Network Coding based Wireless Broadcast with Performance Guarantee

Peng Wang, *Student Member, IEEE,* Guoqiang Mao, *Senior Member, IEEE,* Zihuai Lin, *Senior Member, IEEE,* Xiaohu Ge, *Senior Member, IEEE,* and Brian D. O. Anderson, *Life Fellow, IEEE*

***Abstract*—Wireless broadcast has been increasingly used to deliver information of common interest to a large number of users. There are two major challenges in wireless broadcast: the unreliable nature of wireless links and the difficulty of acknowledging the correct reception of every broadcast packet by every user when the number of users becomes large. In this paper, by resorting to stochastic geometry analysis, we develop a network coding based broadcast scheme that allows a base station (BS) to broadcast a given number of packets to a large number of users, without user acknowledgment, while being able to provide a performance guarantee on the probability of successful delivery. Further, the BS only has limited statistical information about the environment including the spatial distribution of users (instead of their exact locations and number) and the wireless propagation model. Performance analysis is conducted. On that basis, an upper and a lower bound on the number of packet transmissions required to meet the performance guarantee are obtained. Simulations are conducted to validate the accuracy of the theoretical analysis. The technique and analysis developed in this paper are useful for designing efficient and reliable wireless broadcast strategies.**

***Index Terms*—Rateless codes; wireless broadcast; stochastic geometry; reliability; latency**

I. Introduction

Broadcasting has been used widely in wired and wireless networks to disseminate information of common interest, e.g. safety warning messages, emergency information and weather information, to a large number of users. There are two major challenges in wireless broadcast. The first one is caused by the unreliable nature of wireless communications and the second one is acknowledging the correct reception of every broadcast packet by ever receiver, particularly when the number of receivers is large. Due to the unreliable nature of wireless communications, qualities of wireless links often vary temporally and spatially. Automatic Repeat reQuest (ARQ) is a common solution to combat the challenge of unreliable wireless communications. With ARQ, feedbacks are transmitted back to the transmitter, e.g. a base station (BS), after each broadcast using either acknowledgements (ACKs) if the packets are correctly received or negative acknowledgements (NACKs) if the packets are deemed erroneous. If NACKs are received or ACKs are not received within a predesignated amount of time, the BS will retransmit the packets. However several drawbacks appear when using packet acknowledgment. Firstly, the overhead incurred when gathering acknowledgment information from multiple receivers increases with the number of receivers. In other words, when the number of receivers is large, packet acknowledgement may cause significant delay and bandwidth consumption [1]. Consequently, using ARQ for wireless broadcast is not scalable [2]. This is particularly true for highly dynamic networks where the user population and the users' locations change dramatically with time. Take vehicular network as an example, due to the mobility of vehicles, it is difficult to obtain the exact location of each vehicle and the exact channel state of each vehicle-BS channel. But the density of vehicles at a particular time period of a day can typically be obtained with much less effort. Therefore, it is highly desirable to design a wireless broadcast scheme that a) uses minimal information about network environment, not relying on information such as the exact number of receivers, the exact location of each receiver and the channel state of each receiver-BS channel, b) reliably delivers information to a large number of users, c) does not rely on user acknowledgment, and d) is able to provide a guaranteed performance on the probability of successful delivery.

In this paper we tackle the above challenges by resorting to the network coding (NC) technique [3], [4], [5] and the stochastic geometry analysis. Recent work has shown that NC can significantly improve both the transmission efficiency and the reliability of transmission [6], [7]. Particularly, in [6] Dong *et al.* proposed several NC based broadcast schemes [6]. It was shown that NC based retransmission scheme performs better than its counterpart using ARQ only. However, their NC based retransmission strategy relies on the use of feedback information from receivers. In [8], Luby developed a class of rateless codes (RCs), i.e., LT codes, to improve the transmission efficiency. RCs are a special class of forward error correction

This research is supported by Australian Research Council Discovery Projects: DP110100538, DP120102030 and DP120100405. The work of X. Ge was supported in part by the NFSC Major International Joint Research Project under Grant 61210002, by the Chinese Ministry of Science and Technology Grant 2014DFA11640 and under Grant 0903, by the Fundamental Research Funds for the Central Universities under Grant 2013ZZGH009, by the EU FP7-PEOPLE-IRSES through Project S2EuNet under Grant 247083, by Project WiNDOW under Grant 318992, and by Project CROWN under Grant 610524.

Peng Wang is with the School of Electrical and Information Engineering, The University of Sydney, Australia and National ICT Australia (NICTA) Sydney (e-mail: thomaspeng.wang@sydney.edu.au).

Guoqiang Mao is with the School of Computing and Communication, The University of Technology Sydney, Australia and National ICT Australia (NICTA) Sydney (e-mail: g.mao@ieee.org).

Zihuai Lin is with the School of Electrical and Information Engineering, The University of Sydney, Australia (e-mail: zihuai.lin@sydney.edu.au).

Xiaohu Ge (corresponding author) is with the Department of Electronics and Information Engineering, Huazhong University of Science and Technology, Wuhan 430074, Hubei, China (email: xhge@mail.hust.edu.cn).

B. D. O. Anderson is with Research School of Engineering, The Australian National University, Australia and National ICT Australia (NICTA), Canberra (e-mail: brian.anderson@anu.edu.au).

codes. Compared with other forward error correction codes with finite length, such as Reed-Solomon code, Block code and Convolutional code, RCs can automatically adapt to the channel conditions and avoid the need for a feedback channel [7], [8], [9]. RCs can generate a potentially limitless stream of coded packets. A sufficient number of successfully received coded packets can lead to successful decoding of all source packets. Due to these salient advantages of RCs, in this paper we choose RCs for use in our broadcast strategy design.

Specifically, in this paper, we develop a network coding based broadcast scheme that allows a BS to broadcast a given number of packets to an unknown number of receivers, without requiring the receivers to acknowledge the correct receipt of broadcast packets and in the meantime being able to provide a performance guarantee on the probability of successful delivery. Further, we assume that the BS only has limited prior knowledge about the network environment, which includes the spatial distribution of the receivers, i.e. the receiver density λ, and the wireless propagation model. However the BS may not know the exact number of receivers and their locations. The above assumption is due to the consideration that in some highly dynamic networks, particularly vehicular networks, the receiver density in the coverage area of a BS is relatively stable and easy to estimate however the receivers in the coverage area may be quickly changing. Compared with the broadcast scheme without NC, the RCs technique can facilitate information dissemination by reducing the number of transmissions while providing a guaranteed performance on the probability of successful delivery. The performance of the proposed RCs based broadcast scheme is validated both analytically and via simulations. The following is a detailed summary of our contributions:

1) A RCs based broadcast scheme is proposed, which broadcasts a given number of packets from a BS to a large number of users with a priori knowledge about the spatial distribution of the receivers and the wireless propagation model only. The scheme does not need users' acknowledgment and is able to provide a performance guarantee on the probability of successful delivery.
2) The performance of the proposed scheme is analyzed. Firstly, an upper and a lower bound on the decoding success probability for a single BS and receiver pair using RCs are obtained. To our knowledge, these bounds are the first such analytical bounds for RCs. On that basis, an upper and a lower bound on the probability that all receivers in a bounded area successfully receive or decode all source packets from the BS are derived.
3) On the basis of the above results, the minimum number of transmissions required for a guaranteed performance on the probability of successful delivery is obtained.
4) Simulations are conducted which validate both the accuracy of the analysis and the performance improvement of the proposed scheme.

The technique and analysis presented in this paper can be useful for designing broadcast strategies to deliver information of common interest to a large number of users efficiently and reliably.

The rest of the paper is organized as follows. Section II reviews the related work. Section III describes the system model and problem formulation. In Section IV, we analyze the probability that a receiver can successfully decode all source packets conditioned on the event that the receiver has successfully received a known number of coded packets from the BS. The analysis in Section IV forms the theoretical basis for later performance analysis. In Section V, we carry out performance analysis of the proposed RCs based broadcast scheme and present a technique to estimate the number of transmissions required to meet the performance objective on the probability of successful delivery. In Section VI, we validate our analytical results using simulations. Section VII concludes the paper.

II. Related work

In this section, we review related work on the study of rateless codes (RCs), on the analysis of the corresponding decoding success probability, and on wireless broadcast schemes.

The first practical digital fountain code is Luby Transform (LT) codes [8], which was invented by Luby. In LT codes, the source packet length can be arbitrary. To transmit a traffic session containing k source packets, each coded packet is independently generated by the BS, and the entire session can be recovered from any $n = k + O(\sqrt{k}\log^2(k/\delta))$ coded packets with a probability of $1-\delta$. Based on [8], Shokrollahi [9] developed "Raptor codes" which have less encoding and decoding complexities than LT codes.

It was shown in [9] that LT codes can deliver excellent performance when the value of k is large. In reality, a traffic session may contain a small numbers of packets only. Under this scenario, a large packet overhead, which is defined as $\gamma = \frac{n-k}{k}$ and is a key parameter related to the error-performance of LT codes, is however reported [10]. Hyytia et al. [10] optimized the configuration of the degree distribution for LT codes when the number of packets is small. However, as presented in [10], their proposed methods are not scalable and can only handle the situation when the number of source packets k is around 10. The authors in [11] proposed a new algorithm for decoding. Using this algorithm, the packet overhead γ is reduced.

The above work on RCs focuses on the study of the transmission between single transmitter and single receiver. Although significant work has been done on studying better degree distribution of RCs such that less encoded packets are required to decode all the source packets in the single transmitter and single receiver scenario, not all work related to RCs focuses on transmission between single transmitter and single receiver. In [7] Nguyen et al. investigated the benefits of applying fountain codes (FCs) on improving the transmission efficiency of broadcast between single transmitter and multiple receivers, but they did not consider the decoding success probability. In [12], Xiao et al. developed and analyzed two new data dissemination protocols employing RCs. Their work did not analyze performance of RCs. In comparison, our work uses RCs for packet broadcast between single transmitter and multiple receivers without the use of feedback information from the receivers and provides theoretical analysis of the decoding success probability of RCs.

A major challenge in analyzing the performance of RCs is that the decoding success probability of RCs is difficult to analyze. In [13], the authors proposed a method to recursively compute the decoding success probability of RCs. The detailed proof of their method was presented in [14]. The recursion involved in the computation makes it very difficult to derive a closed-form analytical result for the decoding success probability. It is worth noting that in [11], a decoding algorithm called full-rank decoding was employed and on that basis theoretical analysis was conducted on the decoding success probability of the proposed algorithm. However the analysis in [11] was incomplete to the extent that no rigorous analysis was presented to support some results presented in the paper and the analytical result presented on the decoding success probability was in fact an approximation only, which will be discussed in further details in the analysis of Section IV. In this paper, we advance the work in [13], [14], [11] by providing rigorous upper and lower bounds on the decoding success probability of RCs.

In [15], Tukmanov et al. studied the effect of cooperation on broadcast and derived analytical results characterizing the performance of non-cooperative broadcast scheme and cooperative broadcast scheme respectively. In their schemes, network coding technique was not employed. In [6], Dong et al. compared the efficiency of the network coding based broadcast scheme and traditional ARQ based schemes. Their network coding based broadcast scheme relies on the feedback information provided by the receivers.

