Uncoordinated Cooperative Communications in Highly Dynamic Wireless Networks

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***Abstract*—Cooperative communication techniques offer significant performance benefits over traditional methods that do not exploit the broadcast nature of wireless transmissions. Such techniques generally require advance coordination among the participating nodes to discover available neighbors and negotiate the cooperation strategy. However, the associated discovery and negotiation overheads may negate much of the cooperation benefit in mobile networks with highly dynamic or unstable topologies (e.g. vehicular networks). This paper discusses *uncoordinated* cooperation strategies, where each node overhearing a packet decides *independently* whether to retransmit it, without any coordination with the transmitter, intended receiver, or other neighbors in the vicinity. We formulate and solve the problem of finding the optimal uncoordinated retransmission probability at every location as a function of only *a priori* statistical information about the local environment, namely the node density and radio propagation model. We show that the solution consists of an optimal *cooperation region* which we provide a constructive method to compute explicitly. Our numerical evaluation demonstrates that uncoordinated cooperation offers a low-overhead viable alternative, especially in high-noise (or low-power) and high node density scenarios.**

***Index Terms*—Cooperative communication, coordination, random networks.**

I. Introduction

Cooperative communication in wireless networks has attracted considerable research attention in recent years. Unlike the traditional layered approach, where the route between a source and destination is determined in advance and each node along the route is solely responsible for delivering the respective packets to its next hop, cooperative communication methods allow additional nodes in the vicinity of the route that overhear the transmitted data to assist in delivering it to its destination, leveraging the broadcast nature of the medium to provide diversity against time-varying link fades and outages.

The literature on cooperative communication can be broadly classified into physical-layer and network-layer techniques. Physical-layer cooperation, or *cooperative relaying*, includes either amplification of the source signal (*amplify-and-forward*) or retransmission of the packet after decoding it (*decode-and-forward*) [1]; in the case of multiple-relay cooperation, the relay signals may be multiplexed on orthogonal CDMA [2] or TDMA [3] subchannels. Network-layer cooperation provides *path diversity* to the route of each packet via *opportunistic routing* where each packet is broadcast at every hop and the next-hop node is then decided opportunistically among the successful recipients of the broadcast. This decision can be based on an explicit handshake of acknowledgments or RTS/CTS exchanges [4], or take an implicit form with nodes waiting for a random period before forwarding the packet, and discarding it if they overhear it being forwarded from another neighbor in the meantime [5].

A common feature in existing cooperative techniques is the *coordination* required among the participating neighbors. For physical-layer cooperation, the initial set-up generally requires the discovery of neighbors in the vicinity, the collection of channel information to these neighbors, and the selection of the best neighbor(s) whose cooperation will maximize the performance improvement. The optimization of the relay selection and coordination process have been the subject of several recent studies [6]–[8]. For opportunistic routing, coordination is required at every hop to decide the node that will serve as the packet's next hop towards the destination. Approaches such as in [5], which trade coordination message overhead for a higher latency, are effective when employed on large batches of packets that the latency can be amortized over. However, such large batches are generally impractical in highly dynamic ad hoc networks, which are the focus of this paper.

Due to associated coordination overheads, existing cooperative methods are suitable mostly for mesh or sensor networks with static or relatively stable topologies. They are not useful when the topology is very dynamic, due to either a high velocity (e.g. vehicular networks) or a high density of the nodes (e.g. networks of mobile devices carried by people on a busy street or conference hall). Indeed, if a network is highly dynamic, the coordination overheads are incurred too frequently to be practical even just to maintain an up-to-date view of the neighbor topology, let alone an up-to-date channel state information to the neighbor nodes. Motivated by the above observation, we consider an *uncoordinated cooperative retransmission* framework, where cooperative nodes overhearing a packet make retransmission decisions *independently* — with no prior coordination or measurement of real-time channel information to other nearby nodes, and even without being aware of their existence (apart from the transmitter and

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receiver of the packet). Thus, the decisions at each node (e.g. whether or not to retransmit an overheard packet, and the transmission power or modulation to use) may only be based on the location of that node, the locations of the sender and receiver, and some limited prior statistical knowledge about the local environment, namely, the spatial distribution of the nodes and radio propagation characteristics.

