Section IV presents simulation results and analyzes the causes of the discrepancies between the analytical results and the simulation results. Section V presents a method to correct the discrepancies, which gives a more accurate results. Finally, Section VI concludes this paper and discusses future research directions.

II. Probability that any two sensors are two-hop neighbors

In this paper we consider a WSN whose sensors are randomly and independently distributed in a bounded area according to a homogeneous Poisson process with node density ρ and the underlying graph of the sensor network is a unit disk graph with a uniform transmission radius r_0. The transmission range of one sensor is defined as the circle of radius r_0 centered at this sensor.

Obviously, $\Phi_1(x) = 1$ when $x \leq r_0$ and $\Phi_1(x) = 0$ when $x > r_0$. For $k = 2$, it means that the two sensors have no direct link between each other but can communicate through at least one intermediate sensor. Therefore at least one intermediate sensor must lie in the intersectional area of the transmission ranges of the two sensors, i.e., the shaded area **A** in Fig. 1, to act as a relay sensor. Ignoring the boundary effect, the

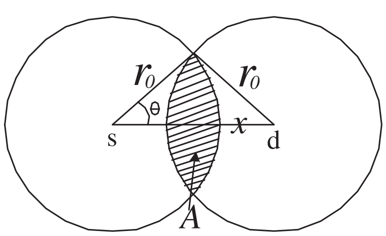

Fig. 1. An illustration of two-hop neighbors. Sensors s and d are two random sensors, separated by a known distance x ($r_0 < x \leq 2r_0$).

probability $\Phi_2(x)$ can be found as the probability that there is at least one sensor located in the region **A** in Fig. 1.

Since sensors are Poissonly distributed, the probability that there is no sensor located in **A** is $exp(-\rho A)$, where A is the size of the area **A**, given by

$$A = 2r_0^2 \arcsin(\sqrt{1-\frac{x^2}{4r_0^2}}) - xr_0\sqrt{1-\frac{x^2}{4r_0^2}}, \quad r_0 < x \leq 2r_0. \tag{1}$$

Hence, the probability $\Phi_2(x)$ can be readily obtained:

$$\Phi_2(x) = 1 - Pr\{\texttt{no sensor in } \mathbf{A}\} = 1 - e^{-\rho A}. \tag{2}$$

When $x \leq r_0$, the two sensors can connect directly with each other, so $\Phi_2(x) = 0$; when $x > 2r_0$, we have $A = 0$, so that, $\Phi_2(x) = 0$. Therefore,

$$\Phi_2(x) = \begin{cases} 1 - e^{-\rho A} & r_0 < x \leq 2r_0; \\ 0 & \texttt{otherwise.} \end{cases} \tag{3}$$

III. Probability that any two sensors are k-hop neighbors

For $k > 2$, it can be easily understood through the geometry that the derivation of the probability is rather tedious and it is very hard to obtain an exactly closed form equation for $\Phi_k(x)$. In this section, we shall evaluate the probability $\Phi_k(x)$ for $k > 2$, continuing to ignore the boundary effect. Consider two random sensors s and d which are separated by a known distance x, as shown in Fig. 2. Sensor d is a k-hop neighbor of s iff sensor d is not a m-hop neighbor of s for any $m < k$ *and* there is at least one sensor within the transmission range of d which is a $k-1$ hop neighbor of s.

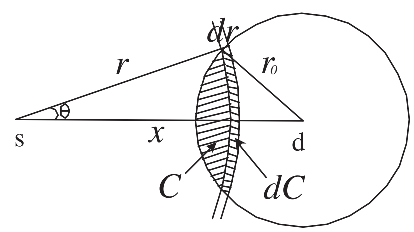

Fig. 2. An illustration of k-hop ($k > 2$) neighbors. Sensors s and d are two random sensors separated by a known distance x ($r_0 < x \leq kr_0$).