In this paper, we present theoretical analysis on the decoding success probability for single BS and receiver pair using RCs. This analysis is subsequently used to calculate the overall transmission success probability, i.e. the probability that all receivers successfully receive all broadcast packets. Finally, the minimum number of transmissions required by the BS to meet a pre-designated target on the overall transmission success probability is determined.

III. System Model and Problem Formulation

A. System Model

In this paper, a cellular network with one BS and an unknown number of receivers is considered. Receivers are distributed across a two dimensional disk, denoted by $D\left(o,R\right)$, according to a homogeneous Poisson point process (PPP) Φ with intensity λ where $D\left(o,R\right)$ represents a disk centered at the origin o and with a radius R. The BS is located at the origin. Let $\{\boldsymbol{x}_i\}$ denote the set of receivers on $D\left(o,R\right)$ and we refer to a receiver by its location $\boldsymbol{x}_i$.

We assume that the channel between the BS and a receiver and the channel between the BS and another distinct receiver are independent [1]. For the data transmission from the BS located at o to a receiver located at $\boldsymbol{x}_i$, the SNR of the received signal is written as:

$$\mathbf{SNR}_i = \frac{P_t h_i \left\|\boldsymbol{x}_i\right\|^{-\alpha}}{N_o} \tag{1}$$

where P_t is the transmitting power of the BS, α is the path loss exponent and $\|\boldsymbol{x}_i\|$ represents the Euclidean norm of $\boldsymbol{x}_i$. The noise is additive and random with constant power N_o, but our analysis does not rely on the noise to assume any specific distribution. Parameter h_i is a random positive number modeling the small scale fading and shadowing between the BS and $\boldsymbol{x}_i$ and is assumed to be exponentially distributed with a mean value of 1 [15].

The BS broadcasts coded packets to all receivers where the source packets are coded using rateless codes (RCs). A (coded) packet is considered to be successfully delivered from the BS to the receiver $\boldsymbol{x}_i$ when the instantaneous SNR is greater than or equal to a designated threshold δ. Denote by P_i the probability of successful packet delivery for the receiver $\boldsymbol{x}_i$. It follows that

$$P_i = \Pr[\mathbf{SNR}_i \geq \delta] \tag{2}$$

Further, for each receiver, we assume that the event that a (coded) packet is successfully received and the event that another (coded) packet is received are independent.

B. Problem Formulation

The metric of interest is the number of transmissions by the BS, denoted by L, required to deliver k source packets of equal length to all receivers in $D\left(o,R\right)$ such that the probability of successful delivery of all k packets to all receivers is above a predesignated threshold $1-\epsilon$, where ϵ is a small positive constant.

Denote by η_i the event that all k source packets have been received, i.e. successfully decoded from the coded packets received from the BS, by receiver $\boldsymbol{x}_i$. Let

$$\eta \triangleq \bigcap\nolimits_{i\in\Gamma}\eta_i \tag{3}$$

where Γ denotes the set of indices of all receivers. Obviously $\Pr(\eta)$ depends on the number of (coded) packets broadcast by the BS. Denote by m the number of packets broadcast from the BS and we also write η as $\eta\left(m\right)$ to emphasize the dependence of η on m when necessary. Parameter L can be defined more rigorously as:

$$L \triangleq \min_{m} \Pr\left(\eta\left(m\right)\right) \geq 1-\epsilon \tag{4}$$

[1]The assumption of channel independence has been widely used and is also supported by some measurement studies although we acknowledge that in some environment channel correlations can be a major concern. For example, in [16] it was shown that the coherence distance in an omnidirectional Rayleigh channel is: $\frac{9\lambda}{16\pi}$ [16, Eq. (5.116)] where λ is the wavelength and the value for a non-omnidirectional channel is only slightly different [16, Eq. (5.117)]. In a more recent work it was shown [17] that if a pair of receivers are separated by more than λ, their received signals from a common transmitter can be considered independent [16, p. 243] (with a correlation coefficient less than 0.15). At 800 MHz $\lambda = 0.375$ m, thus the requirement on the separation of receivers (in order for the channels to be considered independent) can be easily met.

In this paper, we shall quantitatively characterize the value of L. This is done by first deriving an upper and a lower bound on the decoding success probability $\Pr(\eta_i)$ for single BS and receiver pair using RCs. On that basis, an upper and a lower bound on the probability $\Pr(\eta)$ that all receivers successfully decode all source packets from the BS are derived. Consequently, an upper and a lower bound on L are obtained which allows us to draw conclusion on the number of (coded) packets that the BS needs to transmit with RCs to guarantee that $\Pr(\eta) \geq 1-\epsilon$.

Fig. 1 illustrates the system model.

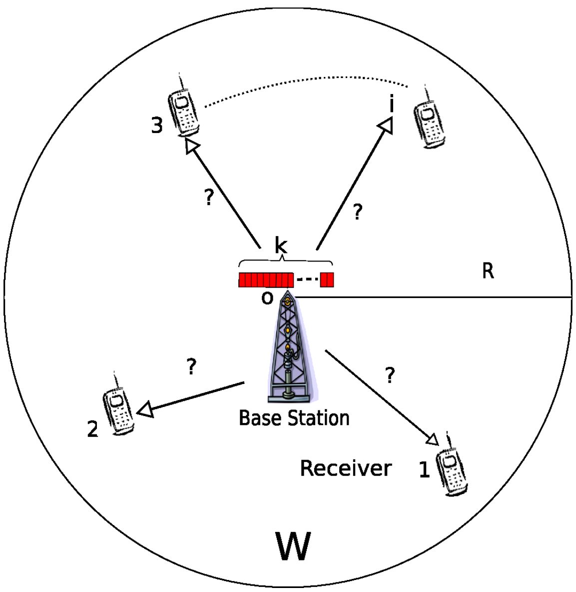


Figure 1. An illustration of the system model

IV. Analysis on the Decoding Success Probability of Rateless Codes

Denote by R_n^k the event that a receiver can successfully decode all k source packets conditioned on the event that the receiver has successfully received n coded packets from the BS. In this section, we shall analyze the probability of R_n^k. Particularly an upper and a lower bound on $\Pr\left(R_n^k\right)$ will be derived.

When RCs are used by the BS to broadcast k source packets, the following encoding rule is utilized to generate each coded packet: firstly a positive integer d (often referred to as the "degree" [8] of coded packets) is drawn from the set of integers $\{1,...,k\}$ according to a probability distribution $\mathbf{\Omega} = (\Omega_1,...,\Omega_k)$ where Ω_d is the probability that d is picked and $\sum_{d=1}^{k}\Omega_d = 1$. This probability distribution is the degree distribution of the RCs, and is obtained from [8]. Degree distribution optimization of RCs has been very well investigated. The degree distribution will affect the decoding success probability. However, few work has been done to derive the analytical method to calculate the decoding success probability of RCs. Our work focus on deriving the decoding success probability of RCs that is applicable for any degree distribution. Our analysis is also useful to obtain the optimal degree distribution of RCs with full-rank decoding algorithms [11]. Then, d distinct source packets are selected randomly and independently from the k source packets, where each source packet is selected with equal probability. These d source packets are then encoded using XOR operation to generate the coded packet [8], [9]. Finally, the coded packet is broadcast to all receivers.

A typically used decoding process for RCs is the so-called "LT process" [8], but it is well known that the LT process is not able to decode all source packets which can be possibly recovered from information contained in the received coded packets. For example, LT process relies on the existence of a degree-one coded packet to be received in order to start the decoding process. Therefore in this paper, we use a different decoding algorithm called the full-rank decoding [11] to decode the source packets. More specifically, let n $(n \geq k)$ be the number of coded packets that have already been successfully received by a receiver. We use a $1 \times k$ row vector to represent the information contained in a coded packet, where the j^{th} entry of the row vector is 1 if the corresponding coded packet is a result of XOR operation on the j^{th} source packet (and other source packets); otherwise the j^{th} entry equals to 0. Thus, a random row vector in this paper refers to the row vector of a randomly chosen coded packet where the coded packet is generated using the RCs encoding process. In this way, the information contained in the n coded packets can be represented by a $n \times k$ matrix, denoted by $\mathbf{G}_{n\times k}$. We say that the receiver can recover all k source packets from the n coded packets if and only if $\mathbf{G}_{n\times k}$ is a full rank matrix, i.e. its rank equals to k. Note that in this paper, all algebraic operations and the associated analysis are conducted in a binary field. Obviously the event that $\mathbf{G}_{n\times k}$ is a full rank matrix is equivalent to the event R_n^k.

The main result of this section is summarized in the following theorem:

Theorem 1. *When the BS generates coded packets using the rateless codes and the coded packets received at a receiver are decoded using the full-rank decoding, the probability that a receiver can successfully decode all k source packets from n received coded packets with $n \geq k$, denoted by R_n^k, satisfies*

$$\Pr\left[R_n^k\right] \leq \mathbf{e}_k \left(\mathbf{X}\right)^{n-1} \mathbf{R}(1) \tag{5}$$

where $\mathbf{e}_k$ is a $1 \times k$ row vector with the k^{th} entry equal to 1 and all other entries equal to 0,

$$\mathbf{X} = \begin{pmatrix} 1-O_1^1 & 0 & \cdots & 0 & 0 \\ O_1^1 & 1-O_2^2 & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1-O_{k-1}^{k-1} & 0 \\ 0 & 0 & \cdots & O_{k-1}^{k-1} & 1-O_k^k \end{pmatrix}$$

$\mathbf{R}(1) = (1,0,\ldots,0)^T$ and

$$O_m^m = \frac{\Pr\left[R_{m+1}^{m+1}\right]}{\Pr\left[R_m^m\right]}$$

Further,

$$\Pr\left[R_m^m\right] = \prod_{q=2}^{m}\left[(1-I_q)^{\binom{m}{q}}\right]$$

where I_q is given by:

$$I_{q,\, q\geq 2} = (Q_{10}, Q_{20}, \ldots, Q_{k0})\mathbf{Tr}^{q-2}(\Omega_1, \Omega_2, \ldots, \Omega_k)^T$$

and $\mathbf{Tr}$ in the above equation is given by

$$\mathbf{Tr} = \begin{pmatrix} Q_{11} & \cdots & Q_{(k-1)1} & Q_{k1} \\ Q_{12} & \cdots & Q_{(k-1)2} & Q_{k2} \\ \vdots & \ddots & \vdots & \vdots \\ Q_{1k} & \cdots & Q_{(k-1)k} & Q_{kk} \end{pmatrix}$$

and

$$Q_{ij} = \begin{cases} \sum_{\substack{0\leq a\leq \min(k-j,i) \\ b=j-i+a}} \Omega_{a+b}\frac{\binom{i}{a}\binom{k-i}{b}}{\binom{k}{a+b}}, & i<j \\ \sum_{\substack{1\leq a\leq \min(k-j,i) \\ b=j-i+a}} \Omega_{a+b}\frac{\binom{i}{a}\binom{k-i}{b}}{\binom{k}{a+b}}, & i=j \\ \sum_{\substack{i-j\leq a\leq \min(k-j,i) \\ b=j-i+a}} \Omega_{a+b}\frac{\binom{i}{a}\binom{k-i}{b}}{\binom{k}{a+b}}, & i>j \end{cases}$$

In addition to the above upper bound, a lower bound of $\Pr\left[R_n^k\right]$ can also be obtained:

$$\Pr\left[R_n^k\right] \geq \mathbf{e}_k \begin{pmatrix} 1-u_1 & \cdots & 0 & 0 \\ u_1 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 1-u_{k-1} & 0 \\ 0 & \cdots & u_{k-1} & 1-u_k \end{pmatrix}^{n-1} \mathbf{R}(1) \tag{6}$$

where

$$u_z = \max_{0\leq i\leq k-z}\{\sum_{d=0}^{z-1}[\binom{z-1}{d}P_g(d+i-z+1)] + \sum_{d=1}^{z-1}[\binom{z-1}{d}P_g(d)]\}$$

and $P_g(d) = \frac{\Omega_d}{\binom{k}{d}}$.