The concept of uncoordinated probabilistic forwarding arises in many other networking contexts, such as probabilistic flooding or *gossip* protocols [9] and opportunistic forwarding in delay-tolerant networks [10]. It frequently leads to distributed optimization of global performance objectives (e.g. delivery probability or latency) by tuning the transmission probabilities at the individual nodes. A discussion and a detailed solution approach of this distributed optimization problem is provided in [11] in the context of stochastic routing, which is closely related to the framework of this paper. However, the convergence process to the optimal solution in [11] requires a number of information exchanges between neighboring nodes (increasing in the size of the network), which renders it unsuitable for highly dynamic scenarios where the topology may change before the algorithm can converge. On the other hand, our framework in this paper focuses on solving such optimization problems and finding optimal transmission probabilities using *a priori* information only, with no exchanges of location and connectivity information among the cooperative nodes in real time.

Our contributions in this paper are as follows. We introduce an optimization framework for cooperative communication when the instantaneous topology and channel states are not known to the nodes (beyond an *a priori* stochastic model). We present a detailed analysis of the optimal uncoordinated cooperation (i.e. one that maximizes the probability of successful packet delivery to the receiver) where each node applies the cooperation probability independently. Our analysis reveals that the optimal cooperation strategy is for any node to retransmit a packet if and only if it belongs to a certain *cooperation region*, determined by a threshold of the link quality from that node to the receiver, and provides a constructive method to calculate that threshold. Finally, we evaluate the performance of uncoordinated cooperation in a variety of scenarios based on a realistic propagation model, and demonstrate the viability of the proposed technique, especially in the regime of high noise (or low transmission power) and high node density.

The rest of this paper is structured as follows. Section II describes the network model and formulates the functional optimization problem, which is then analytically solved in Section III. The numerical study that demonstrates the performance of the proposed method is presented in Section IV. Finally, Section V concludes the paper.

II. Network Model and Problem Formulation

We consider an ad-hoc network where nodes are randomly distributed in a given region V, following a generalized Poisson point process with a known node density $\rho(v), v \in V$. Here V and $\rho(v)$ can be either two- or three-dimensional, and $\rho(v)$ may or may not be constant throughout the region V. We assume that the density distribution is time-stationary and ergodic; this is a standard assumption that generally holds under broad assumptions, e.g., for random direction mobility models [12], [13].

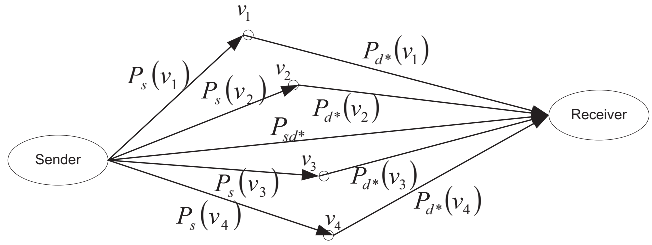


Fig. 1. The system model and notation of link quality parameters.

We focus on the transmission of a packet from a sender towards a receiver at a known location within a one-hop distance. The receiver's location is provided by the higher-layer protocol that initiates the transmission; e.g., it can be available naturally in the case of geocasting, or obtained in advance via some prior handshaking process. In any case, we assume the locations of the sender and the receiver do not change significantly during the short cooperative transmission period, which starts from the initial transmission of a packet by the sender and ends with its successful reception at the receiver, either directly from the sender or from one of the cooperative neighbors. Further, we assume that each node knows its own location; this can be obtained either from an embedded GPS receiver, which is becoming increasingly ubiquitous in many mobile devices and vehicles, or through a localization technique based on signal strength or angle-of-arrival measurements with nearby nodes. From the above assumptions, the locations of both the sender and receiver can be readily advertised to nodes overhearing the transmission, by piggybacking the locations within the transmitted packet.

Figure 1 illustrates a simple two-dimensional example scenario for the proposed retransmission model. A sender attempts to deliver a packet to a receiver, without success. The packet can be overheard by some randomly located nodes, who will then retransmit it with a location-dependent probability $\tau(v)$ independent of the retransmission decisions of other nodes, where v denotes the location of a node. We refer to $\tau(v)$ as the *strategy* function. We assume that each node has limited prior knowledge about the network environment, which includes its own location, the spatial distribution of other nodes in its vicinity, i.e. the node density $\rho(v)$, and the wireless propagation model. The goal of our analysis will be to find the optimum function $\tau(v)$ that maximizes the probability of successful delivery to the receiver, and depends only on the limited information that is locally available at every node.