Let us first consider the probability that there is at least one sensor within the transmission range of d which is a $k-1$ hop neighbor of s. An approximation, termed the *independence assumption*, has to be used in order to obtain an analytical solution, i.e., the event that one arbitrary sensor located within the transmission range of d is a $k-1$ hop neighbor of s is independent of the event that another arbitrary sensor located within the transmission range of d is a $k-1$ hop neighbor of s. The implication of this approximation will be discussed in the next section. Consider the area C in Fig. 2 which is the intersectional area of a circle of radius r centered at s and a circle of radius r_0 centered at d. A differential increment dr on r gives a differential area dC and the size of the differential area dC is $2r\theta dr$, where θ is given by,

$$\theta = \arccos\frac{x^2 + r^2 - r_0^2}{2xr}. \tag{4}$$

Since dr is a very small value, the probability that there exists more than one sensor within dC can be ignored and the probability that there exists a single sensor in the differential area dC is given by $2\rho r\theta dr$. Given $\Phi_{k-1}(r)$, the probability that there is a sensor within dC which is also a $k-1$ hop neighbors of s is given by $2\Phi_{k-1}(r)\rho r\theta dr$.

Let $f(C)$ denote the probability that there is *no* sensor in C which is a $k-1$ hop neighbor of s. Using the *independence assumption*, the probability $f(C)$ is independent of the probability that there is no sensor in dC which is a $k-1$ hop neighbor of s, hence

$$f(C+dC) = f(C)(1 - 2\Phi_{k-1}(r)\rho r\theta dr). \quad (5)$$

Eq. 5 readily leads to the conclusion that

$$df(C) = -2\Phi_{k-1}(r)\rho r\theta f(C)dr. \quad (6)$$

Therefore the probability that there is *no* sensor within the transmission range of d which is a $k-1$ hop neighbor of s at distance x is given by

$$g(x) = exp(\int_{x-r_0}^{x+r_0} -2\Phi_{k-1}(r)\rho r\theta dr). \quad (7)$$

The probability that a sensor d at a distance x to s is not a m-hop neighbor of s for any $m < k$ is given by

$$1 - \Phi_1(x) - \Phi_2(x) - ... - \Phi_{k-1}(x) = 1 - \sum_{i=1}^{k-1}\Phi_i(x). \quad (8)$$

Therefore the probability that sensor d is a k-hop neighbor of s is given recursively as:

$$\Phi_k(x) = (1-\sum_{i=1}^{k-1}\Phi_i(x))(1-\exp(-\int_{x-r_0}^{x+r_0} 2\Phi_{k-1}(r)\rho r\theta dr)), \quad (9)$$

where $r_0 < x \le kr_0$; $\Phi_k(x) = 0$ if $x \le r_0$ or $x > kr_0$.

For $k = 2$, when $x \le r_0$ or $x > 2r_0$, $\Phi_2(x) = 0$; when $r_0 < x \le 2r_0$, it can be shown that

$$-\int_{x-r_0}^{x+r_0} 2\Phi_1(r)\rho r\theta dr = -\rho\int_{x-r_0}^{r_0} 2r\theta dr = -\rho A, \quad (10)$$

where A is given in Eq. 1, and $1-\Phi_1(x) = 1$, the expression for $\Phi_2(x)$ agrees with that in Eq. 3.

A. Discussion

Let $\alpha \doteq x/r_0$ and $\beta \doteq \pi r_0^2\rho$. The parameter α is the normalized distance and the parameter β is the average vertex degree of a unit disk graph. It can be shown by *mathematical induction* that the probability $\Phi_k(x)$, which is parameterized by ρ, x and r_0, is a function of α and β only.

Proof: For $k = 1$, it can be readily shown that $\Phi_1(x)$ is a function of α and β only.