The rest of this section is devoted to the proof of Theorem 1. Because of the close connection between the event R_n^k and the event that $\mathbf{G}_{n\times k}$ is a full rank matrix, the analysis of $\Pr\left(R_n^k\right)$ is conducted by analyzing the rank of $\mathbf{G}_{n\times k}$.

A. Analysis of the rank of a random matrix

In this subsection, we give procedure on computing the probability that $\mathbf{G}_{n\times k}$ is a full rank matrix, where $n \geq k$.

Let R_n^r be the event that the rank of the encoding coefficient matrix $\mathbf{G}_{n\times k}$ is r and let $\Pr[R_n^r]$ be its probability. Define the rank profile of $\mathbf{G}_{n\times k}$ to be a vector $\mathbf{R}(n) = \left(\Pr[R_n^1], \Pr[R_n^2], \ldots, \Pr[R_n^k]\right)^T$. Noting that the decoding success probability is equal to the probability that the rank of the encoding coefficient matrix $\mathbf{G}_{n\times k}$ equals k, i.e. $\Pr[R_n^k]$, our analysis on the decoding success probability relies on a recursive computation of $\mathbf{R}(n)$ as n increases.

When $n = 1$, it can be readily shown that $\mathbf{R}(1) = (\Pr[R_1^1], \Pr[R_1^2], \ldots, \Pr[R_1^k])^T = (1, 0, \ldots, 0)^T$. For $n > 1$, the rank profile of $\mathbf{G}_{n\times k}$ can be obtained from the rank profile of $\mathbf{G}_{(n-1)\times k}$ recursively. Particularly, $\mathbf{G}_{n\times k}$ can be considered as $\mathbf{G}_{(n-1)\times k}$ with an additional row $\mathbf{x}$ added into $\mathbf{G}_{(n-1)\times k}$. The degree of $\mathbf{x}$, i.e. the number of non-zero elements of $\mathbf{x}$, is chosen according to the pre-defined degree distribution $\mathbf{\Omega} = (\Omega_1, ..., \Omega_k)$ and each non-zero element is then placed randomly and uniformly into $\mathbf{x}$. Let $rk(\mathbf{G})$ be the rank of the matrix $\mathbf{G}$ and let $Im(\mathbf{G})$ be the row vector space generated by a matrix $\mathbf{G}$. That is, $Im(\mathbf{G})$ is the vector space formed by all linear combinations of the rows of $\mathbf{G}$. Note that it may possibly occur that $Im(\mathbf{G}_{n\times k}) = Im(\mathbf{G}_{m\times k})$ where $m \neq n$. If a row vector $\mathbf{x}$ can be expressed as a linear combination of the row vectors of $\mathbf{G}$, we say that $\mathbf{x} \in Im(\mathbf{G})$; otherwise $\mathbf{x} \notin Im(\mathbf{G})$. For $k \geq r \geq 2$, it can be shown that

$$\begin{aligned} &\Pr\left[rk(\mathbf{G}_{n\times k}) = r\right] \\ =&\Pr\left[rk(\mathbf{G}_{(n-1)\times k}) = r\right] \times \\ &\Pr\left[\mathbf{x} \in Im(\mathbf{G}_{(n-1)\times k}) \mid rk(\mathbf{G}_{(n-1)\times k}) = r\right] \\ &+ \Pr\left[rk(\mathbf{G}_{(n-1)\times k}) = r-1\right] \times \\ &\Pr\left[\mathbf{x} \notin Im(\mathbf{G}_{(n-1)\times k}) \mid rk(\mathbf{G}_{(n-1)\times k}) = r-1\right] \end{aligned} \tag{7}$$

For convenience let $O_{n-1}^{r-1} = \Pr\left[\mathbf{x} \notin Im(\mathbf{G}_{(n-1)\times k}) \mid R_{n-1}^{r-1}\right]$. It follows from the equation (7) that:

$$\Pr\left[R_n^r\right] = \Pr\left[R_{n-1}^r\right](1 - O_{n-1}^r) + \Pr\left[R_{n-1}^{r-1}\right]O_{n-1}^{r-1} \tag{8}$$

Based on (8), the following equation can be obtained by recursion:

$$\begin{aligned} &\mathbf{R}(n) \\ =&\begin{pmatrix} 1-O_{n-1}^1 & \cdots & 0 & 0 \\ O_{n-1}^1 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 1-O_{n-1}^{k-1} & 0 \\ 0 & \cdots & O_{n-1}^{k-1} & 1-O_{n-1}^k \end{pmatrix}\mathbf{R}(n-1) \\ =&(\prod_{m=1}^{n-1}\mathbf{X}_m)\mathbf{R}(1) \end{aligned} \tag{9}$$

where

$$\mathbf{X}_m = \begin{pmatrix} 1-O_m^1 & 0 & \cdots & 0 & 0 \\ O_m^1 & 1-O_m^2 & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1-O_m^{k-1} & 0 \\ 0 & 0 & \cdots & O_m^{k-1} & 1-O_m^k \end{pmatrix}$$

The probability that $\mathbf{G}_{n\times k}$ is of full rank, hence all k source packets can be successfully decoded, can be calculated by:

$$\begin{aligned} \Pr\left[R_n^k\right] &= \begin{pmatrix} 0 & 0 & \cdots & 0 & 1 \end{pmatrix}\mathbf{R}(n) \\ &= \mathbf{e}_k(\prod_{m=1}^{n-1}\mathbf{X}_m)\mathbf{R}(1) \end{aligned} \tag{10}$$

where $\mathbf{e}_i$, $1 \leq i \leq k$, is a $1 \times k$ row vector with the i^{th} entry equal to 1 and all other entries equal to 0.

The above recursive way of computing the rank profile of $\mathbf{G}_{n\times k}$ and the probability that $\mathbf{G}_{n\times k}$ is a full rank matrix relies on the knowledge of the parameters $O_{n-1}^z =$

$\Pr\left[\mathbf{x} \notin Im(\mathbf{G}_{(n-1)\times k}) \mid R^z_{n-1}\right], 1 \leq z \leq k$. In the following paragraphs, we give analysis on the computation of $\Pr\left[\mathbf{x} \in Im(\mathbf{G}_{(n-1)\times k}) \mid R^z_{n-1}\right]$.

For convenience let A_{n-1} be the event that $\mathbf{x} \notin Im(\mathbf{G}_{(n-1)\times k})$ and $\overline{A_{n-1}}$ be the complement of event A_{n-1}. Temporarily assuming that $rk(\mathbf{G}_{(n-1)\times k}) = z, 1 \leq z \leq k$ and noting that $\mathbf{G}_{(n-1)\times k}$ is a random matrix, under the above two conditions, let V^z be a row vector space formed by all linear combinations of the rows of *an instance* of $\mathbf{G}_{(n-1)\times k}$. Of course the dimension of V^z equals to z, hence the superscript. Further, let $\mathcal{E}^z$ be the set of *all possible* and *distinct* V^zs: $\mathcal{E}^z \triangleq \{V^z\}$. When $z = k$, the row vector space whose dimension **equals to** k is unique. However when $1 \leq z < k$, there are multiple distinct row vector spaces with dimension z. For convenience, we number the elements of $\mathcal{E}^z$ sequentially and denote by Γ^z_v be the set of indices of all V^z satisfying $V^z \in \mathcal{E}^z$. Denote by V^z_i the i^{th} element of $\mathcal{E}^z$. As noted in the last paragraph, the coding coefficient matrix $\mathbf{G}$ and the vector space formed by the row vectors of $\mathbf{G}$ have independent significance in the sense that for two positive integers $m, n \geq z$ and $m \neq n$, it may happen that $V^z_i = Im(\mathbf{G}_{n\times k}) = Im(\mathbf{G}_{m\times k})$. That is, the vector space and its existence does not depend on some details of the coding coefficient matrix, e.g. number of rows in the coding coefficient matrix and a particular instance of the coding coefficient matrix.

Let $F^z_{i,n-1}$ be the event $Im(\mathbf{G}_{(n-1)\times k}) = V^z_i$. It can be readily shown that: 1) $R^z_{n-1} = \cup_{i\in\Gamma^z_v} F^z_{i,n-1}$, i.e. event that the rank of the encoding coefficient matrix $\mathbf{G}_{n\times k}$ is z equals to the joint events that $Im(\mathbf{G}_{(n-1)\times k}) = V^z_i$ for all $i, i \in \Gamma^z_v$; 2) $F^z_{i,n-1} \cap F^z_{j,n-1} = \emptyset$ for $i \neq j$. Using the definitions of the two events R^z_n and $F^z_{i,n-1}$, Bayes' formula and the above two results, we have

$$\begin{aligned}
&\Pr\left[\mathbf{x} \in Im(\mathbf{G}_{(n-1)\times k}) \mid rk(\mathbf{G}_{(n-1)\times k}) = z\right] \\
&= \Pr\left[\overline{A_{n-1}} \mid R^z_{n-1}\right] = \frac{\Pr\left[\overline{A_{n-1}} \cap R^z_{n-1}\right]}{\Pr\left[R^{r-1}_{n-1}\right]} \\
&= \frac{\Pr\left[\overline{A_{n-1}} \cap (\cup_{i\in\Gamma^z_v} F^z_{i,n-1})\right]}{\Pr\left[\cup_{i\in\Gamma^z_v} F^z_{i,n-1}\right]} = \frac{\sum_{i\in\Gamma^z_v} \Pr\left[\overline{A_{n-1}} \cap F^z_{i,n-1}\right]}{\sum_{i\in\Gamma^z_v} \Pr\left[F^z_{i,n-1}\right]} \\
&= \frac{\sum_{i\in\Gamma^z_v} \Pr\left[\overline{A_{n-1}} \mid F^z_{i,n-1}\right] \Pr\left[F^z_{i,n-1}\right]}{\sum_{i\in\Gamma^z_v} \Pr\left[F^z_{i,n-1}\right]} \qquad (11)
\end{aligned}$$

Let $\overline{B^z_i}$ be the event that $\mathbf{x} \in V^z_i$. Conditioned on the event $F^z_{i,n-1}$ and noting that $\mathbf{x}$ is drawn randomly and independently of the row vectors of $\mathbf{G}_{(n-1)\times k}$, we have

$$\overline{A_{n-1}} \mid F^z_{i,n-1} \Leftrightarrow \overline{B^z_i} \mid F^z_{i,n-1} \qquad (12)$$

Because each row vector is drawn *independently* of other row vectors, the two events $\mathbf{x} \in V^z_i$ and $Im(\mathbf{G}_{(n-1)\times k}) = V^z_i$ are independent. It follows using the definitions of $\overline{B^z_i}$ and $F^z_{i,n-1}$ that $\Pr\left[\overline{B^z_i} \mid F^z_{i,n-1}\right] = \Pr\left[\overline{B^z_i}\right] = \Pr\left[\mathbf{x} \in V^z_i\right]$.

