Achieving Bi-Channel-Connectivity with Topology Control in Cognitive Radio Networks

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***Abstract*—In cognitive radio networks (CRNs), secondary users (SUs) must vacate the spectrum when it is reclaimed by the primary users (PUs). As such, multiple SUs transmitting on the same channel will be affected when the channel is requested by the PUs, thereby resulting in a possible network partition of CRNs. Therefore, how to maintain the connectivity of CRNs considering the activity of PUs is a critical problem. In this paper, we propose a centralized and a distributed topology control algorithm respectively to address this problem. Particularly, we combine power control and channel assignment to construct a bi-channel-connected and conflict-free topology using the minimum number of channels. In the power control phase, we tailor the topology for the channel assignment in the second phase. In the channel assignment phase, we utilize the graph coloring algorithm to achieve conflict-free transmission by assigning a channel to each SU. Theoretical analysis and simulation study show that the derived topology can maintain connectivity in the event of any single channel interruption by PUs. Simulation results also demonstrate that the proposed algorithms can efficiently reduce the average number of required channels for achieving bi-channel-connectivity and conflict-free transmission and ensure that the minimum power paths in the original network preserved in the final topology.**

***Index Terms*—Bi-channel-connectivity, cognitive radio networks, topology control.**

I. Introduction

DUE to the proliferation of ubiquitous services provided by mobile devices, a dramatic increase in demand for radio resources has put great pressure on the already crowded frequency spectrum. On the other hand, Federal Communications Commission reports that large portions of licensed spectrum remain under utilized in spatio-temporal domains due to the current static spectrum management strategies [1]. Therefore, the limited spectrum availability and the under-utilization in licensed bands necessitate a new communication paradigm. Consequently, cognitive radio (CR) has emerged as a promising technique to alleviate spectrum scarcity and improve spectrum utilization by implementing opportunistic spectrum access over the licensed spectrum [2], [3].

In a cognitive radio network (CRN), there are a collection of secondary users (SUs) coexisting with primary users (PUs). PUs are the license holder of the radio spectrum, whereas SUs have no license for accessing spectrum but are allowed to temporarily access the licensed bands according to the spectrum availability. To avoid harmful interference to PUs, SUs must vacate the spectrum when it is reclaimed by PUs. As such, multiple SUs in the CRN have to cease their data transmissions or switch their transmissions to other unoccupied spectrum if the channel they operate on becomes occupied by a PU. Therefore, the emergence of one or more active PUs is possible to partition the CRN, especially during the spectrum switching, resulting in packet loss or substantial packet delay for the SUs.

Then, a natural question arises: How to maintain the connectivity of the CRN considering the activities of PUs? An effective approach to addressing this problem is controlling the topology of the CRN [4]–[6]. However, topology control in CRNs has some challenges that differentiate it from the topology control in classical ad hoc or cellular networks. First, due to the large interference region of PUs, the emergence of an active PU is likely to affect multiple SUs transmitting on the same channel and located in the interference region of the PU. In other words, multiple SUs will be removed simultaneously upon the appearance of a PU in the CRN. Comparatively, the removal of nodes due to node failure is independent or has low spatial correlation with each other in the classical ad hoc network. Second, the channel availability in the CRN varies over time because of the activity of PUs. Also, the difficulty in the prediction of PUs activity makes this problem even more challenging.

In addition to the interference between PUs and SUs, the interference among SUs will also cause network performance degradation. In particular, when multiple SUs access the same channel, the nearby simultaneous transmissions will introduce severe interference. To avoid such interference, a good channel assignment scheme is desired such that the conflict-free transmissions are possible. For example, if the channel assigned to an SU is different from that assigned to its conflicting neighbors, then this SU can transmit without interfering these

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conflicting neighbors. Moreover, in order to keep SUs from accessing the channels that are frequently reclaimed by PUs, it is crucial to minimize the number of channels required to achieve conflict-free transmissions in CRNs. Therefore, SUs are able to operate on the channels with high idle probabilities, thereby alleviating frequently transmission interruption and spectrum switching.

Based on these observations, in this paper we shall construct an underlying topology robust against interruptions from PUs activities and produce a spectrum assignment in a CRN to achieve bi-channel-connectivity and conflict-free transmission. A CRN is called bi-channel connected if the network remains connected when any channel currently being used by an SU becomes occupied by a PU. Obviously a bi-channel-connected CRN is more robust against channel interruptions caused by PU activities. Hence, bi-channel-connectivity is a highly desirable property to achieve in CRNs. In the meantime, it is also highly desirable to have other properties, i.e., preserving the minimum power paths, realizing conflict-free transmission, in addition to have bi-channel-connectivity.

In particular, we define a topology control problem with the dual objectives of minimizing the number of channels used by SUs and preserving the minimum power paths while satisfying the bi-channel-connectivity and conflict-free constraints. Here, we define the minimum power path as the path with the minimum path weight (i.e., the power consumption) among all the paths connecting two given nodes. Note that, both the constraint of bi-channel-connectivity and the constraint of conflict-free channel assignment will affect the number of required channels, however, their effects are contradictory. Take a k-vertex connected network as an example, in a sparse network with a small k, less channels are required to achieve conflict-free transmissions while more channels are needed to maintain bi-channel-connectivity, and the converse in a dense network with a large k. Therefore, in order to minimize the total number of required channels, we have to strike a balance between these two constraints by preparing a good topology for the channel assignment.

Among the two constraints, conflict-free property is relatively easy to achieve by using a graph coloring algorithm. Nonetheless, it is challenging to guarantee bi-channel-connectivity with a computationally efficient algorithm. An intuitive solution is to construct a k-vertex ($k \geq 2$) connected topology and assign channels to SUs under the constraint of bi-channel-connectivity, i.e., check whether the CRN is bi-channel-connected or not when assigning a channel to an SU. The complexity of this method is very high, and it is difficult to find an appropriate k to reduce the number of required channels. Moreover, this method is not easy to implement in a distributed way. On the other hand, we note that, when we assign channels using a graph coloring algorithm to ensure conflict-free transmissions, every color class[1] forms an independent set. When PUs reclaim a channel, a (sub-)set of SUs in the corresponding color class are removed. Therefore, a sufficient condition to maintain the connectivity of the CRN is that any independent set is not the vertex-cut set of the CRN. This is the main idea of this paper.

The main contribution of this paper is threefold.

- We prove the sufficient condition and the necessary condition for the existence of a feasible topology control algorithm to achieve bi-channel-connectivity and conflict-free transmission in CRNs. The NP-hardness of the topology control problem is proved as well.
- We reveal the sufficient condition to achieve bi-channel-connectivity in CRNs. Specifically, we only need to ensure that all the independent sets (i.e., group of SUs that transmit using the same channel) are not vertex-cut sets of the CRN. With this guideline, we propose a centralized topology control algorithm with joint power control and channel assignment. In particular, we construct a topology S such that, for each node u, the nodes in its conflict neighbor set are connected in $S \setminus u$. On that basis, a greedy coloring algorithm is used to assign channels to guarantee conflict-free transmissions and bi-channel-connectivity simultaneouly. The analysis and simulation show that the proposed algorithm has lower computational complexity and reduces the number of required channels.
- Based on the centralized topology control algorithm, we design a distributed algorithm where each SU only uses local information to construct the topology and select its channel. We observe that most of nodes only require its 2-hop neighborhood information in a 2-vertex connected network with randomly distributed nodes. The correctness and the message complexity of the distributed algorithm are given as well.

The remainder of this paper is organized as follows. In Section II, we review related work. Section III presents the network model and the problem definition. We elaborate the centralized and distributed topology control algorithm in Section IV and Section V, respectively. The Section VI demonstrates the simulation results. Finally, concluding remarks are given in Section VII.

II. Related Work

There exists considerable previous work on topology control algorithms to reduce interference in ad hoc networks [7]–[14]. We roughly classify these algorithms into three types. The first type of algorithms control topology by controlling the transmission power of each node where these nodes operate on a single shared channel, such as Low Interference Spanner Establisher (LISE) [7] and Min-Max Link/Node Interference with a Property (MMLIP or MMNIP) [8]. The second type of algorithms assign the channel to each link of the network to to achieve pre-designated connectivity and other performance targets, such as interference-aware topology control [9], Connected Low Interference Channel Assignment (CLICA) [10], and Resource-Minimized Channel Assignment (RMCA) [11]. They typically deal with a network with multiple channels. The third type of algorithms combine power control and channel assignment to achieve interference-free connections using the minimum number of channels, such as Local Random Sequential and δ-Improvement Algorithm (LOCAL RS-DIA) [13] and joint processing scheme of the topology control and

[1]Each color class is a subset of vertices assigned to the same color. Here, one color is equivalent to a channel.

channel assignment (JSTCCA) [14]. Furthermore, in order to provide fault tolerance to battery depletion and hardware failure, robust topology control algorithms have been proposed to guarantee k-vertex connectivity or k-edge connectivity in ad hoc networks [15]–[17].