Suppose $\Phi_n(x)$ can be expressed as a function of α and β for $n \le k$, i.e., $\Phi_n(x) = \Upsilon_n(\alpha, \beta)$. Then when $n = k+1$, the first term on the right side of Eq. 9 is

$$1 - \sum_{i=1}^{k}\Phi_i(x) = 1 - \sum_{i=1}^{k}\Upsilon_i(\alpha, \beta). \quad (11)$$

Let $\mu = r/r_0$, the integral in the second term on the right side of Eq. 9 becomes

$$\int_{x-r_0}^{x+r_0} 2\Upsilon_k(\frac{r}{r_0}, \beta)\rho r\theta dr \quad (12)$$

$$= \int_{x/r_0-1}^{x/r_0+1} 2\Upsilon_k(\mu, \beta)\rho\mu r_0\theta r_0 d\mu \quad (13)$$

$$= \frac{\beta}{\pi}\int_{\alpha-1}^{\alpha+1} 2\Upsilon_k(\mu, \beta)\arccos\frac{\alpha^2+\mu^2-1}{2\alpha\mu}\mu d\mu. \quad (14)$$

From Eq. 9, 11 and 14, we have $\Phi_{k+1}(x) = \Upsilon_{k+1}(\alpha, \beta)$, hence, the hypothesis is also valid when $n = k+1$. This completes the proof.

The above discussion leads to the insight that *under the independence assumption, the probability that two random sensors are k-hop neighbors is only determined by the normalized distance between the two sensors and the average vertex degree.*

Note that in this paper, we consider the ideal case that the wireless link between any two sensors does not suffer from shadow fading. The transmission range of a sensor is modeled by a disk with radius r_0 centered at this sensor. The analysis in this paper can be extended to consider shadow fading environment, which is typically modeled by a log-normal model.

IV. Simulation and Discrepancy Analysis

In this section, we use simulations to establish the accuracy of the theoretical analysis in the presence of boundary effects and the shortcomings of the independence assumption. In the simulation, sensors are distributed in a square of size $a \times a$, where $a = 20$, according to a homogenous Poisson process with node density ρ. We vary the average vertex degree (i.e., $\pi r_0^2\rho$) while keeping the node density ρ fixed, each value of the average vertex degree represents a different scenario. Each scenario is repeated 100 times and the average result is shown.

Figs. 3 shows the probability that any two sensors separated by a distance x are k-hop neighbors for $k = 2, 3$ and 4. For $k = 2$, we can see that the simulation results and the analytical results agree very well, which indicates that Eq. 3 is an accurate expression of $\Phi_2(x)$. However, for $k = 3$ and $k = 4$, there are slight discrepancies between the analytical results and the corresponding simulation results, as shown in Fig. 3. The figure also shows that for $k = 3$ and $k = 4$, when $(k-1)r_0 < x \le kr_0$, the analytical results are always larger than the corresponding simulation results. The discrepancies are attributable to the boundary effect and the independence assumption we used, as will be discussed below.

A. Boundary Effect

For any two sensors s_i and s_j which are close to the border, the intersectional region of the transmission ranges of the two sensors may be located partially outside the network area, which causes an error in computing $\Phi_k(x)$. This effect is the boundary effect. The impact of the boundary effect will reduce as the network area becomes larger compared to r_0.

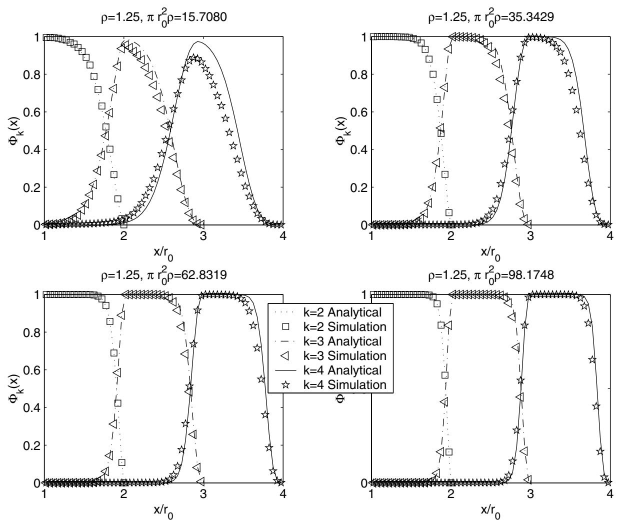

Fig. 3. Probability that two random sensors separated by a known distance x are k-hop neighbors for $k = 2, 3$ and 4.