For the other term $\Pr\left[F^z_{i,n-1}\right]$ in (11), we recall that $F^z_{i,n-1}$ is the event $Im(\mathbf{G}_{(n-1)\times k}) = V^z_i$. Let $E^z_{i,n-1}$ be the event $V^z_i \subseteq Im(\mathbf{G}_{(n-1)\times k})$ and obviously $F^z_{i,n-1} \subseteq E^z_{i,n-1}$. Conditioned on the event $E^z_{i,n-1}$, without loss of generality, let $\{\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_z\}$ be the row vectors of $\mathbf{G}_{(n-1)\times k}$ that forms a basis of V^z_i. The set of row vectors of $\mathbf{G}_{(n-1)\times k}$ that forms a basis of V^z_i may not be unique. Let $\{\mathbf{w}_1, \mathbf{w}_1, ..., \mathbf{w}_{n-z-1}\}$ be the remaining row vectors of $\mathbf{G}_{(n-1)\times k}$. Further note that each row vector of $\mathbf{G}_{(n-1)\times k}$ is formed *independently* of other row vectors. Noting that $F^z_{i,n-1} \subseteq E^z_{i,n-1}$, it can be shown that

$$\begin{aligned}
&\Pr\left[F^z_{i,n-1}\right] \\
&= \Pr\left[F^z_{i,n-1} | E^z_{i,n-1}\right] \Pr\left[E^z_{i,n-1}\right] \\
&= \Pr\left[\mathbf{w}_1 \in V^z_i \cap \cdots \cap \mathbf{w}_{n-z-1} \in V^z_i | E^z_{i,n-1}\right] \Pr\left[E^z_{i,n-1}\right] \\
&= \left(\Pr\left[\mathbf{w}_1 \in V^z_i | E^z_{i,n-1}\right]\right)^{n-z-1} \Pr\left[E^z_{i,n-1}\right] \\
&= \left(\Pr\left[\overline{B^z_i}\right]\right)^{n-z-1} \Pr\left[E^z_{i,n-1}\right] \qquad (13)
\end{aligned}$$

where the last step results because the two events $\mathbf{w}_1 \in V^z_i$ and $E^z_{i,n-1}$ are independent. Combining the three equations (11), (12), and (13), conclusion follows that

$$\begin{aligned}
&\Pr\left[\overline{A_{n-1}} \mid R^z_{n-1}\right] \\
&= \frac{\sum_{i\in\Gamma^z_v} \Pr\left[\overline{A_{n-1}} \mid F^z_{i,n-1}\right] \Pr\left[F^z_{i,n-1}\right]}{\sum_{i\in\Gamma^z_v} \Pr\left[F^z_{i,n-1}\right]} \\
&= \frac{\sum_{i\in\Gamma^z_v} \left(\Pr\left[\overline{B^z_i}\right]\right)^{n-z} \Pr\left[E^z_{i,n-1}\right]}{\sum_{i\in\Gamma^z_v} \left(\Pr\left[\overline{B^z_i}\right]\right)^{n-z-1} \Pr\left[E^z_{i,n-1}\right]} \qquad (14)
\end{aligned}$$

As manifested in equation (14), the computation of $\Pr\left[\overline{A_{n-1}} \mid R^z_{n-1}\right]$, which is required for computing the rank profile of $\mathbf{G}_{n\times k}$ and the probability that $\mathbf{G}_{n\times k}$ is a full rank matrix, relies on the knowledge of $\Pr\left[\overline{B^z_i}\right]$ and $\Pr\left[E^z_{i,n-1}\right]$. These parameters can be difficult to obtain when k is large. Therefore in the rest of this section, we devote our efforts to finding an upper and a lower bound of $\Pr\left[\overline{A_{n-1}} \mid R^z_{n-1}\right]$, which will be shown later using simulations to be reasonably tight.

1) Derivation of An Upper Bound of $\Pr\left[R^k_n\right]$*:* Let $a_{i,n-1} = \Pr\left[E^z_{i,n-1}\right]$ and $b_{i,z} = \Pr\left[\overline{B^z_i}\right]$ for notational convenience. Equation (14) can be rewritten as:

$$\Pr\left[\overline{A_{n-1}} \mid R^z_{n-1}\right] = \frac{\sum_{i\in\Gamma^z_v} a_{i,n-1} b^{n-z}_{i,z}}{\sum_{i\in\Gamma^z_v} a_{i,n-1} b^{n-z-1}_{i,z}} \qquad (15)$$

Next we shall evaluate the monoticity of $\Pr\left[\overline{A_{n-1}} \mid R^{r-1}_{n-1}\right]$ as a function of n. It can be shown that :

$$\begin{aligned}
&\Pr\left[\overline{A_n} \mid R^z_n\right] - \Pr\left[\overline{A_{n-1}} \mid R^z_{n-1}\right] \\
&= \frac{\sum_{i\in\Gamma^z_v} a_{i,n} b^{n-z+1}_{i,z}}{\sum_{i\in\Gamma^z_v} a_{i,n} b^{n-z}_{i,z}} - \frac{\sum_{i\in\Gamma^z_v} a_{i,n-1} b^{n-z}_{i,z}}{\sum_{i\in\Gamma^z_v} a_{i,n-1} b^{n-z-1}_{i,z}} \\
&= \frac{\sum_{i\in\Gamma^z_v} a_{i,n} a_{i,n-1} b^{2n-2z}_{i,z} - \sum_{i\in\Gamma^z_v} a_{i,n} a_{i,n-1} b^{2n-2z}_{i,z}}{\sum_{i\in\Gamma^z_v} a_{i,n} b^{n-z}_{i,z} \sum_{i\in\Gamma^z_v} a_{i,n-1} b^{n-z-1}_{i,z}} \\
&+ \frac{\sum_{j\in\Gamma^z_v} \sum_{i\in\Gamma^z_v} a_{i,n} a_{j,n-1} b^{n-z+1}_{i,z} b^{n-z-1}_{j,z}}{\sum_{i\in\Gamma^z_v} a_{i,n} b^{n-z}_{i,z} \sum_{i\in\Gamma^z_v} a_{i,n-1} b^{n-z-1}_{i,z}} \\
&- \frac{\sum_{j\in\Gamma^z_v} \sum_{i\in\Gamma^z_v} a_{i,n} a_{j,n-1} b^{n-z}_{i,z} b^{n-z}_{j,z}}{\sum_{i\in\Gamma^z_v} a_{i,n} b^{n-z}_{i,z} \sum_{i\in\Gamma^z_v} a_{i,n-1} b^{n-z-1}_{i,z}} \\
&= \frac{\sum_{j\in\Gamma^z_v} \sum_{i\in\Gamma^z_v} a_{i,n} a_{j,n-1} b^{n-z-1}_{i,z} b^{n-z-1}_{j,z} (b_{i,z} - b_{j,z})^2}{\sum_{i\in\Gamma^z_v} a_{i,n} b^{n-z}_{i} \sum_{i\in\Gamma^z_v} a_{i,n-1} b^{n-z-1}_{i}} \geq 0 \qquad (16)
\end{aligned}$$

As a result of the above analysis, we can conclude that the conditional probability $\Pr\left[\overline{A_{n-1}} \mid R_n^z\right]$ is a monotonically increasing function of n and $\Pr\left[\overline{A_n} \mid R_n^z\right] \geq \Pr\left[\overline{A_{n-1}} \mid R_{n-1}^z\right] \geq \cdots \geq \Pr\left[\overline{A_z} \mid R_z^z\right]$

Let

$$\mathbf{X} = \begin{pmatrix} 1-O_1^1 & 0 & \cdots & 0 & 0 \\ O_1^1 & 1-O_2^2 & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1-O_{k-1}^{k-1} & 0 \\ 0 & 0 & \cdots & O_{k-1}^{k-1} & 1-O_k^k \end{pmatrix}$$

We can then obtain that

$$\begin{aligned} \mathbf{e}_k(\prod_{m=1}^{n-1}\mathbf{X}_m)\mathbf{R}(1) &\leq \mathbf{e}_k(\mathbf{X})^{n-1}\mathbf{R}(1) \\ \Pr\left[R_n^k\right] &\leq \mathbf{e}_k(\mathbf{X})^{n-1}\mathbf{R}(1) \end{aligned} \tag{17}$$

Now an upper bound of the decoding success probability is derived and this upper bound was shown as exact value of the decoding success probability in [11], which supports the statement in the *Introduction Section* that the analysis in [11] was incomplete. Further, this expression of upper bound relies on the knowledge of $O_z^z, 1 \leq z \leq k$. The technique to obtain $O_z^z, 1 \leq z \leq k$ was not explained in [11]. In the following paragraphs, we present analysis leading to the computation of $O_z^z, 1 \leq z \leq k$. Noting that when $1 \leq z \leq k$, $\mathbf{x} \notin Im(\mathbf{G}_{z\times k}) \cap rk(\mathbf{G}_{z\times k}) = z \Leftrightarrow rk(\mathbf{G}_{(z+1)\times k}) = z+1$, it can be shown that

$$\begin{aligned} O_z^z &= \Pr\left[\mathbf{x} \notin Im(\mathbf{G}_{z\times k}) \mid rk(\mathbf{G}_{z\times k}) = z\right] \\ &= \frac{\Pr\left[\mathbf{x} \notin Im(\mathbf{G}_{z\times k}) \cap rk(\mathbf{G}_{z\times k}) = z\right]}{\Pr\left[rk(\mathbf{G}_{z\times k}) = z\right]} \\ &= \frac{\Pr\left[rk(\mathbf{G}_{(z+1)\times k}) = z+1\right]}{\Pr\left[rk(\mathbf{G}_{z\times k}) = z\right]} = \frac{\Pr\left[R_{z+1}^{z+1}\right]}{\Pr\left[R_z^z\right]} \end{aligned} \tag{18}$$

where $\Pr\left[R_z^z\right]$ represents the probability that a random (encoding coefficient) matrix $\mathbf{G}_{z\times k}$, $z \leq k$, is of full rank. The method to calculate $\Pr\left[R_z^z\right]$ is provided in the following lemma.