We now introduce the notation to describe the wireless environment in the vicinity of a given sender and receiver. Assuming that a node exists at some location $v \in V$, it will overhear the transmission from the source with probability $P_s(v)$. We then assume that the wireless link from v to the destination can fall under one of the following three categories:

- *No link*, i.e., a transmission made by v will not be received (or received with a negligible power) by the destination;
- *Interfering link*, i.e., a transmission made by v will be received by the destination at a power that is not sufficient

to decode the information, but high enough to prevent the destination from receiving a concurrent transmission from any other node;

- *Strong link*, i.e., a transmission made by v (in the absence of other concurrent transmissions) will be received with a sufficient power to be successfully decoded.

We denote the probability of the wireless link from location v to the destination node to be nonexistent, interfering, or strong by $P_{d0}(v)$, $P_{d1}(v)$, and $P_{d2}(v)$, respectively, where $P_{d0}(v) + P_{d1}(v) + P_{d2}(v) = 1$ for all $v \in V$. Note that the overall probability of the destination node to receive the retransmitted packet successfully will depend on the *collective* behavior of multiple nodes, i.e. on the strategy function $\tau(v)$. Specifically, a successful reception will require one transmission from a node with a strong link, and no other simultaneous transmissions from nodes with either a strong or an interfering link. The formula connecting between the aforementioned probabilities will be derived in the next subsection.

Remark. There are two interference models that are widely used in the literature on wireless networks. Our definitions of "interfering" and "strong" links correspond to the *protocol model* [14]; the alternative *physical model* defines a transmission to be successful if the signal-to-interference-and-noise ratio (SINR) at the receiver is above a certain threshold. While the physical model is considered more realistic, since it accounts for the accumulation of interference, it is typically much harder to analyze. Several recent studies have confirmed that insights and solutions based on the protocol model are adequate in many application scenarios under rather general conditions [14]–[16]. Accordingly, we focus on the protocol model in this paper, and leave the necessary extensions for the SINR-based model to future work.

The above notation is quite generic and provides a framework that can be applied in a wide variety of scenarios. In the most common application, the functions may be used to describe general propagation models that assume only minimal knowledge of the environment. For example, a simple double-disk distance-based model might be described by $P_{d2}(v) = 1$ for any v within a certain "transmission range" from the receiver (i.e. in a sphere around the receiver node), and $P_{d2}(v) = 0$ elsewhere; $P_{d1}(v) = 1$ within some further "interference range" beyond the transmission range, and finally $P_{d0}(v) = 1$ beyond the maximum interference range. More generally, these probabilities may take any values between 0 and 1, and need not necessarily be isotropic (i.e. based solely on distance from the destination). Note that a link can fall under the "interfering" category even if the received signal is strong but distorted for any reason, such as Doppler shift or large delay spread due to multi-path fading.

We emphasize that the functions $P_s(v)$ and $P_{d*}(v)$[†] describe the *a priori* wireless propagation characteristics of the environment, and represent the *long-term*, time-averaged values, i.e. probability values when there is no additional information about the instantaneous or recent state of the respective wireless channels. These functions do not use any local information that is only available to the nodes in real time, such as channel state information (CSI) or the actual locations of nearby relay nodes. Accordingly, they can be determined in advance and are assumed to be commonly known to the nodes without the need for any real-time coordination. Alternatively, the calculations described henceforth of the optimal retransmission strategy, based on the above functions, can be performed at the source node and conveyed within the transmitted packet itself. We discuss the practical implications of our approach further in subsection III-C.

[†]We will use the wildcard notation $P_{d*}(v)$ as an abbreviation to refer to the functions $P_{d0}(v)$, $P_{d1}(v)$, $P_{d2}(v)$ as a group.

A. Problem formulation

Consider the success probability of a cooperative retransmission attempt in the above model, i.e. the probability that the packet can be successfully decoded by the receiver. Since all retransmissions immediately follow an unsuccessful one by the source and are uncoordinated, collisions are possible, resulting in an ultimate failure even if some of the individual retransmissions are made over strong links. Accordingly, we now derive the functionals that show how the success probability depends on the retransmission strategy $\tau(v)$. As $\tau(v)$ is itself a function of a continuous variable, this will therefore lead to the functional optimization problem of finding the strategy $\tau(v)$ that maximizes the retransmission success probability.