However, these previous topology control algorithms consider neither the dynamic availability of the spectrum nor the protection of PUs, and hence can not be used for CRNs directly. Particularly, since any single channel interruption by PUs will force multiple SUs to vacate the channel, neither k-vertex connectivity nor k-edge connectivity can prevent the partition of CRNs unless the value of k is very large. To address these new challenges, several topology control algorithms have been proposed in [4]–[6], [18]. In [18], a distributed prediction-based cognitive topology control (PCTC) scheme was designed to construct an efficient and reliable topology based on the link prediction. However, PCTC can be applied only to single-channel CRNs. In multi-channel CRNs, different robust topology control algorithms have been proposed in [4]–[6] to mitigate the impacts of dynamic spectrum availability. [4], [5] designed channel assignment algorithms to maintain network connectivity when PU appears and minimize the co-channel interference due to simultaneous transmissions. While the underlying topology is fixed in [4], [5] because of the constant transmission power of SUs, [6] proposed a centralized algorithm combining power control and channel assignment for CRNs. The objective of [6] is to minimize the number of required channels while maintaining network connectivity and conflict-free property. However, the algorithm remains inadequate for minimizing the number of required channels and reducing the complexity. Moreover, it is difficult to extend this algorithm to distributed implementation. To this end, we propose a low-complexity topology control algorithm which jointly designs power control and channel assignment. Not only can the derived topology preserve connectivity upon PU appearance, but the topology control algorithm can also be implemented distributedly.

III. Network Model and Problem Definition

In this section, we first describe the network model and then define the topology control problem.

A. Network Model

We consider a cognitive radio network consisting of n SUs, as shown in Fig. 1, where each SU has a unique identifier and all of them can access the same set of available channels. Particularly, each SU is equipped with a radio which is able to receive on all the channels simultaneously but only allowed to transmit on one assigned channel. Due to the half-duplex constraint, one SU cannot transmit and receive on the same channel at the same time. The transmission power of an SU, P_u, can be adjusted in a continuous way but is limited by a maximum value P_{max}, i.e., $0 \leq P_u \leq P_{max}$. We consider additive white Gaussian noise channels and assume that a packet can be correctly received if the received signal strength exceeds the receiver's sensitivity β. Therefore, SU v is said to be in the transmission range of SU u if $P_u d_{u,v}^{-\alpha} \geq \beta$, where $d_{u,v}$ is the Euclidean distance between SU u and SU v

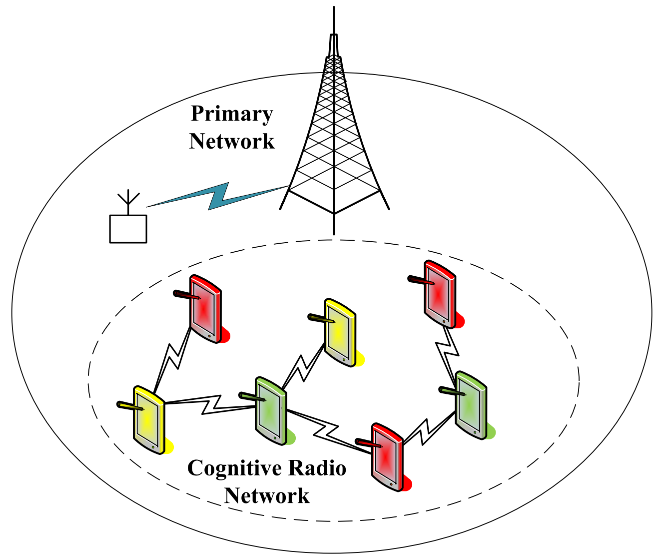

Fig. 1. Illustration of a cognitive radio network with $n = 7$, where the SUs with the same color transmit on the same channel.

and α is the path loss exponent. Accordingly, the maximum transmission range of an SU is $R_{max} = (\frac{P_{max}}{\beta})^{\frac{1}{\alpha}}$.

We model the topology of the CRN as a directed graph $G = (V(G), E(G))$, where $V(G)$ is the set of SUs in the network and $E(G)$ is the set of directed edges representing the wireless links. Note that the terms "SU" and "node" are used interchangeably hereafter. A directed edge (u, v) exists if v is in the transmission range of u and hence v is defined as a neighbor of u. Such neighbor relation is denoted by $u \rightarrow v$. Each directed edge (u, v) in $E(G)$ is assigned a weight $\overrightarrow{w}(u, v)$, which is the minimum transmission power for node u to reach node v. Specifically, $\overrightarrow{w}(u, v) = \beta d_{u,v}^{\alpha}$. It is easy to see that $\overrightarrow{w}(u, v) = \overrightarrow{w}(v, u)$. Moreover, there exists bi-directed link between u and v (denote as $u \leftrightarrow v$) if and only if both u and v are in the transmission range of each other, i.e., $u \rightarrow v$ and $u \leftarrow v$. Note that, when all the SUs transmit at the maximum power, the topology of the CRN is an undirected graph, referred to as maximum power topology and denoted by G_{max}.

The primary network has C license channels and can influence all the SUs in the cognitive radio network.[2] We assume that the PU occupies only one channel at one time and stays on that channel for a random duration. The occupancy probability of each channel is denoted by $p_i, i \in \{1, 2, \ldots, C\}$. Once the PU reclaims a channel, the SUs that transmit on the same channel should vacate the channel and the outgoing links from these SUs will be removed, possibly resulting in a network partition. Note that the communication is available between two SUs if their is a bi-directed link connecting them, since the messages can be sent and acknowledged on the forward and reverse links, respectively. As a result, G is said to be connected if and only if any two SUs are connected by either a bi-directed link or a bi-directed path $(u = q_0, q_1, \ldots, q_{m-1}, q_m = v)$ such that $q_i \in V(G), i = 0, 1, \ldots, m$ and $q_j \leftrightarrow q_{j+1} \in E(G), j = 0, 1, \ldots, m-1$.

[2]The scenario where one primary network only influences part of SUs is beyond the scope of this paper and will be studied in the future work.

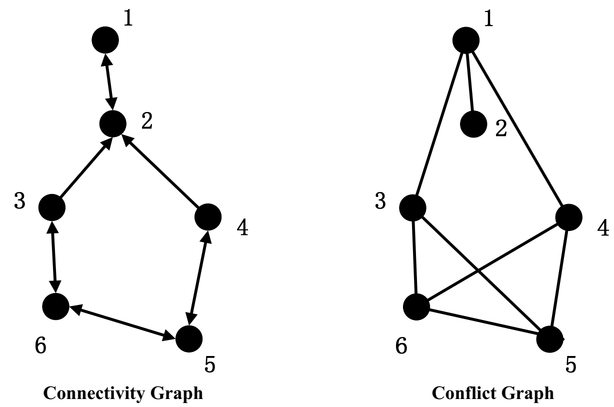

Fig. 2. Example of a connectivity graph and the corresponding distance-2 conflict graph, where the number associated with a node represents the node ID.

Further, G is *bi-channel connected* if the remaining network is connected when any channel is reclaimed by PU and becomes unavailable to SUs.

Besides, in CRNs, SUs that transmit on the same channel can potentially interfere with one another. We model the interference by transforming the connectivity graph G to a distance-2 conflict graph $U_G = (V(U_G), E(U_G))$, where $V(U_G) = V(G)$ and $E(U_G)$ is the set of undirected edges representing potential interference between two SUs [19]. Specifically, an undirected edge is placed between SU u and SU v in U_G if any one of the following two conditions is satisfied: i) u and v are connected by a bi-directed link; ii) u and v are two-hop apart and there exists an SU z such that u and z are bi-connected and z is a neighbor of v, i.e., $u \leftrightarrow z, v \rightarrow z$, or v and z are bi-connected and z is a neighbor of u, i.e., $v \leftrightarrow z, u \rightarrow z$.[3] An example of a connectivity graph G and the corresponding distance-2 conflict graph U_G is shown in Fig. 2. According to the conflict graph, the interference can be avoided by assigning different channels to conflicting SUs. Particularly, for a channel assignment, the network is said to be conflict-free if any two conflicting SUs (i.e.,$(u, v) \in E(U_G)$) have been assigned different channels.