Lemma 2. *Let $\mathbf{v}_i$ be the i^{th} row vector of $\mathbf{G}_{z\times k}$. Denote by I_q (whose value will be determined later in Lemma 3) the probability of the event that $\sum_{i=1}^{q}\mathbf{v}_i = \mathbf{0}$, conditioned on that the summation of any w row vectors of $\mathbf{G}_{z\times k}$ is not equal to $\mathbf{0}$, where $\mathbf{0}$ is a $1\times k$ row vector with all elements equal to 0, $w \in \mathbb{Z}^+,\ 1 < w < q$. $F(z)$ can be determined by:*

$$\Pr\left[R_z^z\right] = \prod_{q=2}^{z}\left[(1-I_q)^{\binom{z}{q}}\right] \tag{19}$$

Proof: See Appendix A ■

Now we shall derive I_q which is required in Lemma 2. To obtain I_q, we must first evaluate the degree transition probability Q_{ij}, i.e. the probability that the row vector $\mathbf{S}_q$ produced by summing q row vectors has degree j given that the row vector $\mathbf{S}_{q-1}$ generated by summing the first $q-1$ row vectors of the above q row vectors has degree i. We can derive Q_{ij}[11] as:

$$Q_{ij} = \begin{cases} \sum\limits_{\substack{0\leq a\leq \min(k-j,i) \\ b=j-i+a}} \Omega_{a+b}\dfrac{\binom{i}{a}\binom{k-i}{b}}{\binom{k}{a+b}}, & i < j \\ \sum\limits_{\substack{1\leq a\leq \min(k-j,i) \\ b=j-i+a}} \Omega_{a+b}\dfrac{\binom{i}{a}\binom{k-i}{b}}{\binom{k}{a+b}}, & i = j \\ \sum\limits_{\substack{i-j\leq a\leq \min(k-j,i) \\ b=j-i+a}} \Omega_{a+b}\dfrac{\binom{i}{a}\binom{k-i}{b}}{\binom{k}{a+b}}, & i > j \end{cases} \tag{20}$$

where Ω_d, $1 \leq d \leq k$ is the degree distribution of RCs, which is defined in Section IV.

Now we are ready to analyze I_q.

Lemma 3. *Let $\mathbf{Tr}$ be a $k\times k$ transition matrix with dimension $k\times k$ whose $(j,i)^{th}$ element equal to Q_{ij}. The matrix $\mathbf{Tr}$ can be expressed as:*

$$\mathbf{Tr} = \begin{pmatrix} Q_{11} & \cdots & Q_{(k-1)1} & Q_{k1} \\ Q_{12} & \cdots & Q_{(k-1)2} & Q_{k2} \\ \vdots & \ddots & \vdots & \vdots \\ Q_{1k} & \cdots & Q_{(k-1)k} & Q_{kk} \end{pmatrix}$$

the probability I_q is given by:

$$I_{q,\ q\geq 2.} = (Q_{10}, Q_{20}, \ldots, Q_{k0})\mathbf{Tr}^{q-2}\cdot(\Omega_1, \Omega_2, \ldots, \Omega_k)^T \tag{21}$$

Proof: See Appendix B ■

Using (17), (18) and Lemmas 2 and 3, an upper bound on $\Pr\left[R_n^k\right]$ can be computed, which completes the first part of the proof of Theorem 1 on the upper bound.

2) Derivation of A Lower Bound of $\Pr\left[R_n^k\right]$*:* In addition to the upper bound derived earlier in the section, a lower bound on the decoding success probability can also be obtained:

$$\begin{aligned} \Pr\left[\overline{A_n} \mid R_n^z\right] &= \frac{\sum_{i\in\Gamma_v^z} a_{i,n} b_{i,z}^{n-z+1}}{\sum_{i\in\Gamma_v^z} a_{i,n} b_{i,z}^{n-z}} \leq \max_{i\in\Gamma_v^z}\{b_{i,z}\} \\ &\leq \max_{i\in\Gamma_v^z}\{\Pr\left[\overline{B_i^z}\right]\} \end{aligned} \tag{22}$$

Thus we can obtain that

$$\begin{aligned} \mathbf{e}_k(\mathbf{X}_{min})^{n-1}\mathbf{R}(1) &\leq \mathbf{e}_k(\prod_{m=1}^{n-1}\mathbf{X}_m)\mathbf{R}(1) \\ \Pr\left[R_n^k\right] &\geq \mathbf{e}_k(\mathbf{X}_{min})^{n-1}\mathbf{R}(1) \end{aligned} \tag{23}$$

where $\mathbf{X}_{min}$ is given in (24).

The above lower bound relies on the knowledge of $\max_{i\in\Gamma_v^z}\{\Pr\left[\overline{B_i^z}\right]\}, 1 \leq z \leq k$. In the following analysis, we give analysis that leads to the computation of $\max_{i\in\Gamma_v^z}\{\Pr\left[\overline{B_i^z}\right]\}$.

Note that a particular row vector with degree d occurs with probability

$$P_g(d) = \frac{\Omega_d}{\binom{k}{d}} \tag{25}$$

where Ω_d is the probability that a (any) row vector with degree d is chosen and $\binom{k}{d}$ is the total number of degree d vectors among all $1\times k$ binary vectors. Recall that the degree of a

$$\mathbf{X}_{min} = \begin{pmatrix} 1-\max_{i\in\Gamma_v^1}\{\Pr\left[\overline{B_i^1}\right]\} & \cdots & 0 & 0 \\ \max_{i\in\Gamma_v^1}\{\Pr\left[\overline{B_i^1}\right]\} & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 1-\max_{i\in\Gamma_v^{k-1}}\{\Pr\left[\overline{B_i^{k-1}}\right]\} & 0 \\ 0 & \cdots & \max_{i\in\Gamma_v^{k-1}}\{\Pr\left[\overline{B_i^{k-1}}\right]\} & 1-\max_{i\in\Gamma_v^k}\{\Pr\left[\overline{B_i^k}\right]\} \end{pmatrix} \quad (24)$$

vector is the number of non-zero elements in it. Recall that $\mathbf{e}_i$ is a $1\times k$ row vector with the i^{th} entry equal to 1 and all other entries equal to 0. Obviously $\{\mathbf{e}_1,\ldots,\mathbf{e}_k\}$ forms a set of orthogonal basis vectors where *any row vector,* hence a row vector in any V_i^z, $i\in\Gamma_v^z$, in the coding coefficient matrix can be represented as a linear combination of these basis vectors. Let us focus now on a z dimensional subspace formed by $\{\mathbf{e}_1,\ldots,\mathbf{e}_z\}$, denoted by $V_{\{\mathbf{e}_1,\ldots,\mathbf{e}_z\}}$. Using some straightforward combinatorial argument and further noting that we are working in a binary field, it can be shown that the number of degree d, $d\le z$, vectors in $V_{\{\mathbf{e}_1,\ldots,\mathbf{e}_z\}}$ is given by $\begin{pmatrix} z \\ d\end{pmatrix}$. Therefore $\Pr\left[\mathbf{x}\in V_{\{\mathbf{e}_1,\ldots,\mathbf{e}_z\}}\right]=\sum_{d=1}^{z}\left[\begin{pmatrix} z \\ d\end{pmatrix}P_g\left(d\right)\right]$. Denote by Ω_i^z any other z dimensional vector space whose basis vectors are the row vectors of a matrix obtainable by reshuffling the columns of the matrix $\{\mathbf{e}_1,\ldots,\mathbf{e}_z\}^T$ (or equivalently any other z dimensional vector space whose basis vectors are obtained by randomly choosing z vectors from $\{\mathbf{e}_1,\ldots,\mathbf{e}_k\}$). Because the number of non-zero elements are uniformly and independently distributed in a row vector, it follows that $\Pr\left[\mathbf{x}\in V_{\{\mathbf{e}_1,\ldots,\mathbf{e}_z\}}\right]=\Pr\left[\mathbf{x}\in\Omega_i^z\right]$.

Now let us consider a z dimensional vector space formed by the basis vectors $\{\mathbf{e}_1,\ldots,\mathbf{e}_{z-1},\mathbf{e}_z+\mathbf{e}_{z+1}\}$. Except for the last basis vector which has degree 2, all other basis vectors have degree 1 only. Using some straightforward combinatorial argument, the number of vectors in $V_{\{\mathbf{e}_1,\ldots,\mathbf{e}_{z-1},\mathbf{e}_z+\mathbf{e}_{z+1}\}}$ containing $\mathbf{e}_z+\mathbf{e}_{z+1}$ *and* having a degree $d+2$ is given by $\begin{pmatrix} z-1 \\ d\end{pmatrix}$; the number of vectors in $V_{\{\mathbf{e}_1,\ldots,\mathbf{e}_{z-1},\mathbf{e}_z+\mathbf{e}_{z+1}\}}$ *not* containing $\mathbf{e}_z+\mathbf{e}_{z+1}$ *and* having a degree d is given by $\begin{pmatrix} z-1 \\ d\end{pmatrix}$. Therefore

$$\begin{aligned} &\Pr\left[\mathbf{x}\in V_{\{\mathbf{e}_1,\ldots,\mathbf{e}_{z-1},\mathbf{e}_z+\mathbf{e}_{z+1}\}}\right] \\ &=\sum_{d=0}^{z-1}\left[\left(\begin{matrix} z-1 \\ d\end{matrix}\right)P_g\left(d+2\right)\right]+\sum_{d=1}^{z-1}\left[\left(\begin{matrix} z-1 \\ d\end{matrix}\right)P_g\left(d\right)\right] \end{aligned} \quad (26)$$

Similarly, denote by Ω_i^z any other z dimensional vector space whose basis vectors are the row vectors of a matrix obtainable by reshuffling the columns of the matrix $\{\mathbf{e}_1,\ldots,\mathbf{e}_{z-1},\mathbf{e}_z+\mathbf{e}_{z+1}\}^T$. It can be shown that $\Pr\left[\mathbf{x}\in V_{\{\mathbf{e}_1,\ldots,\mathbf{e}_{z-1},\mathbf{e}_z+\mathbf{e}_{z+1}\}}\right]=\Pr\left[\mathbf{x}\in\Omega_i^z\right]$.

Continuing with the above discussion for $V_{\{\mathbf{e}_1,\ldots,\mathbf{e}_{z-1},\mathbf{e}_z+\mathbf{e}_{z+1}+\mathbf{e}_{z+2}\}}$,...,$V_{\{\mathbf{e}_1,\ldots,\mathbf{e}_{z-1},\mathbf{e}_z+\cdots+\mathbf{e}_k\}}$, it can be shown that

$$\begin{aligned} &\Pr\left[\mathbf{x}\in V_{\{\mathbf{e}_1,\ldots,\mathbf{e}_{z-1},\mathbf{e}_z+\cdots+\mathbf{e}_i\}}\right] \\ &=\sum_{d=0}^{z-1}[(\begin{matrix} z-1 \\ d\end{matrix})P_g\left(d+i-z+1\right)]+\sum_{d=1}^{z-1}[(\begin{matrix} z-1 \\ d\end{matrix})P_g\left(d\right)] \end{aligned} \quad (27)$$

where $0\le i\le k-z$. Because we are working in the binary field, it can be shown that the above discussion covers all occurrences of z dimensional spaces.

Summarizing the above discussion, it follows that

$$\max_i\{\Pr\left[\overline{B_i^z}\right]\}=\max_{0\le i\le k-z}\Pr\left[\mathbf{x}\in V_{\{\mathbf{e}_1,\ldots,\mathbf{e}_{z-1},\mathbf{e}_z+\cdots+\mathbf{e}_{z+i}\}}\right] \quad (28)$$

where the values of $\Pr\left[\mathbf{x}\in V_{\{\mathbf{e}_1,\ldots,\mathbf{e}_{z-1},\mathbf{e}_z+\cdots+\mathbf{e}_{z+i}\}}\right]$ is given by (27).