To evaluate the impact of the boundary effect, we keep $\pi r_0^2 \rho$ constant and vary the ratio r_0/a to calculate the mean absolute difference (MAD) between the analytical results and the simulation results. MAD is the average value of the absolute differences, i.e., $MAD = \frac{1}{N}\sum_{i=1}^{N}|Ana_i - Sim_i|$, where Ana_i and Sim_i are the i-th analytical result and its corresponding simulation result respectively, and N is the number of results selected to calculate MAD. The larger MAD is, the greater the discrepancy is. The result is shown in Fig. 4. When the ratio r_0/a is small enough, the boundary

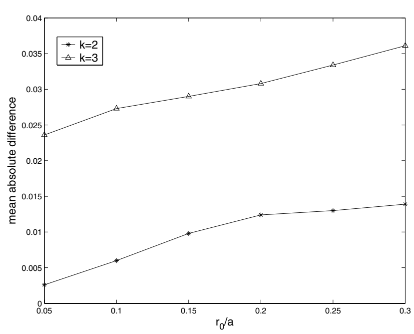

Fig. 4. MAD for different values of r_0/a; $\pi r_0^2 \rho$ is kept constant at 15.7080. When r_0/a increases, a larger percentage of sensors are located near the boundary and the impact of the boundary effect is more pronounced. Thus the discrepancy between the analytical and the simulation results increases.

effect can be ignored. Moreover, the boundary effect can be eliminated in simulations by using *toroidal distance metric* [6].

B. Dependence Problem

A second cause of the error is the independence assumption in section III. The assumption is true for $k = 2$, but (and this is a subtle point) it is not absolutely valid for $k > 2$. Now we take $k = 3$ as an example to explain the difference. In Fig. 5, sensor d is a 3-hop neighbor of s iff sensor d is not an m-hop neighbor of s for any $m < 3$ *and* there is at least one sensor within the transmission range of d which is a 2-hop neighbor of s. For sensor d_1, it is a 2-hop neighbor of s iff there is at least one sensor within $C_1 \cup C_2$; and for sensor d_2, it is a 2-hop neighbor of s iff there is at least one sensor within $C_2 \cup C_3$. We call C_2 the dependent area of sensors d_1 and d_2, and call C_1 and C_3 the independent areas. If there is a sensor in C_2, this sensor will form both a part of the path from d_1 to s and a part of the path from d_2 to s. Consequently, the probability that d_1 is a 2-hop neighbor of s and the probability that d_2 is a 2-hop neighbor of s are no longer independent. This correlation between the two probabilities constitutes a violation of the independence assumption and thus contributes to the error in our analytical results.

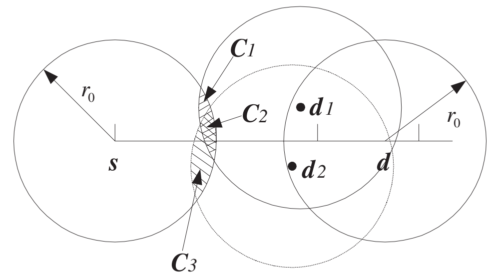

Fig. 5. Dependence problem for $k = 3$. s and d are the source and the destination respectively with a distance x apart. Sensors d_1 and d_2 are two sensors randomly located within the transmission range of d; $C_1 \cup C_2$ is the intersectional area of the transmission ranges of s and d_1; $C_3 \cup C_2$ is the intersectional area of the transmission ranges of s and d_2. The probability that d_1 is a 2-hop neighbor of s is not independent of the probability that d_2 is a 2-hop neighbor of s because there is nonzero probability of having a sensor in C_2.