B. Problem Definition

Circumventing the network partition upon PUs appearance and avoiding the interference among SUs are two major concerns during the topology control process. We adjust the transmission power and assign different channels to SUs to satisfy these two constraints with the aim of preserving the minimum power paths and minimizing the number of required channels. As such, SUs can prolong the network lifetime and access fewer channels with small occupancy probabilities, thereby alleviating the impact of dynamic channel availability. Specifically, the topology control problem can be defined as follows.

[3]Here, we include two cases by stating z is a neighbor of v. The first case is that z is a neighbor of v but v is not a neighbor of z, i.e., $v \rightarrow z$. The second case is that z is a neighbor of v and v is a neighbor of z, i.e., $v \leftrightarrow z$. Similar explanation is applied to the statement that z is a neighbor of u.

Definition 1 (Topology Control Problem)**.** Given a cognitive radio network G and a set of C available channels, the topology control problem is to construct a subgraph of G preserving the minimum power paths and seek a channel assignment with the minimum number of required channels such that the induced topology is bi-channel connected and conflict-free.

Before we examine the solvability of the above problem, we first give the sufficient condition and the necessary condition for the feasibility of the topology control problem in the following two theorems.

Theorem 1. *If G_{max} is k-vertex connected*[4] *with $k \geq 2$ and the number of licensed channels is no less than the number of SUs, i.e., $C \geq N$, then there exists a topology control algorithm that can achieve bi-channel-connectivity and conflict-free property.*

Proof: Given any at least 2-vertex connected G_{max} and $C \geq N$, one such topology control algorithm is to assign each node with a distinct channel. As a result, there is no conflict between any pair of nodes. When any assigned channel is reclaimed by PUs, only one SU is removed and hence the CRN remains connected, thereby achieving bi-channel-connectivity. ■

Theorem 1 presents the sufficient feasibility condition for the topology control problem. Note that for most of k-vertex connected G_{max} with $k \geq 2$, there exists a feasible topology control algorithm that requires less than N number of channels, and hence C can be less than N. However, in the worst case, i.e., G_{max} is a ring topology consisting of N nodes, it needs exact N number of channels to achieve bi-channel-connectivity and conflict-free property.

Theorem 2. *If there exists a topology control algorithm that achieves bi-channel-connectivity and conflict-free property, then G_{max} is at least 2-vertex connected and $C \geq 3$.*

Proof: We first assume that G_{max} is k-vertex connected with $k \leq 1$. If G_{max} is disconnected ($k = 0$), it is obvious that the topology control problem is infeasible. Then we consider the case that G_{max} is 1-vertex connected. Given a topology control algorithm that achieves bi-channel-connectivity and conflict-free property, there is at least one SU transmitting on each assigned channel. Therefore, when any assigned channel is reclaimed by the PU, at least one SU will vacate the channel and be removed from the derived topology, possibly resulting a network partition. Since the topology control algorithm ensures bi-channel-connectivity, the derived topology remains connected when PUs appears. This contradicts the assumption that G_{max} is 1-vertex connected. Therefore, no feasible topology control algorithm exists when G_{max} is 1-vertex connected.

Then we consider that G_{max} is k-vertex connected with $k \geq 2$. The above analysis also indicates that a topology derived by a feasible topology control algorithm is at least 2-vertex connected. To achieve conflict-free property, we assign channels according to the distance-2 conflict graph. Since the

[4]A graph is said to be k-vertex connected if the removal of any $(k-1)$ nodes leaves the graph connected.

derived topology is at least 2-vertex connected, the degree of each node is at least 2. Therefore, the clique number of the distance-2 conflict graph is at least 3. Since the number of channel required to achieve conflict-free property is lower bounded by the clique number of the distance-2 conflict graph, the number of licensed channels C is no less than 3, otherwise, there is no feasible topology control algorithm that can achieve conflict-free property. ■

Theorem 2 gives the necessary feasibility condition for the topology control problem. Hereafter, we assume that the maximum power topology of the CRN, G_{max}, is 2-vertex connected and $C \geq 3$. Now we have the following theorem concering the NP-hardness of the topology control problem.

Theorem 3. *The topology control problem is NP-hard.*

Proof: The proof is based on the reduction from graph K-colorability problem which is known to be NP-complete for $K \geq 3$. The graph K-colorability problem is to decide whether a graph G is K-colorable, i.e., is there a function $f : V(G) \rightarrow [1, 2, \ldots K]$ such that $f(u) \neq f(v)$ whenever $(u, v) \in E(G)$.

According to Definition 1, one of the sub-problems of the topology control problem is the channel assignment problem. Given a 2-vertex connected graph G and $C (C \geq 3)$ channels, the number of required channels is at least 3 according to the proof in Theorem 2. This makes the channel assignment problem equivalent to graph K-colorability problem with $K \geq 3$. Therefore, the topology control problem is NP-hard. ■

IV. Centralized Topology Control Algorithm

In this section, we first introduce a simple example so that readers can grasp the design philosophy before delving into the full details of algorithm. Next, the centralized topology control algorithm is proposed and elaborated, followed by a correctness proof and a complexity analysis of the proposed algorithm.

A. Design Philosophy

To solve the defined topology control problem, an existing approach is to construct a k-vertex connected graph with $k \geq 2$ and assign different channels to SUs using a graph coloring algorithm. During the channel assignment, every time when assigning a channel c to an SU u, a connectivity test has to be performed on the entire network to check whether the CRN remains connected if channel c is reclaimed. Since the connectivity test is frequently perform to check the connectivity of the entire network, the computational complexity is very high and it is not easy to implement this test in distributed systems.

Another major drawback is that this simple approach can not effectively reduce the number of required channels due to the difficulty of determining an appropriate k. In fact, there is a tradeoff between the bi-channel-connectivity constraint and the conflict-free constraint in terms of the number of required channels. To illustrate this point, we show an example in Fig. 3. Here, we use a simplified conflict definition, i.e., one node conflicts with its all 1-hop and 2-hop neighbors, for ease of understanding, and hence, a conflict graph can be

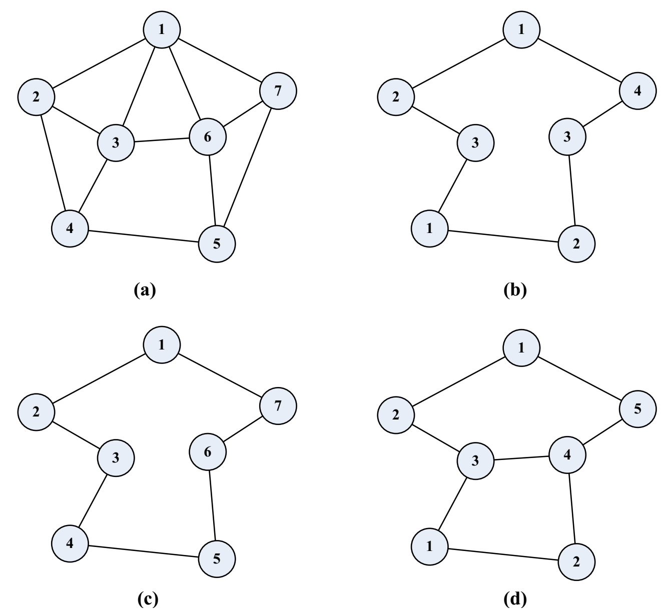

Fig. 3. Illustration of different connectivity graphs, where the number associated with a node indicates the channel assigned to this node.

generated from each connectivity graph in Fig. 3 according to this simplified definition.

Fig. 3(a) illustrates a dense topology with $k = 3$, where any pair of nodes conflict with each other. In order to avoid the interference among SUs, we have to assign each SU with a distinct channel, as shown in Fig. 3(a). Since this topology is 3-vertex connected, the bi-channel-connectivity is also guaranteed without requiring additional channels. Therefore, 7 channels are utilized in this case. To reduce the number of required channels due to the conflict-free constraint, we construct a sparse topology with $k = 2$ as shown in Fig. 3(b), where only 4 channels are needed to avoid interference among SUs. However, this topology is not bi-channel connected. Particularly, when any of the first three channels is reclaimed, a network partition will occur. This is because each of the first three color class constitutes a vertex-cut set. In order to avoid a network partition, we have to assign more than one colors to a vertex-cut set. Note that, on the other hand, a color class is an independent set in the conflict graph. Since any independent set with at least two nodes in the conflict graph is a vertex-cut set in this topology, no two nodes can share a same channel. Therefore, we need another 3 channels to guarantee bi-channel-connectivity, resulting in 7 channels in total as shown in Fig. 3(c). This example indicates that, although we can reduce the number of channel required to ensure the conflict-free constraint by lowering k, the number of channels required by the bi-channel-connectivity constraint increases simultaneously.