Combining equations (23), (25), (27) and (28), the second part of the proof of Theorem 1 on the lower bound is also completed.

V. Analysis on the Overall Success Probability for Multiple Receivers

On the basis of the analysis in the last section, which investigated the decoding success probability of a single receiver who have successfully received n coded packets from the BS, in this section, we continue to analyze the overall success probability that all receivers have successfully received all k packets, i.e. $\Pr(\eta)$ where the event η is defined in equation (3).

For convenience, let $\phi(n)$, $\phi_l(n)$ and $\phi_u(n)$ be the exact value, the upper and the lower bound of $\Pr\left[R_n^k\right]$ as suggested in Theorem 1 respectively. According to Theorem 1, $\phi(n)\ge\phi_l(n)=\mathbf{e}_k(\mathbf{X}_{min})^{(n-1)}\mathbf{R}(1)$ and $\phi(n)\le\phi_u(n)=\mathbf{e}_k(\mathbf{X})^{(n-1)}\mathbf{R}(1)$. Denote by L the total number of transmissions required on the BS in order to meet the objective $\Pr\left(\eta\right)\ge 1-\epsilon$. The probability that all the k source packets can be successfully decoded by all the receivers after m transmissions by the BS, denoted by $\Pr\left(\eta\left(m\right)\right)$, can be expressed as:

$$\Pr\left(\eta\left(m\right)\right)=\sum_{j=0}^{\infty}\Pr\left[\eta\left(m,j\right)\mid N=j\right]\Pr\left[N=j\right] \quad (29)$$

where $\eta\left(m,j\right)$ is the event that all k source packets have been received, i.e. successfully decoded from the m coded packets broadcast by the BS, by all j receivers in the coverage area of the BS $D\left(o,R\right)$ and N is the total number of receivers in $D\left(o,R\right)$. Parameter N is a Poissonly distributed non-negative integer with mean $\lambda\pi R^2$:

$$\Pr(N=j) = \frac{(\lambda\pi R^2)^j\exp(-\lambda\pi R^2)}{j!} \quad (30)$$

As an easy consequence of the Poisson distribution of receivers [18] and the independence of channels between the BS and the receivers, it can be obtained that

$$\Pr\left[\eta\left(m,j\right)\mid N=j\right]=\prod_{i=1}^{j}\Pr\left[\eta_i\left(m\right)\right]=\left(\Pr\left[\eta_i\left(m\right)\right]\right)^{j} \quad (31)$$

where $\eta_i\left(m\right)$ represents the event that the i^{th} receiver (which is randomly drawn from the set of all receivers) can successfully decode all k source packets when the BS broadcasts m coded packets.

For the same receiver, each coded packet is successfully received independent of other coded packet broadcast by the BS. Let r_i be the (random) distance between the i^{th} receiver and the BS and $r_i=\|\boldsymbol{x}_i\|$. It readily follows that

$$\begin{aligned}&\Pr\left[\eta_i\left(m\right)\mid r_i=y\right]\\ =\ &\sum_{n=k}^{m}\tbinom{m}{n}\left\{P_i(y)\right\}^{n}\left\{1-P_i(y)\right\}^{m-n}\rho(n)\end{aligned} \quad (32)$$

where the term $\binom{m}{n}\left\{P_i(y)\right\}^{n}\left\{1-P_i(y)\right\}^{m-n}$ represents the probability that out of m coded packets broadcast by the BS, n coded packets are received by the i^{th} receiver and $P_i(y)$ represents the probability that a coded packet is successfully received by the i^{th} receiver conditioned on that $r_i=y$. According to the definition in Section III, $P_i(y)$ can be expressed as:

$$P_i(y) = \Pr[\mathbf{SNR}_i(y)\geq\delta] \quad (33)$$

where $\mathbf{SNR}_i(y)$ is instantaneous SNR of the channel between the BS and i^{th} receiver. Using equation (1) and that h_i is exponentially distributed with mean value 1, equation (33) can be rewritten as:

$$P_i(y) = \Pr[h_i\geq\frac{N_o\delta y^{\alpha}}{P_t}]=\exp(-\frac{N_o\delta y^{\alpha}}{P_t}) \quad (34)$$

Inserting equation (34) into equation (32) we obtain:

$$\begin{aligned}&\Pr\left[\eta_i\left(m\right)\mid r_i=y\right]\\
=&\sum_{n=k}^{m}\tbinom{m}{n}\left\{P_i(y)\right\}^{n}\left\{1-P_i(y)\right\}^{T-n}\phi(n)\\
=&\sum_{n=k}^{m}\tbinom{m}{n}\phi(n)\left[\exp(-\frac{N_o\delta y^{\alpha}}{P_t})\right]^{n}\\
\times&\left[1-\exp(-\frac{N_o\delta y^{\alpha}}{P_t})\right]^{m-n}\\
=&\sum_{n=k}^{m}\tbinom{m}{n}\phi(n)\sum_{i=0}^{m-n}\tbinom{m-n}{i}(-1)^{(m-n-i)}\\
\times&\left[\exp(-\frac{N_o\delta y^{\alpha}}{P_t})\right]^{(m-n-i)}\left[\exp(-\frac{N_o\delta y^{\alpha}}{P_t})\right]^{(n)}\\
=&\sum_{n=k}^{m}\tbinom{m}{n}\phi(n)\sum_{i=0}^{m-n}\tbinom{m-n}{i}(-1)^{m-n-i}\left[\exp(-\frac{N_o\delta y^{\alpha}}{P_t})\right]^{(m-i)}\end{aligned} \quad (35)$$

Owing to the property of Poisson process, conditional on the number of receivers $N=j$, each receivers i.i.d on $D\left(o,R\right)$ following a uniform distribution. Therefore the cumulative distribution function of r_i can be easily obtained:

$$\Pr[r_i\leq y]=\frac{y^2}{R^2},y\in[0,R] \quad (36)$$

and the probability density function of r_i is given by $\frac{2y}{R^2}$.

Using the total probability theorem, we can now derive $\Pr\left[\eta_i\left(m\right)\right]$ as:

$$\begin{aligned}\Pr\left[\eta_i\left(m\right)\right]=&\int_{y=0}^{y=R}\Pr\left[\eta_i\left(m\right)\mid r_i=y\right]\frac{2y}{R^2}dy\\
=&\sum_{n=k}^{m}[\frac{2\binom{m}{n}\phi(n)}{R^2}]\sum_{i=0}^{m-n}\tbinom{m-n}{i}(-1)^{m-n-i}\\
\times&\int_{y=0}^{y=R}y\left[\exp(-\frac{(m-i)N_o\delta y^{\alpha}}{P_t})\right]dy\end{aligned} \quad (37)$$

Further, the integral inside equation (37) can be computed:

$$\begin{aligned}&\int_{y=0}^{y=R}y\left[\exp(-\frac{(m-i)N_o\delta y^{\alpha}}{P_t})\right]dy\\
=&\left[\frac{\Gamma[\frac{2}{\alpha},\frac{(m-i)N_o\delta y^{\alpha}}{P_t}]}{\alpha\left(\frac{(m-i)N_o\delta}{P_t}\right)^{\frac{2}{\alpha}}}\right]_0^R\\
=&\frac{\Gamma[\frac{2}{\alpha},\frac{(m-i)N_o\delta R^{\alpha}}{P_t}]-\Gamma[\frac{2}{\alpha},0]}{\alpha\left(\frac{(m-i)N_o\delta}{P_t}\right)^{\frac{2}{\alpha}}}\end{aligned} \quad (38)$$

where $\Gamma(n,x)$ is the incomplete Gamma function.

Inserting the results of equations (30), (31), (37) and (38), into equation (29), we can obtain an upper bound and a lower bound on $\Pr(\eta\left(m\right))$, which is given in (39) and (40). Particularly, using the lower bound on $\Pr(\eta\left(m\right)$ in (40), the minimum number of transmissions required by the BS in order to meet the performance guarantee that $\Pr\left(\eta\right)\geq 1-\epsilon$ can be determined.

VI. Simulation Results

In this section, we use simulations to validate the accuracy of the analytical results and the tightness of the bounds. The simulations are conducted in a simulator written in Matlab. Each point shown in the figures is the average value obtained from 10000 simulations. The 95% confidence interval is shown in the figures too. The radius R is chosen to be 2.5 km. The receiver density is varied from $\lambda=10$ nodes/km^2 to $\lambda=100$ nodes/km^2. The number of source packets is chosen to be 5. The degree distribution of the RCs follows the widely used Luby's Ideal Soliton distribution [8]. Path-loss exponent is set to be $\alpha=2$. The transmitting power of the transmitter (BS) P_t is set to be 10 dBm and the thermal noise power density N_o is -80 dBm. The SINR threshold δ is set to be 0 dB. For comparison, the scenario that the BS broadcasts without using network coding is also shown in some figures. When the BS broadcasts without using network coding, the BS broadcasts the k source packets sequentially and repeat the process when the last source packet is broadcast. Theoretical analysis for the scenario that the BS broadcasts without using network coding is trivial compared with that using network coding and hence is not presented in the paper.

Analytical and simulation results are presented in Fig. 2 on the probability that all receivers successfully receive all 5

$$\Pr(\eta\,(m)) \leq \exp\left\{\lambda 2\pi \sum_{n=k}^{m} \binom{m}{n}\phi_u(n) \sum_{i=0}^{m-n} \binom{m-n}{i}(-1)^{(m-n-i)} \left[\frac{\Gamma[\frac{2}{\alpha}, \frac{(m-i)N_o\delta R^\alpha}{P_t}] - \Gamma[\frac{2}{\alpha}, 0]}{\alpha\left(\frac{(m-i)N_o\delta}{P_t}\right)^{\frac{2}{\alpha}}}\right] - \lambda\pi R^2\right\} \tag{39}$$

$$\Pr(\eta\,(m)) \geq \exp\left\{\lambda 2\pi \sum_{n=k}^{m} \binom{m}{n}\phi_l(n) \sum_{i=0}^{m-n} \binom{m-n}{i}(-1)^{(m-n-i)} \left[\frac{\Gamma[\frac{2}{\alpha}, \frac{(m-i)N_o\delta R^\alpha}{P_t}] - \Gamma[\frac{2}{\alpha}, 0]}{\alpha\left(\frac{(m-i)N_o\delta}{P_t}\right)^{\frac{2}{\alpha}}}\right] - \lambda\pi R^2\right\} \tag{40}$$

source packets as a function of the number of transmissions by the BS. As shown in Fig. 2, our analytical results, i.e., upper and lower bound, match the simulation results very well, which validate the accuracy of the analysis in this paper. However there is still a gap between the upper (lower) bounds and simulation results in the figures. The gap between the exact value and the upper bound is caused by the approximation used in equation (5) and the gap between the exact value and the lower bound is caused by equation (6).