Now we explain the exact implication of the above discussion on our analysis and remedy for it. Let Λ_i denote the event that an arbitrary differential area dS_i in the transmission range of d has one sensor s_i in it[1] and this sensor s_i is also a $k-1$ hop neighbor of s, and let $Pr\{\Lambda_i\}$ denote the corresponding probability of event Λ_i. Then,

$$Pr\{\Lambda_i\} = \Phi_{k-1}(r_i) \cdot \rho dS_i, \quad (15)$$

where r_i is the distance between sensors s and s_i. The integral in the second term on the right side of Eq. 9 is actually the sum of $Pr\{\Lambda_i\}$, i.e.,

$$\int_{x-r_0}^{x+r_0} 2\Phi_{k-1}(r)\rho r\theta dr = \sum_i Pr\{\Lambda_i\}. \quad (16)$$

Due to the dependency problem as discussed before, in the second term on the right side of Eq. 9, it should be the probability of the union of all possible Λ_i, i.e., $Pr\{\bigcup_i \Lambda_i\}$, instead of $2\Phi_k(r)\rho r\theta_0 dr = \sum_i Pr\{\Lambda_i\}$. We define a new variable $\Phi'_k(x)$ to be the correct probability considering the dependency problem, then,

$$\Phi'_k(x) = (1 - \sum_{i=1}^{k-1}\Phi'_i(x))(1 - \exp(-Pr\{\bigcup_i \Lambda_i\})). \quad (17)$$

[1] Since dS_i is a very small value, the probability that there is more than one sensor within dS_i can be ignored.

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Since the two events Λ_i and Λ_j $(i \neq j)$ are not necessarily independent, we have,

$$Pr\{\bigcup_i \Lambda_i\} \leq \sum_i Pr\{\Lambda_i\}, \quad (18)$$

Therefore, the true value of the second term on the right side of Eq. 9 is overestimated, i.e.,

$$1 - \exp(Pr\{\bigcup_i \Lambda_i\}) \leq 1 - \exp(\int_{x-r_0}^{x+r_0} 2\Phi_{k-1}(r)\rho r\theta dr). \quad (19)$$

When $(k-1)r_0 < x \leq kr_0$, $\Phi'_m(x) = \Phi_m(x) = 0$ for all $m < k$, therefore, by Eq. 17

$$\begin{aligned}\Phi'_k(x) &= 1 - \exp(-Pr\{\bigcup_i \Lambda_i\}), \\ &\leq 1 - \exp(-\int_{x-r_0}^{x+r_0} 2\Phi_{k-1}(r)\rho r\theta dr) = \Phi_k(x). \quad (20)\end{aligned}$$

Therefore, *when* $(k-1)r_0 < x \leq kr_0$*, Eq. 9 actually gives a over bound for the real probability* $\Phi'_k(x)$. This explains why the analytical results yield values which are always larger than those from the simulation results when $(k-1)r_0 < x \leq kr_0$ as shown in Fig. 3. Note however that when ρ goes to infinity, $\Phi_k(x) \rightarrow \Phi'_k(x)$, i.e., the impact of the dependency problem vanishes.

V. Correction to the Analytical Recursive Equation

Though Eq. 17 is more accurate than Eq. 9, $Pr\{\bigcup_i \Lambda_i\}$ in the second term on the right side of Eq. 17 is very hard to compute. In this section, we present an empirical method to correct for the error caused by the independence assumption.

Given two events E and F, there exists a real number ζ, $\zeta \in [0, 1]$, such that,

$$\begin{aligned} Pr\{E \cup F\} &= Pr\{E\} + Pr\{F\} - Pr\{E \cap F\}, \quad (21) \\ &= \zeta \cdot (Pr\{E\} + Pr\{F\}). \quad (22)\end{aligned}$$

Therefore, we assume (and this *is* an assumption) that there exists a function $\xi_k(\beta)$, $\xi_k(\beta) \in [0, 1]$ such that,

$$Pr\{\bigcup_i \Lambda_i\} = \xi_k(\beta) \cdot \sum_i Pr\{\Lambda_i\}. \quad (23)$$

where β is defined in subsection III-A. Then, the probability $\Phi'_k(x)$ becomes,

$$\begin{aligned}\Phi'_k(x) &= (1 - \sum_{i=1}^{k-1} \Phi'_i(x)) \\ &\cdot (1 - \exp(-\xi_k(\beta) \int_{x-r_0}^{x+r_0} 2\Phi'_{k-1}(r)\rho r\theta dr)). \quad (24)\end{aligned}$$