To strike a balance between these two constraints, we construct a topology by reducing the number of smallest vertex-cut sets as shown in Fig. 3(d), where any independent set in the corresponding conflict graph is not a vertex-cut set in the connectivity graph. Therefore, we are able to decrease the number of channels required by the conflict-free constraint to the most extent without using any additional channel to fulfill the bi-channel-connectivity requirement. In particular, we can achieve conflict-free property with 5 channels in this topology. Meanwhile, bi-channel-connectivity is guaranteed without any additional channel since the number of channels assigned to the nodes in a vertex-cut set is at least 2. Thus, the total number of required channels is 5, which is lower than previous examples. Moreover, since the realization of conflict-free transmission in this topology also ensure the bi-channel-connectivity, the connectivity test is dispensed with in the channel assignment, thereby reducing the computational complexity.

As illustrated in the above examples, our design philosophy is to tailor the network topology to meet the need of channel assignment. In particular, we construct a topology such that any independent set in the corresponding conflict graph does not partition the network. As such, any conflict-free channel assignment can guarantee the bi-channel-connectivity constraint simultaneously with low complexity and high spectrum efficiency.

B. Algorithm Description

The centralized topology control algorithm consists of two phases, namely, topology construction and channel assignment. In the first phase, based on the maximum power topology, we properly assign transmission power to SUs so as to prepare a good topology for the channel assignment phase. In particular, we construct a subgraph S which is able to preserve minimum power paths and is robust to the removal of any independent set in the corresponding conflict graph, i.e., any independent set in the conflict graph is not a vertex-cut set of S. Based on this tailored topology, in channel assignment phase, we dispense with the connectivity test which is used for checking whether the bi-channel-connectivity constraint is guaranteed or not. Instead, only a simple graph coloring algorithm is needed to sequentially assign a channel to each node so as to build a conflict-free and bi-channel connected topology. The details of both phases are described in the sequel and summarized in Algorithm 1.

Algorithm 1 Centralized topology control algorithm

Input:
The maximum power topology G_{max};
Output:
The induced topology S and the channel assignment $\mathcal{A}$;

Phase I: Topology construction
$V(S) \Leftarrow V(G_{max})$, $E(S) \Leftarrow \emptyset$;
Find the shortest paths between every pair of nodes u, v in G_{max};
Construct the subgraph S by including all the undirected edges in the obtained shortest paths;
Sort all nodes in order of non-descending degree in G_{max};
for each node $u \in V(S)$ in the order **do**
  Find the conflict neighbor set of node u, CN_u, in the distance-2 conflict graph U_S;
  $V(CS_u) \Leftarrow CN_u$,
  $E(CS_u) \Leftarrow \{(x, y) \mid x, y \in CN_u, (x, y) \in E(G_{max})\}$;
  if CS_u is connected **then**
    Call Algorithm 2 to construct a local spanning subgraph T_u of CS_u;
  else
    Construct a Steiner tree T_u in $G_{max} \setminus u$;
  end if
  $E(S) \Leftarrow E(S) \cup E(T_u)$, $LCN_u \Leftarrow V(T_u)$;
end for
$P_u \Leftarrow \max\{\beta d_{u,v}^{\alpha} \mid (u, v) \in S, v \in V(S)\}$ for $u \in V(S)$;

Phase II: Channel assignment
Generate the distance-2 conflict graph U_S and modify it by adding the edges between an SU u and its logical conflict neighbors in LCN_u;
Sort all nodes in order of non-ascending conflict degree in U_S;
Sort all channels in order of non-descending occupancy probability;
for each node u in the order **do**
  Find the conflict neighbor set of node u, $\overline{CN}_u$, in the modified conflict graph;
  for each channel c in the order **do**
    if c is not assigned to $\overline{CN}_u$ **then**
      Assign channel c to node u;
      break;
    end if
  end for
end for

In the topology construction phase, we first build a spanning subgraph $S = (V(S), E(S))$ which preserves the minimum power paths in the maximum power topology. Particularly, we find the minimum power paths between every pair of nodes u, v in G_{max} using Floyd-Warshall algorithm and add all the edges in the obtained paths to $E(S)$. Then, we sort all nodes in non-descending order of degree in G_{max}. According to this order, we find the conflict neighbor set CN_u of u which consists of the nodes that are directly connected with node u in the distance-2 conflict graph U_S. Based on CN_u, we obtain a local conflict neighbor subgraph of node u, $CS_u = (V(CS_u), E(CS_u))$, where $V(CS_u) = CN_u$ and $E(CS_u)$ is the set of edges between the conflict neighbors of node u in $E(G_{max})$, i.e., $E(CS_u) \Leftarrow \{(x, y) \mid x, y \in CN_u, (x, y) \in E(G_{max})\}$. Note that $u \notin V(CS_u)$.

Then, we check whether CS_u is connected or not. If CS_u is connected, we construct a local spanning subgraph T_u over CS_u using the algorithm described in Algorithm 2, where the distance weight is Euclidean distance.[5] Otherwise, we build a Steiner tree T_u in $G_{max} \setminus u$. Note that Steiner tree problem is to determine a minimum cost subgraph spanning a set of specified vertexes (i.e., basic vertexes) [20]. In order to achieve this minimum cost subgraph, additional vertexes (i.e., Steiner vertexes) may be included. With the objective of guarantee bi-channel-connectivity, we aim at connecting all the conflict neighbors of node u when u vacates the spectrum. Therefore,

[5]Note that the construction of a local spanning subgraph is similar to Kruskal's algorithm which is used to build the minimum spanning tree (MST) of a given graph. In particular, when $E(T_u) = \emptyset$ in line 1 in Algorithm 2, the procedure of line 2 to 8 is exactly Kruskal's algorithm.

Algorithm 2 Local spanning subgraph

```
V(T_u) ⇐ V(S_u),
E(T_u) ⇐ {(x, y) | x, y ∈ CN_u, (x, y) ∈ E(S)};
Ē_u ⇐ {(x, y) | (x, y) ∈ E(CS_u), (x, y) ∉ E(S)};
Sort all edges in Ē_u in non-descending order of distance weight;
for each edge (x, y) in the order do
    if x is not connected to y in T_u then
        E(T_u) ⇐ E(T_u) ∪ {(x, y)}
    end if
end for
```

Steiner tree is utilized to connect the nodes in CS_u which itself is not connected by including additional nodes. In our problem, the Euclidean distance is the cost and the nodes in CN_u are basic vertices, while the Steiner vertices is yet to be determined. We use the TMR algorithm in [21] to construct the Steiner tree. In particular, we first find the Steiner vertices in $V(G_{max})\setminus(u \cup V(CS_u))$ such that the subgraph consisting of the basic vertices and Steiner vertices is connected. Then, we find the MST of this subgraph and prune it to ensure all leaves are basic vertices.

By constructing a local spanning subgraph or a Steiner tree over CN_u, the conflict neighbors of node u are able to connect to one another with the minimum transmission power when node u vacates the channel to avoid interference to PU. Afterward, all bi-directed edges in $E(T_u)$ are supplemented to $E(S)$ and all nodes in $V(T_u)$ are recorded in LCN_u as the logical conflict neighbors of u.[6] Note that LCN_u may contain the nodes that are more than 2 hops away from u on S. The procedure (Lines 6-13 in Algorithm 1) is repeated until all nodes in $V(S)$ have been traversed consecutively. Finally, each node selects all nodes that are bi-directionally connected one hop away on the spanning subgraph S as its logical neighbors and sets its transmission power to the level such that it can reach the furthest logical neighbor in S.

In the channel assignment phase, we first generate the distance-2 conflict graph U_S according to the derived topology S and then modify U_S by adding the edges between an SU u and its logical conflict neighbors in LCN_u to U_S. The conflict neighbor set of node u in this modified conflict graph is denoted by $\overline{CN}_u$. Next, we calculate the conflict degree of each node, i.e., $|\overline{CN}_u|$, and sort all nodes in non-ascending order of conflict degree. All license channels are sorted in order of non-descending occupancy probability as well. Now, we begin to assign distinct channels to SUs. For the node with the largest conflict degree, we assign it the channel with the lowest occupancy probability. Then, the i-th ($2 \leq i \leq n$) node in the order will be assigned the channel with the lowest occupancy probability that is not yet used in set $\overline{CN}_u$. Using this sequential channel assignment, we can obtain a conflict-free topology. The bi-channel-connectivity constraint is also guaranteed without the need of executing the high-complexity connectivity test. Finally, the derived topology S comprises all nodes in $V(G_{max})$ with assigned channels and their individually determined logical neighbor relations.