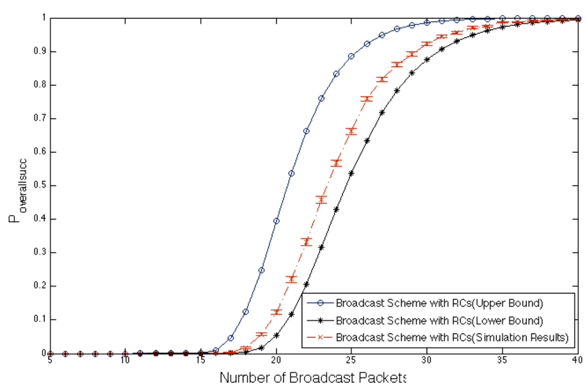


Figure 2. The probability of successfully decoding all 5 source packets by all receivers versus the number of coded packets broadcast by the BS.

In Fig. 3, we further compare the success probabilities of broadcast using RCs and without using network coding. As shown in Fig. 3, it can be seen that the use of RCs yields much better performance in terms of the number of transmitted packets required to meet the same performance objective on the probability of successful delivery (i.e. all receivers receive all source packets). In comparison, without using network coding, the BS needs to transmit more packets to meet the performance objective. For example, when the probability of successful delivery is set to be 0.947, at most 33 transmissions is needed when RCs are used, while 50 broadcasts are required when NC is not used, which represents a saving of 50% transmissions when using RCs.

Fig. 4 shows the system success probabilities of the proposed RCs based broadcast scheme as a function of the node density when the number of broadcast from the BS is fixed at 35. We can see that the simulation results match well with the theoretical results. Further, for all values of the node density, broadcast using network coding offers better performance than broadcast without using network coding. We also can observe that as the node density increases the gaps between the upper

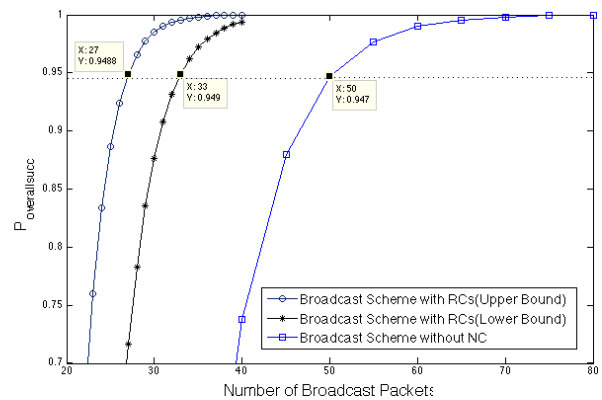


Figure 3. The probabilities of successfully decoding all 5 source packets by all receivers for broadcast scheme using RCs and that without NC as a function of the number of packets broadcast by the BS

and the lower bounds become bigger. This is because that the differentiation of the gap of the bounds is a positive value when $0 \leq \lambda \leq \lambda_c$, where λ_c is a positive number and can be easily calculated.

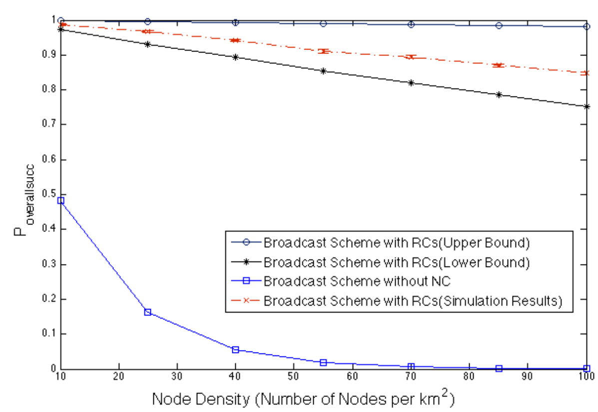
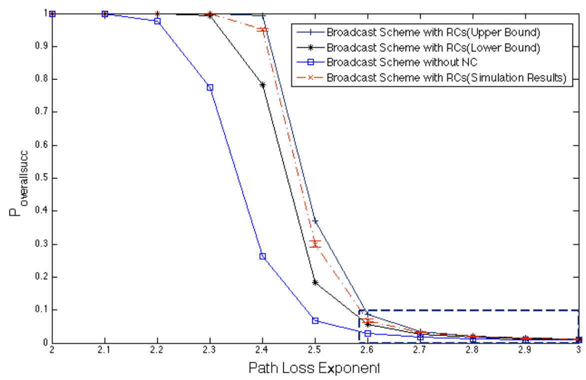


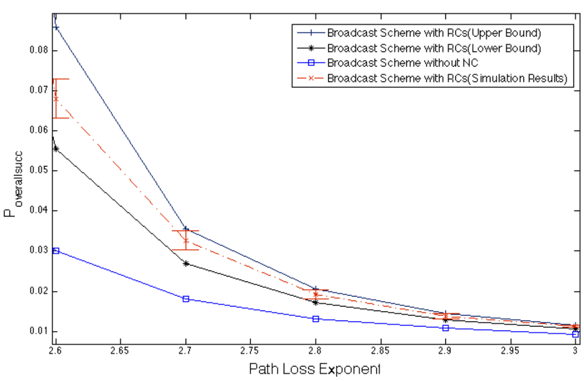
Figure 4. The probabilities of successfully decoding all 5 source packets by all receivers for broadcast scheme using RCs and that without NC as a function of the node density.

The variation of the system success probabilities of the proposed RCs based broadcast scheme with the path loss exponent is demonstrated in Fig. 5(a) and Fig. 5(b). The number of source packets and number of broadcast from the BS are set to be 15 and 75, respectively. The radius R is chosen to be 400 m. The receiver density is set to be

$\lambda = 10$ nodes/km^2. The transmitting power of the transmitter (BS) P_t is set to be -18 dBm and the thermal noise power density N_o is -80 dBm. We can observe that the simulation results lie between the upper and lower bound, i.e., **are** consistent with the theoretical results. Further, for all values of the path loss exponent, broadcast using network coding outweighs the performance of broadcast without using network coding.



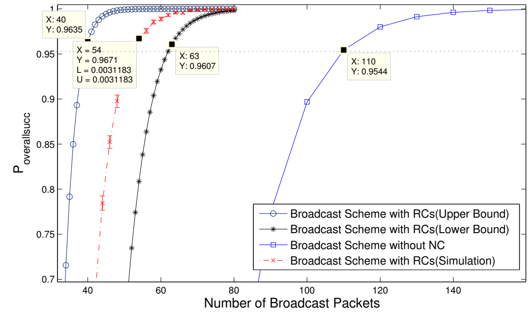
(a) Full Scale



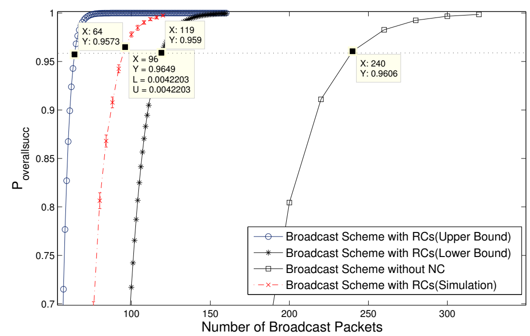
(b) Zoom of the dotted rectangular box in (a)

Figure 5. The probabilities of successfully decoding all 15 source packets by all receivers for broadcast scheme using RCs and that without NC ***vs*** the path loss exponent.

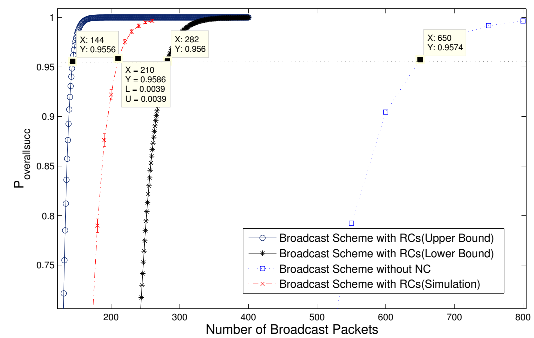
When the number of source packets increases, the conclusion that the use of rateless codes can significantly reduce the number of transmissions required to meet the same performance objective, compared with that without using network coding still hold. As demonstrated in Fig. 6(a), 6(b), 6(c) and 6(d), compared with broadcasting without using network coding, the BS can reduce the number of transmissions required to meet the same performance objective, which leads to reduced transmission latency and energy consumption. When the performance objective, i.e., the probability of successful delivery, is set to 0.954, for k=10, the ratio of the number of packets transmitted without using NC to that using RCs equals 2.037; for k=20, the ratio is 2.5; for k=50, the ratio increases to 3.095; for k=100, the ratio becomes 3.5. It seems that the ratio increases as the number of source packets increases.



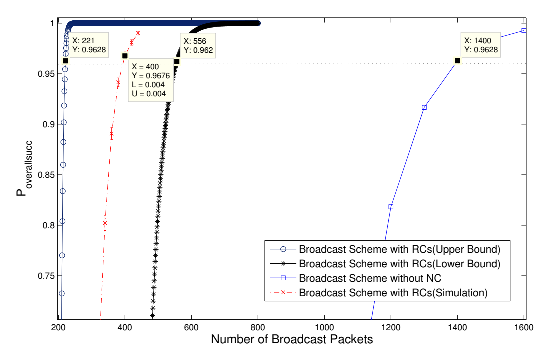
(a) $k = 10$



(b) $k = 20$



(c) $k = 50$



(d) $k = 100$

Figure 6. The probabilities of successfully decoding all source packets by all receivers for broadcast scheme using RCs and that without NC as a function of the number of packets broadcast by the BS

VII. Conclusion and Future Work

In this paper we studied reliable broadcast in a wireless network with a BS and a number of receivers. More specifically, we assume that the BS only has limited statistical information about the environment including the spatial distribution of users (instead of their exact locations and number) and the wireless propagation model. By resorting to stochastic geometry analysis, a rateless code based broadcast scheme was designed that allows the BS to broadcast a given number of source packets to a large number of users, without user acknowledgment, while being able to provide a performance guarantee on the probability of successful delivery. The scheme is based on a rigorous analysis on the probability of successful delivery using rateless codes. An upper and a lower bound on the probability that all receivers successfully decode all source packets from the BS were derived. On that basis, an upper and a lower bound of the number of transmissions required for a guaranteed performance on the probability of successful delivery was obtained. Simulations were conducted to validate the accuracy of the theoretical analysis. It was shown that the use of rateless codes can significantly reduce the number of transmissions required to meet the same performance objective, compared with that without using network coding. The technique and analysis developed in this paper can be useful for designing broadcast strategies to deliver information of common interest to a large number of users efficiently and reliably.

In the future, we plan to explore the optimum degree distribution design of the rateless codes for wireless broadcast and the use of cooperations among receivers and network coding to facilitate broadcast.

Appendix A
Proof of Lemma 2

We observe that $\mathbf{G}_{z\times k}$ being full rank implies that there does *not* exist a set of coefficients $c_1,\ldots,c_r$ such that $\sum_{i=1}^{r} c_i\mathbf{v}_i = 0$. Further, since we are working in a binary field, c_i can be either 1 or 0. It follows that $\mathbf{G}_{z\times k}$ being full rank is a sufficient and necessary condition for that for every integer $2 \le q \le r$, the summation of any q row vectors of $\mathbf{G}_{z\times k}$ is not equal to $\mathbf{0}$. This observation forms the basis of the proof.