Now, we give a simple way to estimate $\xi_k(\beta)$ by $\hat{\xi}_k$. For a given k, first, we substitute $\xi_k(\beta)$ by $\hat{\xi}_k$ in Eq. 24 and obtain the estimated probability $\hat{\Phi}'_k(x)$, i.e.,

$$\begin{aligned}\hat{\Phi}'_k(x) &= (1 - \sum_{i=1}^{k-1} \hat{\Phi}'_i(x)) \\ &\cdot (1 - \exp(-\hat{\xi}_k \int_{x-r_0}^{x+r_0} 2\hat{\Phi}'_{k-1}(r)\rho r\theta dr)). \quad (25)\end{aligned}$$

Second, we select one value of β (e.g., $\beta = 15.7080$) and vary the value of $\hat{\xi}_k$ to calculate the MAD between $\hat{\Phi}'_k(x)$ and the simulation results, such that $\hat{\xi}_k = \arg\min MAD$, hence obtain the optimal value of $\hat{\xi}_k$. To check the impact of β on $\xi_k(\beta)$, we may compare $\hat{\Phi}'_k(x)$ and the corresponding simulation results with different values of β, to see if it works well with different β. As two examples, $\hat{\xi}_3 = 0.76$ and $\hat{\xi}_4 = 0.66$ are derived with $\beta = 15.7080$, then are substituted into three other scenarios (i.e., $\beta = 35.3429, 62.8319$ and 98.1748). The results are shown in Figs. 6 and 7 respectively, we can see that the estimated probabilities and the simulation results always agree very well with different values of β, which indicates that β has marginal impact on $\xi_k(\beta)$.

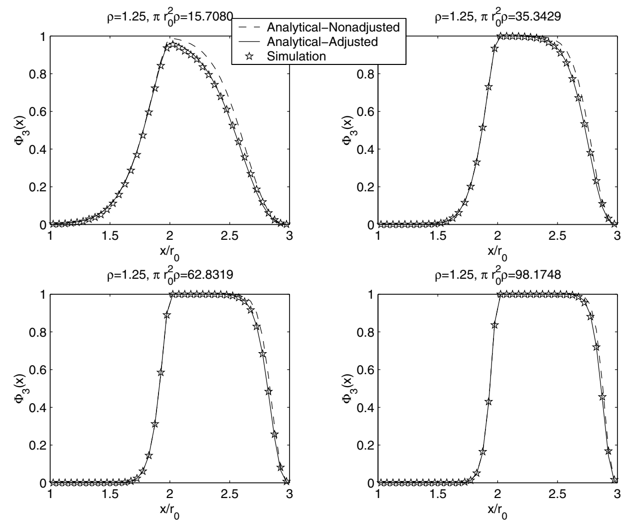

Fig. 6. Probability that two random sensors separated by a known distance x are 3-hop neighbors. $\hat{\xi}_3 = 0.76$.

Our empirical simulation shows that $\xi_k(\beta)$ critically depends on k but is almost independent of the value of β. While this relationship between $\xi_k(\beta)$, k and β needs a further investigation, we offer the following intuitive explanation on why $\xi_k(\beta)$ is less dependent on β through the example in Fig. 5. As explained earlier, the correlation between the probability that d_1 is a 2-hop neighbor of s and the probability that d_2 is a 2-hop neighbor of s is a major cause of the dependence problem, hence the error. Note that this correlation in turn critically depends on the *relative* size of $C_1/\pi r_0^2$, $C_2/\pi r_0^2$ and $C_3/\pi r_0^2$ and is less affected by the average vertex degree β. Therefore one may naturally expect that the correction coefficient $\xi_k(\beta)$ is also less dependent on β than on k.