C. Theoretical Analysis

We first prove the correctness of the proposed centralized topology control algorithm. It is easy to see that the conflict-free transmissions are guaranteed in the final topology, since only the SUs that do not conflict with each other can be assigned with the same channel (Lines 21-26 in Algorithm 1). Therefore, in the following theorem, we only prove that the resultant topology is bi-channel connected.

Theorem 4. *Given a 2-vertex connected CRN G_{max}, the resulting topology S by using our proposed algorithm is bi-channel-connected.*

Proof: It is easy to see that the CRN stays connected if the PU occupy the channel which has not been assigned to any SU. Therefore, we only consider the case in which the channel reclaimed by the PU is assigned to at least one SU. Let $\overline{V}$ denote the set of nodes that vacate the channel reclaimed by the PU, and let $S - \overline{V}$ be the resulting graph by removing the vertex set $\overline{V}$ and the edges incident to nodes in $\overline{V}$. If any pair of nodes in $S - \overline{V}$ is connected, the topology S is bi-channel connected. Therefore, we prove the bi-channel-connectivity of S by showing that any pair of active nodes in $S - \overline{V}$, i.e., nodes that transmit on different channels from the PU, is connected when one channel is reclaimed by the PU.

As we use Floyd-Warshall algorithm in the topology construction phase to find the minimum power paths between every pair of nodes in G_{max}, there exists a bi-directed link or a bi-directed path between any two nodes in S. Consider any two nodes $u, v \in V(S) - \overline{V}$. It is easy to see that nodes u, v are connected in $S - \overline{V}$ if u and v are connected by a bi-directed link in S. Now, we consider the case that u and v are connected by a bi-directed path, $P_a(u, v) = (u = q_0, q_1, \ldots, q_{m-1}, q_m = v)$, in S. We denote the set of nodes that belong to the path $P_a(u, v)$ and meanwhile are influenced by the PU as $\overline{V}_{P_a(u,v)}$, i.e., $\overline{V}_{P_a(u,v)} = \{q_i \mid q_i \in P_a(u, v), q_i \in \overline{V}, i = 1, \ldots, n\}$. For any node $q_i \in \overline{V}_{P_a(u,v)}$, its neighbor nodes q_{i-1} and q_{i+1} in the path are assigned different channels from q_i and do not belong to the set $\overline{V}_{P_a(u,v)}$, since they conflict with node q_i, i.e., $q_{i-1}, q_{i+1} \in CN_{q_i}$. According to the topology construction phase in Algorithm 1, there exists a bi-directed path $P_a(q_{i-1}, q_{i+1})$ from q_{i-1} to q_{i+1} that is internally disjoint with the path (q_{i-1}, q_i, q_{i+1}), since G_{max} is 2-vertex connected. Moreover, all nodes in the path $P_a(q_{i-1}, q_{i+1})$ belong to $\overline{CN}_{q_i}$ and are assigned distinct channels from q_i. Therefore, upon any single channel interruption by PU requests, there is still a bi-directed path from u to v and all nodes in the path remain active in $S - \overline{V}$, if u and v are connected by a bi-directed path in S. As a result, no matter whether u and v are connected by a bi-directed link or a bi-directed path, nodes u, v are connected in $S - \overline{V}$ when one channel is reclaimed by the PU. In other words, the resulting topology S is bi-channel connected. ■

Remark 1. Theorem 4 also indicates that the derived topology

[6] We would like to explain the relationship between node u and the Steiner nodes added when CN_u is not connected. Since G_{max} is assumed to be 2-vertex connected, u and the corresponding Steiner nodes actually constitute a vertex-cut set. By recoding these Steiner nodes as the logical conflict neighbors of u, we assign distinct channel to u from these Steiner nodes. As a result, more than one channels are assigned to this vertex-cut set, leading to a bi-channel-connected topology.

is 2-vertex connected. Hence, the degree of each node is at least 2 and the clique number of the distance-2 conflict graph of S is at least 3. Since the vertexes of a clique require distinct colors, the number of required channel χ is no less than 3. On the other hand, let Λ denote the maximum number of logical conflict neighbors in S. Then, the greedy coloring algorithm can not use more than $\Lambda + 1$ colors. Therefore, the number of required channel χ is no more than $\Lambda + 1$.

Then, the complexity of the proposed topology control algorithm is given in the following theorem.

Theorem 5. *The computational complexity of the proposed algorithm is $O(m|V|^3 + |V|\,|E|\log|V|)$, where $|V|$ and $|E|$ are the number of nodes and links in G_{max}, respectively, and m is the maximum conflict degree in the distance-2 conflict graph transformed from G_{max} .*

Proof: In the topology construction phase, the complexity mainly lies in lines 3, 8, 9 and 11 in Algorithm 1. The operation in line 3 can be realized by using Floyd-Warshall algorithm, costing $O(|V|^3)$. The procedures of lines 8 and 9 can be operated together through constructing a local spanning subgraph using Algorithm 2 with complexity of $O(|E|\log|V|)$. Note that Steiner tree problem is NP-hard. We use the heuristic algorithm TMR in [21] to construct a Steiner tree for an SU u, the complexity of which is $O(|CN_u||V|^2)$. It is worth noting that $|CN_u|$ is no larger than the maximum conflict degree m (i.e., the number of neighbors) in the distance-2 conflict graph transformed from G_{max}. Therefore, in the worst case, the complexity of building a Steiner tree is $O(m\,|V|^2)$. Since every node has to execute the procedure (Lines 6-13 in Algorithm 1), the complexity of this part (Lines 5-14 in Algorithm 1) is $O(|V|(|E|\log|V| + m\,|V|^2))$. Therefore, the total complexity of the first phase is $O(m|V|^3 + |V|\,|E|\log|V|)$. In the channel assignment phase, the complexity of the typical greedy coloring algorithm is $O(|V|^2)$, which is lower than that of the first phase. Therefore, the total complexity of the proposed centralized topology control algorithm is $O(m|V|^3 + |V|\,|E|\log|V|)$. ∎

V. Distributed Topology Control Algorithm

In this section, we propose the distributed version of our centralized topology control algorithm and then present the theoretical analysis of the distributed algorithm. The procedure of the distributed algorithm consists of four phases, namely, information collection, topology construction, channel assignment, and transmission power control. Here, the description of the distributed topology control algorithm assumes the point of view of an SU u.

1) Information Collection: At first, each SU u exchanges information to discover its two-hop neighbors denoted by N_u^2. Particularly, in the first period D_1, each SU randomly chooses a backoff time within D_1 to broadcast a HELLO message on a common control channel with its maximum transmission power.[7] It is worth noting that, in the network model, an SU is allowed to transmit messages over one assigned channel but able to receive messages over all the channels simultaneously. Therefore, when there is no common control channel, each SU

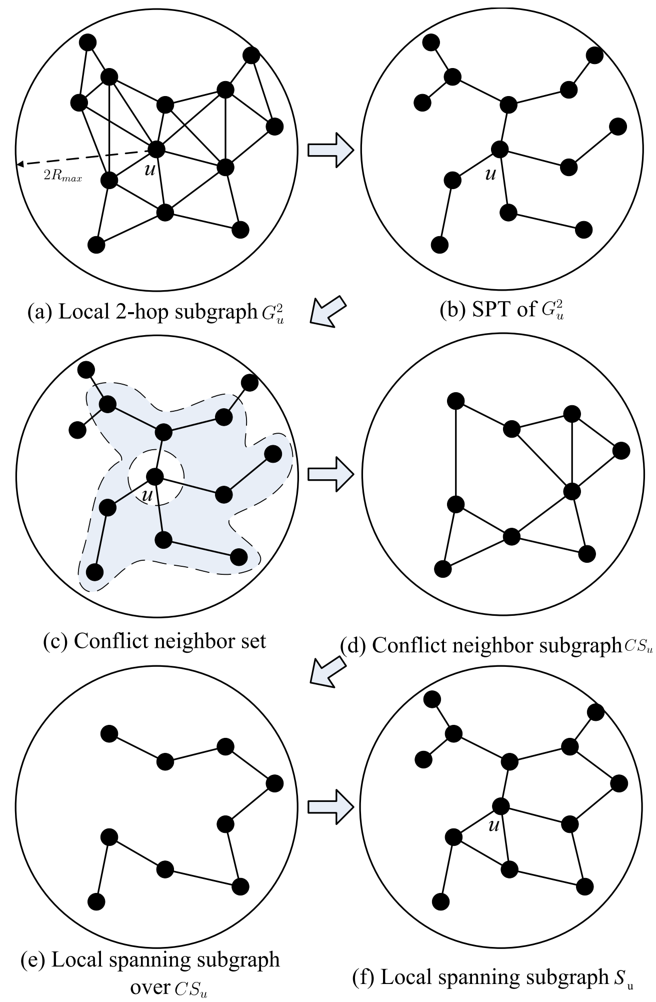

Fig. 4. An example of topology construction.