Let $NZ(q)$ be the event that the summation of any q row vectors in $\mathbf{G}_{z\times k}$ are not equal to $\mathbf{0}$. The probability of $NZ(2)$ can be expressed as $\Pr[NZ(2)] = (1-I_2)^{\binom{r}{2}}$. Further, for every integer q satisfying $3 \le q \le r$,

$$\Pr\left[\cap_{i=2}^{q} NZ(i)\right] = \Pr\left[NZ(q) \mid \cap_{i=2}^{q-1} NZ(i)\right] \Pr\left[\cap_{i=2}^{q-1} NZ(i)\right] \tag{41}$$

With the recursive application of equation (41), we can conclude that the probability that $\mathbf{G}_{z\times k}$, $z \le k$, is of full rank can be obtained as

$$\Pr\left[R_z^z\right] = \Pr(\cap_{i=2}^{z} NZ(i)) = \prod_{q=2}^{z}\left[(1-I_q)^{\binom{z}{q}}\right] \tag{42}$$

Appendix B
Proof of Lemma 3

To obtain I_q, we analyze the degree distribution of row vector $\mathbf{S}_w$ which is the sum of w row vectors. Note that the degree of $\mathbf{S}_w$ should not equal to 0. Let $\mathbf{D}^w = (D_1^w,\ldots,D_k^w)^T$ be the degree distribution of the sum of w (random) row vectors and $w \ge 1$, where D_i^w is the probability that the degree of the row vector $\mathbf{S}_w$ is i, $1 \le i \le k$. When $w=1$, the degree distribution $\mathbf{D}^1$ is obviously $(\Omega_1,\Omega_2,\ldots,\Omega_k)^T$. For $w \ge 2$, the relationship can be analytically described as :

$$D_m^w = (Q_{1m}, Q_{2m}, \ldots, Q_{km})(D_1^{w-1},\ldots,D_k^{w-1})^T \tag{43}$$

From the equation (43), it follows that:

$$\begin{aligned}\mathbf{D}^w &= (D_1^w,\ldots,D_k^w)^T \\ &= \begin{pmatrix} Q_{11} & \cdots & Q_{(k-1)1} & Q_{k1} \\ \vdots & \ddots & \vdots & \vdots \\ Q_{1(k-1)} & \cdots & Q_{(k-1)(k-1)} & Q_{k(k-1)} \\ Q_{1k} & \cdots & Q_{(k-1)k} & Q_{kk}\end{pmatrix}\begin{pmatrix} D_1^{w-1} \\ \vdots \\ D_{k-1}^{w-1} \\ D_k^{w-1}\end{pmatrix} \\ &= \mathbf{Tr}^{w-1}\cdot(\Omega_1,\Omega_2,\ldots,\Omega_k)^T \end{aligned} \tag{44}$$

As an easy consequence of equation (44), I_q can be obtained:

$$\begin{aligned} I_q &= D_0^q = \sum_{i=1}^{k} D_i^{q-1} Q_{i0} = (Q_{10}, Q_{20}, \ldots, Q_{k0})\mathbf{D}^{q-1} \\ &= (Q_{10}, Q_{20}, \ldots, Q_{k0})\mathbf{Tr}^{q-2}\cdot(\Omega_1,\Omega_2,\ldots,\Omega_k)^T \end{aligned} \tag{45}$$

References

[1] I. H. Hou and P. R. Kumar, "Broadcasting delay-constrained traffic over unreliable wireless links with network coding," in Proceedings of the 12th ACM MobiHoc, 2011, pp. 1-10.

[2] S. Katti, H. Rahul, W. Hu, D. Katabi, M. Medard, and J. Crowcroft, "Xors in the air: practical wireless network coding," IEEE/ACM Trans. Netw., vol. 16, no. 3, pp. 497-510, 2008.

[3] S.-C. Liew, S. Zhang and L. Lu "Physical-layer network coding: Tutorial, survey, and beyond", Phys. Commun., 2011 [online] Available: http://arxiv.org/abs/1105.4261

[4] L. Song, G. Hong, B. Jiao and M. Debbah "Joint relay selection and analog network coding using differential modulation in two-way relay channels", IEEE Trans. Veh. Technol., vol. 59, no. 6, pp.2932 -2939, 2010

[5] S. Zhang, and S.-C. Liew, "Joint design of physical layer network coding and channel coding,", Technique Report. 2008 [online] Available::http://arxiv.org/ ftp/arxiv/papers/0807/0807.4770.pdf

[6] N. Dong, T. Tuan, N. Thinh, and B. Bose, "Wireless broadcast using network coding," IEEE Trans. Vehicular Technology, vol. 58, no. 2, pp. 914-925, 2009.

[7] H. D. T. Nguyen, T. Le-Nam, and H. Een-Kee, "On transmission efficiency for wireless broadcast using network coding and fountain codes," IEEE Commu Letters, vol. 15, no. 5, pp. 569-571, 2011.

[8] M. Luby, "LT codes," in Proceedings of the 43rd IEEE FOCS, 2002, pp. 271-280.

[9] A. Shokrollahi, "Raptor codes," IEEE Trans. Inf. Theory, vol. 52, no. 6, pp. 2551-2567, 2006.

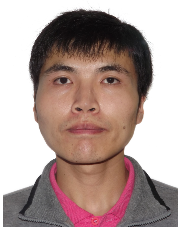
[10] E. Hyytia, T. Tirronen, and J. Virtamo, "Optimal degree distribution for LT codes with small message length," in Proceedings of 26th IEEE INFOCOM, 2007, pp. 2576-2580.

[11] L. Feng, F. Chuan Heng, C. Jianfei, and C. Liang-Tien, "LT codes decoding: Design and analysis," in Proceedings of IEEE ISIT, 2009, pp. 2492-2496.

[12] W. Xiao, S. Agarwal, D. Starobinski and A. Trachtenberg, "Reliable Rateless Wireless Broadcasting With Near-Zero Feedback," IEEE/ACM Trans. Netw. vol. 20, no. 6, pp. 1924-1937, Dec. 2012

[13] R. Karp, M. Luby, and A. Shokrollahi, "Finite length analysis of LT codes," in Proceedings of IEEE ISIT, 2004, p. 39.

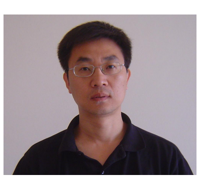
[14] A. Shokrollahi, Theory and applications of Raptor codes. Springer-Verlag, 2009.
[15] A. Tukmanov, D. Zhiguo, S. Boussakta, and A. Jamalipour, "On the broadcast latency in finite cooperative wireless networks," IEEE Transactions on Wireless Communications, vol. 11, no. 4, pp. 1307-1313, 2012.
[16] T. Rappaport, *Wireless Communications: Principles and Practice,* ser. Prentice Hall Communication Engineering and Emerging Technologies. Prentice Hall PTR, 2002.
[17] S. Rajabi, M. Shahabadi, and M. ArdebiliPoor, "Modeling of the correlation coefficients of a receive antenna array in a MIMO multipath channel," in Proceedings of 2nd IEEE/IFIP ICI, 2006, pp. 1-4.
[18] D. J. Daley and D. Vere-Jones, *An Introduction to the Theory of Point Processes,* 2nd ed., ser. Probability and its Applications. Verlag, 2003, vol. I.



Peng Wang received the B.Sc. degree in applied electronics from Beijing University of Aeronautics and Astronautics, Beijing, China, in 2009, and the M.Eng. degree in telecommunications in 2012 from the Australian National University, Canberra, ACT, Australia. He is currently working towards the PhD degree in Engineering at The University of Sydney.

He is also with the Sydney Research Laboratory, National ICT Australia. His research interests include wireless broadcast networks, heterogeneous networks, graph theory, and its application in networking, channel/network coding, 5G cellular systems, etc.

Guoqiang Mao(S'98-M'02-SM'08) received PhD in telecommunications engineering in 2002 from Edith Cowan University. He currently holds the position of Professor of Wireless Networking, Director of Center for Real-time Information Networks at the University of Technology, Sydney. He has published more than 100 papers in international conferences and journals, which have been cited more than 2500 times. His research interest includes intelligent transport systems, applied graph theory and its applications in networking, wireless multihop networks, wireless localization techniques and network performance analysis.



Zihuai Lin received the Ph.D. degree in Electrical Engineering from Chalmers University of Technology, Sweden, in 2006. Prior to this he has held positions at Ericsson Research, Stockholm, Sweden. Following Ph.D. graduation, he worked as a Research Associate Professor at Aalborg University, Denmark and currently at the School of Electrical and Information Engineering, the University of Sydney, Australia. His research interests include graph theory, source/channel/network coding, coded modulation, MIMO, OFDMA, SC-FDMA, radio resource management, cooperative communications, small-cell networks, 5G cellular systems, etc.

Xiaohu Ge (M'09-SM'11) received the Ph.D. degree in communication and information engineering from Huazhong University of Science and Technology (HUST), Wuhan, China, in 2003. From January 2004 to October 2005, he was a Researcher with Ajou University, Suwon, Korea, as well as with Politecnico di Torino, Turin, Italy. From June to August 2010, he was a Visiting Researcher with Heriot-Watt University, Edinburgh, U.K. Since November 2005, he has been with HUST, where he is currently a Professor with the Department of Electronics and Information Engineering, HUST. From January 2013, he was granted as a Huazhong scholarship professor. He is the author of about 90 papers in refereed journals and conference proceedings and holds 15 patents in China. Dr. Ge is a Senior Member of the China Institute of Communications and a member of the National Natural Science Foundation of China and the Chinese Ministry of Science and Technology Peer Review College. He has been actively involved in organizing more the ten international conferences since 2005. He served as the Executive Chair for the 2013 IEEE International Conference on Green Computing and Communications (IEEE GreenCom) and as the Cochair of the Workshop on Green Communication of Cellular Networks at the 2010 IEEE GreenCom. He serves as an Associate Editor for the IEEE ACCESS, Wireless Communications and Mobile Computing Journal (Wiley), the International Journal of Communication Systems (Wiley), etc. Moreover, he served as the guest editor for IEEE Communications Magazine Special Issue on 5G Wireless Communication Systems and ACM/SpringMobile Communications and Application Special Issue on Networking in 5G Mobile Communication Systems. He received a Best Paper Award from the 2010 IEEE Global Communications Conference.

Brian D. O Anderson (M'66-SM'74-F'75-LF'07) was born in Sydney, Australia, and educated at Sydney University in mathematics and electrical engineering, with PhD in electrical engineering from Stanford University in 1966. He is a Distinguished Professor at the Australian National University and Distinguished Researcher in National ICT Australia. His awards include the IEEE Control Systems Award of 1997, the 2001 IEEE James H Mulligan, Jr Education Medal, and the Bode Prize of the IEEE Control System Society in 1992, as well as several IEEE and other best paper prizes. He is a Fellow of the Australian Academy of Science, the Australian Academy of Technological Sciences and Engineering, the Royal Society, and a foreign member of the US National Academy of Engineering. He holds honorary doctorates from a number of universities, including UniversitÃ© Catholique de Louvain, Belgium, and ETH, ZÃŒrich. He is a past president of the International Federation of Automatic Control and the Australian Academy of Science. His current research interests are in distributed control, sensor networks and econometric modelling.