This approximate independence of $\xi_k(\beta)$ and β allows us to estimate $\xi_k(\beta)$ by $\hat{\xi}_k$ whose value can be established via a

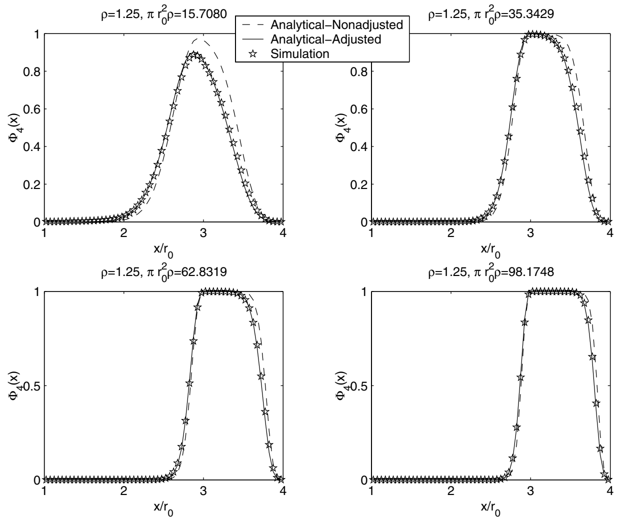

Fig. 7. Probability that two random sensors separated by a known distance x are 4-hop neighbors. $\hat{\xi}_4 = 0.66$.

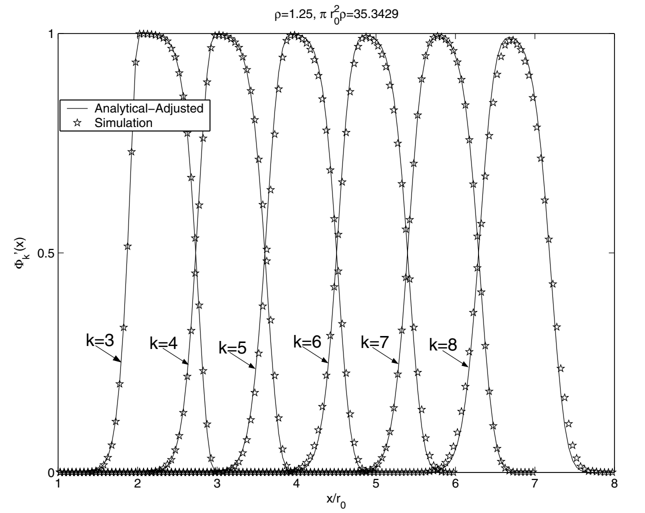

Fig. 8. Probability that two random sensors separated by a known distance x are k-hop neighbors, where k is from 3 to 8.

priori simulation because it is not affected by change in the average vertex degree β.

Table I shows the values of $\hat{\xi}_k$ for k from 3 to 10. The values were obtained by simulating with $\beta = 15.7080$, $\rho = 1.25$, $r_0 = 2$ and $a = 20$. Fig. 8 shows the estimated probability $\hat{\Phi}'_k(x)$ and the simulation results for k from 3 to 8, which shows that the proposed empirical method can effectively correct the error caused by the independence assumption. Note that the value of β in Fig. 8 is 35.3429.

TABLE I

ESTIMATED VALUES OF $\xi_k(\beta)$.

k	3	4	5	6	7	8	9	10
$\hat{\xi}_k$	0.76	0.66	0.76	0.66	0.63	0.60	0.58	0.57

VI. CONCLUSION AND FUTURE WORK

In this paper, we first presented an analytical recursive equation for the probability that any two sensors separated by a known distance x are k-hop neighbors for any positive integer k, using the approximation that the probability that a random sensor is a k-hop neighbor of one sensor is independent of the probability that another random sensor is also a k-hop neighbor of the same sensor. The error in the analytical result was analyzed and an empirical method was presented to correct the error caused by the approximation. Simulation showed a good accuracy of the proposed empirical method.

As part of our future work, we intend to seek an accurate analysis of the relationship between $\xi_k(\beta)$ and β. We also intend to investigate the probability $\Phi_k(x)$ in a log-normal shadowing environment.

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This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE GLOBECOM 2007 proceedings.