randomly chooses an idle channel to transmit a Hello message. Meanwhile, it can receive Hello messages from its neighbor nodes on different channels. The information contained in a HELLO message should at least include the node id and the location information. Upon receiving the hello messages, each SU can obtain its 1-hop neighbors at the end of D_1. Then, similar to the first period, each SU randomly selects a backoff time within the second period D_2 to broadcast its neighbor list, including the id and location of its 1-hop neighbors, with its maximum transmission power. After exchanging the neighbor list with its one-hop neighbors, each SU is able to gain the knowledge of its 2-hop neighbors at the end of D_2. With the location information, it is fairly easy for each node to gather the knowledge of existing edges among its 2-hop neighbors. It is worth noting that, if the location information is not available, the knowledge of the existing edges can still be obtained by using extra rounds of information dissemination at the expense of more communication and computation overhead [23]. Using these edge information, each SU u obtains its local 2-hop subgraph $G_u^2 = (V(G_u^2), E(G_u^2))$, where $V(G_u^2) = N_u^2 \cup \{u\}$ and $E(G_u^2) = \{(x, y) \mid x, y \in V(G_u^2), (x, y) \in E(G_{max})\}$.

2) Topology Construction: According to the local 2-hop subgraph G_u^2, each node u independently builds its local 2-hop spanning subgraph $S_u = (V(S_u),\ E(S_u))$. In particular, node

[7]We use random retransmission to resolve possible collisions [22].

u first calculates the minimum power paths between itself and its 2-hop neighbors in N_u^2 using single-source-shortest-path algorithm, i.e., Dijkstra's algorithm, and adds all the edges in the obtained paths to $E(S_u)$. Then, node u finds the conflict neighbor set CN_u of u which consists of the nodes that are directly connected with node u in the distance-2 conflict graph transformed from S_u. Based on CN_u, node u constructs a conflict neighbor subgraph, $CS_u = (V(CS_u), E(CS_u))$, where $V(CS_u) = CN_u$ and $E(CS_u)$ is the set of edges between the conflict neighbors of node u in $E(G_u^2)$, i.e., $E(CS_u) \Leftarrow \{(x, y) \mid x, y \in CN_u, (x, y) \in E(G_u^2)\}$. Afterwards, node u checks whether CS_u is connected or not. If CS_u is connected, node u constructs a local spanning subgraph over CS_u. Otherwise, node u constructs a Steiner Tree T_u in $G_u^2 \setminus u$. If failed, node u gains knowledge of its h-hop ($h \geq 3$) neighborhood and obtains the local h-hop subgraph G_u^h until a steiner tree can be successfully constructed in $G_u^h \setminus u$. Finally, all bi-directed edges in $E(T_u)$ are supplemented to $E(S_u)$ and all nodes in $V(T_u)$ are recorded in LCN_u as the logical conflict neighbors of u. An example of construction the topology is illustrated in Fig. 4.

Then, each node u broadcasts its logical conflict neighbor set LCN_u and the edge information in $E(S_u)$ to all the nodes in S_u by local flooding. Meanwhile, upon receiving the topology information broadcast by other nodes, node u updates its logical conflict neighbor set and local spanning subgraph S_u accordingly. Note that the resultant topology is over-connected under the distributed algorithm compared with that under the centralized algorithm, since each node independently constructs its local spanning subgraphs only based on G_u^2 without considering the topology constructed by other nodes.

3) Channel Assignment: During the local flooding in the topology construction phase, each node u can obtain knowledge of the conflict degree of other nodes in S_u. The node with larger conflict degree is given higher priority to request channels, since it interferes with more nodes. In other words, node u can execute channel assignment only when all other nodes with larger conflict degrees in S_u have finished their channel assignment. During the channel assignment phase, node u first obtains the channel usage information of its conflict neighbors. Then, it selects the channel with the lowest occupancy probability that is not yet used by its conflict neighbors.

4) Transmission Power Control: Each node u selects all nodes that are bi-directionally connected one hop away on the spanning subgraph S_u as its logical neighbors and sets its transmission power to the level such that it can reach the furthest logical neighbor in S_u.

Remark 2. In the topology construction phase, there may exist some nodes requiring the knowledge of its h-hop ($h \geq 3$) neighborhood because $G_u^2 \setminus u$ is not connected. In the worst case, i.e., G_{max} is a ring topology with N nodes, every node would need the knowledge of its h-hop ($h \geq 3$) neighborhood if N is large. However, we note that most of the nodes only require its 2-hop neighborhood information in a network with randomly distributed nodes. As shown in Fig. 5, we calculate the percentage of nodes that can successfully build the Steiner tree within $G_u^h \setminus u$ in a 1000×1000 m^2 region with $R_{max} = 300$ m. We can see that almost all the nodes only need the knowledge of 2-hop neighborhood to construct the Steiner tree when the number of SUs is large, i.e., $n \geq 130$. Moreover, we compare the probability that the network is 2-vertex connected and the probability that $G_u^2 \setminus u$ is connected in Fig. 6. It can be seen that $G_u^2 \setminus u$ is almost surely connected in a 2-vertex connected network with the connectivity probability being greater than 99%. This also indicates that why most of the nodes only need the knowledge of 2-hop neighborhood to construct the Steiner tree.

Now we prove the correctness and analyze the message complexity of the proposed distributed topology control algorithm in the following theorems.

Theorem 6. *Given a 2-vertex connected cognitive radio network G_{max}, the resulting topology S by using our distributed topology control algorithm is bi-channel-connected.*

Proof: The distributed topology control algorithm mainly differs from the centralized one in the topology construction order and the channel assignment order. In the distributed algorithm, each node executes the operations independently and does not rely on the results of others nodes. Since the correctness proof of the centralized algorithm does not depend on the order, the same procedure can be applied to demonstrate the correctness of the distributed algorithm. ■

To simplify the analysis of message complexity, we assume that there is no contention between any two messages. In addition, since most of the nodes only require its 2-hop neighborhood information, we analyze the message complexity in a scenario where the information exchanges are limited within 2-hop neighborhood.

Theorem 7. *The message complexity of DBCC is $O(|V|(4 + 2\triangle))$, where $|V|$ is the number of nodes and $\triangle$ is the maximum node degree in G_{max}.*

Proof: In the information collection phase, each node exchanges two HELLO packets to get knowledge of the local subgraph G_u^2, and hence the message complexity is $O(2|V|)$. In the topology construction phase, each node informs all the nodes in its constructed spanning subgraph S_u by local flooding. This needs node u and its 1-hop neighbors in G_u^2 to transmit the message, resulting in the message complexity of topology construction is $O(|V|(1 + \triangle))$. Similarly, in the channel assignment phase, a node broadcasts a message to all nodes in S_u after it has selected a channel, which costs $O(|V|(1 + \triangle))$. Therefore, the total packet complexity of DBCC is $O(|V|(4 + 2\triangle))$. ■

VI. Simulation Study

In this section, we present several simulation results to evaluate the performance of our centralized and distributed bi-channel-connected topology control algorithms, which are referred to CBCC and DBCC for short. Particularly, we illustrate the topology derived by the centralized algorithm, and compare the performance of CBCC and DBCC with that of the topology control algorithm only preserving conflict-free property (OCFP) and other two algorithms in [6], referred to

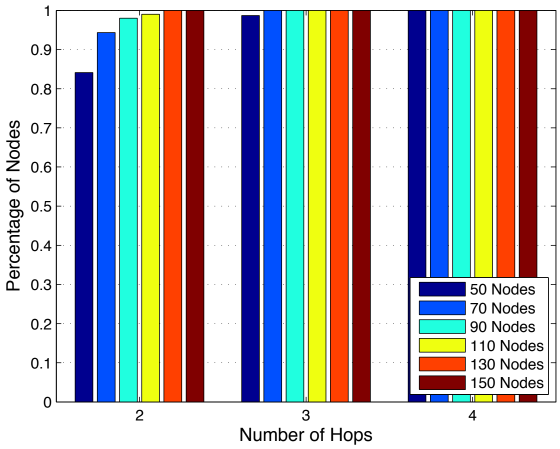

Fig. 5. Percentage of nodes that require the knowledge of no more than h-hop neighborhood when building the Steiner tree ($\alpha = 4$, $\beta = -80$ dBm, $R_{max} = 300$ m, 1000×1000 m^2 region).

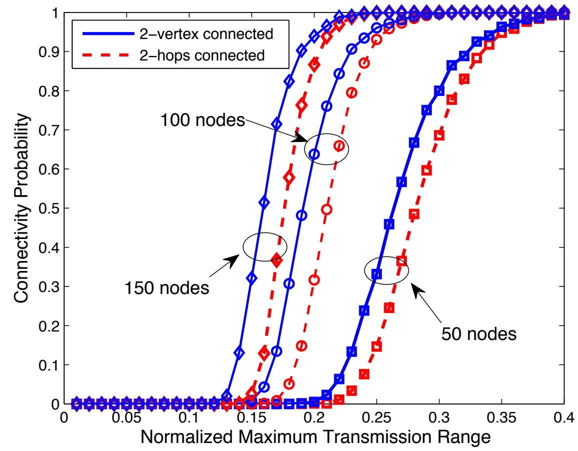

Fig. 6. Comparison between the probability that the network is 2-vertex connected (solid line) and the probability that $G_u^2 \setminus u$ is connected (dashed line).

as GBC, GBC+DC. In OCFP, we build a 2-vertex connected topology which preserves minimum power paths and then assign channels to achieve conflict-free transmissions without bi-channel-connectivity guarantee. Since OCFP considers only one constraint, it serves as a performance lower bound for our algorithms. On the other hand, the main idea of GBC is that it constructs a 2-vertex connected topology by using a pruning-based algorithm at first and then assigns a channel to each node sequentially under the condition that the topology is conflic-free and bi-channel-connected. Furthermore, GBC+DC is a modified version of GBC, which adds degree control at the topology construction stage. The performance of maximum power topology is also shown as a baseline.

At first, we generate a network with 20 SUs uniformly distributed in a 1000×1000 m^2 region for ease of illustration. We set the path loss exponent $\alpha = 4$ and the receiver's sensitivity $\beta = -80$ dBm. The maximum transmission power of each SU is 256 mW and the corresponding maximum transmission range is $R_{max} = 400$ m for all SUs. The maximum power topology and the topology derived by CBCC are illustrated in Fig. 7(a) and (b), respectively. It can be seen that the node degree is reduced substantially in the derived topology, resulting in less potential interference among nodes. This is beneficial to reduce the required number of channels. In particular, 8 channels are used to achieve bi-channel-connectivity and conflict-free property as shown in Fig. 7(b), where the number associated with a node represents the channel assigned to this node. Fig. 7(c) shows the conflict graph transformed from the derived topology in 7(b). Any two nodes that are adjacent to each other in the conflict graph are assigned different channels. This indicates that the derived topology is conflict-free. In addition, the resulting topologies with different channels being occupied by PU are exemplified in Fig. 7(d)-(f). We can see that, although some SUs vacate the channel for PU, the active SUs are still connected in the remaining network. In other words, the topology derived by our proposed algorithm can achieve bi-channel-connectivity.

Then, we compare the performance of CBCC, DBCC, OCFP, GBC, and GBC+DC with respect to several metrics via simulations. We vary the number of nodes from 80 to 150 in a 1000×1000 m^2 region. The values of other parameters, including R_{max}, α, β, and P_{max}, are the same as that in the previous simulation setup. The occupancy probabilities of PU are randomly generated in each trial while satisfying $\sum_{i=1}^{C} p_i = 1$. Resulting values are obtained by averaging over 400 simulation runs.

Fig. 8 plots the power spanner factor of different algorithms. The power spanner factor of the derived topology with respect to the maximum power topology is the maximum over all possible node pairs of the ratio between the weight of the minimum power path in the derived topology and in the maximum power topology. We denote the power spanner factor as ρ. Note that $\rho \geq 1$. In particular, $\rho = 1$ indicates the derived topology preserves the minimum power path in the maximum power topology. It can be seen from Fig. 8 that the topologies derived under CBCC, DBCC, and OCFP preserve the minimum power path.

Fig. 9 and 10 depict the average transmission range and the maximum transmission range of different algorithms, respectively. The transmission range of an SU is the length of a link between the SU and its farthest logical neighbor. Both Fig. 9 and 10 show that the average and the maximum transmission range of maximum power topology are independent of the node density. However, the average and the maximum transmission range of other algorithms decrease with the increasing of the number of nodes, since lower transmission power is required to maintain the network connectivity in the dense network. It can be seen from Fig. 9 that both CBCC and DBCC exhibit larger transmission range than OCFP due to bi-channel-connectivity constraint, but reduce more average transmission power than other algorithms, thereby lowering potential interference and energy consumption. Nonetheless, in Fig. 10, the maximum transmission range of CBCC and DBCC are larger than that of GBC and GBC+DC. This is because GBC and GBC+DC aim at minimizing the maximum transmission power. In contrast, our algorithms preserve the minimum power path, which is helpful to prolong network lifetime. We also note that the performance of DBCC is

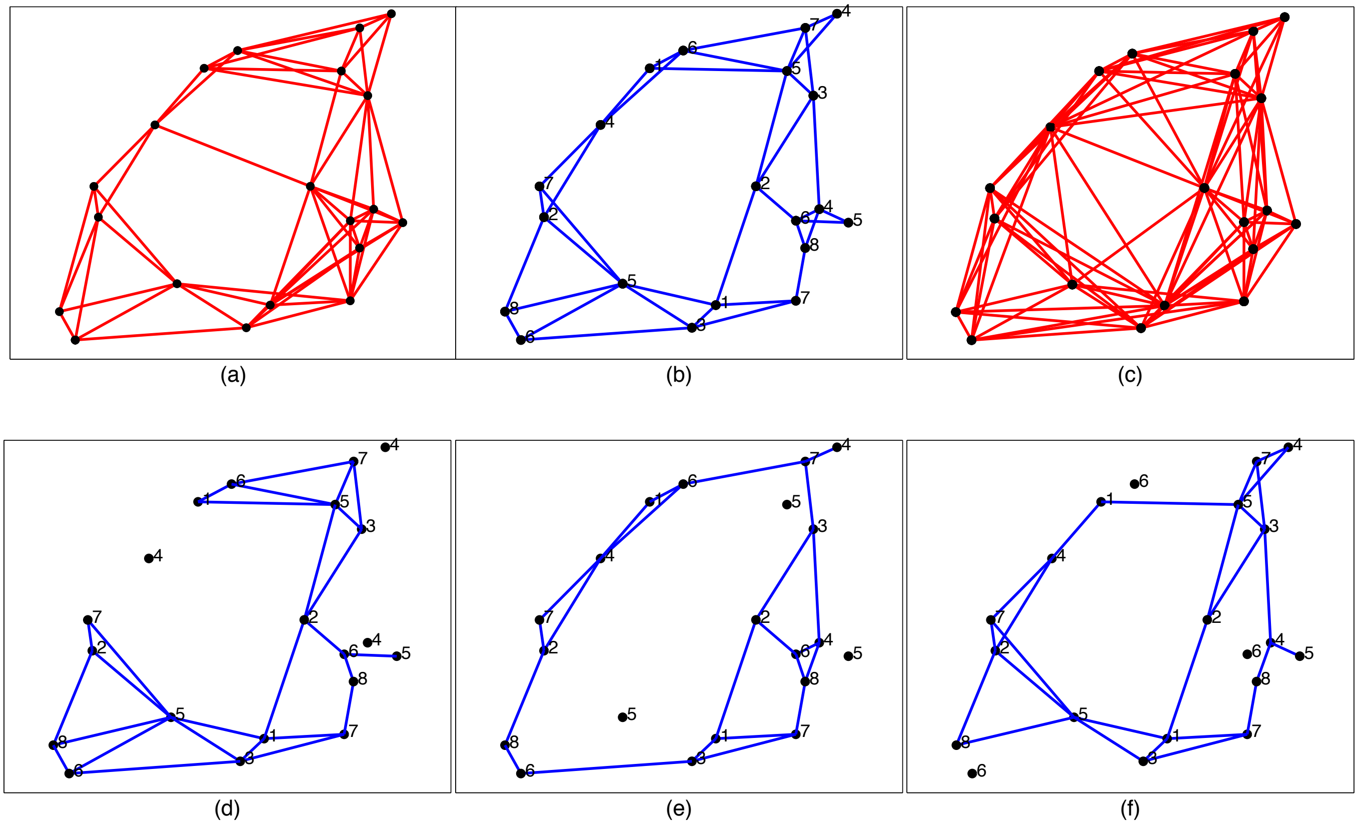

Fig. 7. (a) Maximum power topology. (b) The topology derived by CBCC (the number represents the assigned channel). (c) Conflict Graph transformed from the topology under CBCC. (d) The topology when PU occupies channel 4. (e) The topology when PU occupies channel 5. (f) The topology when PU occupies channel 6.

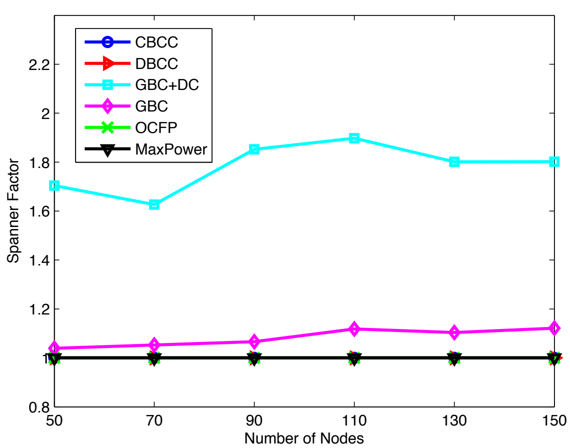

Fig. 8. Comparison of power spanner factor.

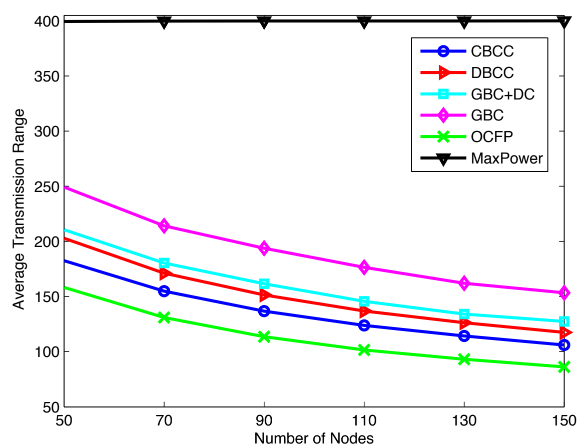

Fig. 9. Comparison of average transmission range.

inferior to CBSS because only local information is available in DBCC.

Fig. 11 and 12 illustrate the average and the maximum number of channels required to construct a conflict-free and bi-channel-connected network with different algorithms. The average number of the required channels of the maximum power topology increases almost linearly with the number of nodes, while that under other topology control algorithms are slightly increased. The reason is that the average and maximum transmission range of other topology control algorithms decrease as the number of nodes increases as shown in Fig. 9 and 10. Therefore, the potential interference slightly increases with the increasing of the number of nodes. It is shown that both the average and the maximum number of channels used in CBCC and DBCC are lower than that in GBC and GBC+DC. In particular, the reduction in the number

TABLE I

THE REDUCTION IN THE NUMBER OF REQUIRED CHANNELS OF OUR ALGORITHMS COMPARED WITH THAT OF GBC

Number of nodes		50	70	90	110	130	150
The average number of required channels	CBCC	48.3%	51.2%	51.8%	54.0%	53.9%	55.1%
	DBCC	38.9%	42.9%	43.7%	45.6%	45.6%	46.6%
The maximum number of required channels	CBCC	68.3%	72.7%	69.2%	70.5%	75.0%	75.5%
	DBCC	63.4%	65.9%	61.5%	63.6%	69.6%	67.9%

TABLE II

THE REDUCTION IN THE NUMBER OF REQUIRED CHANNELS OF OUR ALGORITHMS COMPARED WITH THAT OF GBC+DC

Number of nodes		50	70	90	110	130	150
The average number of required channels	CBCC	27.3%	30.5%	29.3%	31.4%	30.9%	32.7%
	DBCC	14.0%	18.8%	17.3%	18.8%	18.5%	19.9%
The maximum number of required channels	CBCC	64.9%	57.1%	53.8%	58.1%	53.3%	62.9%
	DBCC	59.5%	46.4%	42.3%	48.4%	43.3%	51.4%

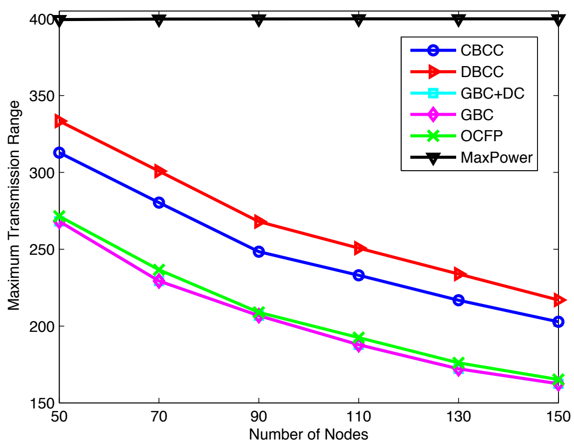

Fig. 10. Comparison of maximum transmission range.

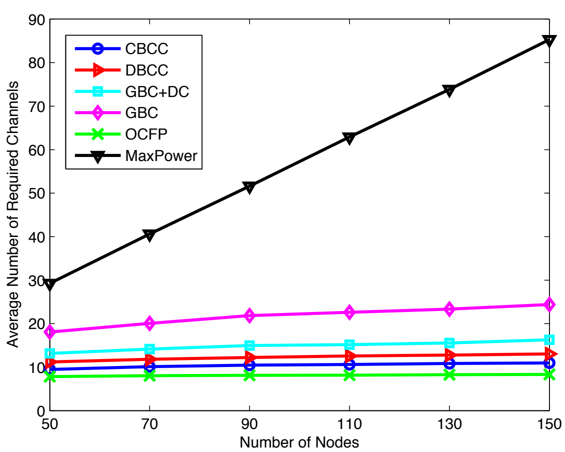

Fig. 11. Comparison of the average number of the required channels.

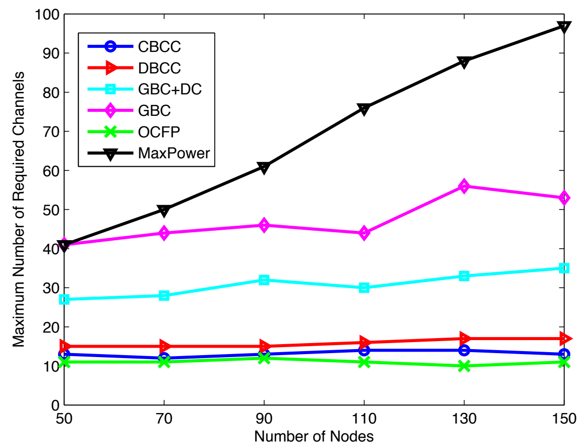

Fig. 12. Comparison of the maximum number of the required channels.

of required channels of our algorithms compared with that of the existing ones is shown in Tables I and II. This is because, 1) in the topology construction phase, our proposed algorithms have lower average transmission range and less potential interference; 2) in the channel assignment phase, only the constraint on conflict-free is considered in our proposed algorithms with the bi-channel-connectivity constraint being guaranteed consequently, while GBC and GBC+DC have to take into account both constraints in this phase. By comparing Fig. 11 and 12, we can see that the difference between the average and the maximum value of our proposed algorithm is very small, which indicates the robustness of our proposed algorithms. It can also be seen from Fig. 9 and 10 that DBCC incurs a performance loss compared with CBCC because the derived topology is over-connected in DBCC with only local information. Moreover, OCFP gives a lower bound on the number of required channels since it only guarantees the conflict-free constraint. We can see that the performance of CBCC is close to the lower bound.

VII. CONCLUSION

In this paper, we studied the topology control problem in CRNs with the objective of minimizing the number of required channels while satisfying conflict-free and bi-channel-connectivity constraints. To solve this NP-hard problem, we proposed a centralized and a distributed topology control algorithm respectively with joint topology construction and channel assignment. Theoretical analysis and simulation study verify the correctness and effectiveness of the proposed algorithm. It is shown that our proposed algorithms obtain a considerable performance gain compared to existing algorithms and approach the lower bound of the average number of required channels.

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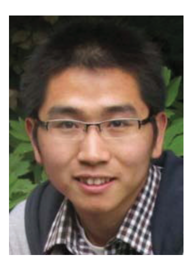

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