To that end, we divide the space V into a lattice of small cubes of size Δv and assume that nodes can only be placed in the centers of the cubes (i.e. in discrete locations), such that the probability of a node existing in the cube containing location v is $\rho(v)\Delta v + o(\Delta v)$. The requirement for a successful cooperative retransmission is that there is precisely one node which: (I) overhears the packet (with probability $P_s(v)$), (II) makes a retransmission (with probability $\tau(v)$), *and* (III) has a strong link to the destination (with probability $P_{d2}(v)$), while all other nodes either do not overhear the packet, do not retransmit it, or have no link to the destination:

$$P_{suc} = \sum_v P_s(v)\tau(v)P_{d2}(v)\rho(v)\Delta v \cdot \prod_{v' \neq v} \left[1 - P_s(v')\tau(v')\left(P_{d1}(v') + P_{d2}(v')\right)\rho(v')\Delta v\right] \quad (1)$$

As $\Delta v \to 0$, we obtain[‡]

$$\begin{aligned}
&\lim_{\Delta v \to 0} \prod_{v' \neq v} \left[1 - P_s(v')\tau(v')\left(P_{d1}(v') + P_{d2}(v')\right)\rho(v')\Delta v\right] \\
&= \lim_{\Delta v \to 0} \exp\left\{ \sum_{v' \neq v} \log\left[1 - P_s(v')\tau(v') \cdot \left(P_{d1}(v') + P_{d2}(v')\right)\rho(v')\Delta v\right] \right\} \\
&= \lim_{\Delta v \to 0} \exp\left\{ - \sum_{v' \neq v} \left[P_s(v')\tau(v')\left(P_{d1}(v') + P_{d2}(v')\right) \cdot \rho(v')\Delta v + o(\Delta v)\right] \right\} \\
&= \exp\left\{ - \int_{v' \in V} P_s(v')\tau(v')\left(P_{d1}(v') + P_{d2}(v')\right)\rho(v')dv' \right\},
\end{aligned}$$

[‡]For the subsequent limits and integral to exist as $\Delta v \to 0$, we henceforth implicitly assume that the functions $\rho(v)$, $\tau(v)$, $P_s(v)$, and $P_{d*}(v)$ are continuous and differentiable almost everywhere (i.e. except in a zero-measure set of points).

i.e. the functional becomes independent of v (reflecting the infinitesimal impact of a single excluded point in a continuous space). Therefore, we can drop the distinction between v and v', and the expression for the success probability (1) becomes

$$P_{suc} = \int_{v\in V} P_s(v)\tau(v)P_{d2}(v)\rho(v)dv \cdot \exp\left\{-\int_{v\in V} P_s(v)\tau(v)\left(P_{d1}(v)+P_{d2}(v)\right)\rho(v)dv\right\}. \quad (2)$$

We now consider the possibility that the original source node takes part in the retransmission as well, immediately after the failure of the initial transmission. To that end, we denote by P_{sd0}, P_{sd1}, and P_{sd2} the probabilities of the direct link between the source and destination to be nonexistent, interfering, or strong at the time of the retransmission (where $P_{sd0}+P_{sd1}+P_{sd2}=1$), and we assume these probabilities are independent of those of all other links. We point out that P_{sd*}† should be interpreted as the probabilities of the direct link state immediately after the known failure of the original transmission. Accordingly, these probability values may also depend on the temporal fading characteristics of the direct source-destination link, and be different from the respective long-term *a priori* probabilities of that link.

With the same requirement as before for the retransmission to succeed, if the retransmission probability (strategy) of the source node is τ_s, the success probability expression becomes

$$P_{suc} = \left[\tau_s P_{sd2} + (1-\tau_s(1-P_{sd0}))\cdot \int_{v\in V} P_s(v)\tau(v)P_{d2}(v)\rho(v)dv\right]\cdot \exp\left\{-\int_{v\in V} P_s(v)\tau(v)\left(P_{d1}(v)+P_{d2}(v)\right)\rho(v)dv\right\}. \quad (3)$$

To summarize, we state the functional optimization problem that is the subject of the analysis in the next section:

Maximize P_{suc} (given by (3))
subject to $0\le\tau_s\le 1$ and $0\le\tau(v)\le 1$, $v\in V$.

III. Optimal Retransmission Strategy Analysis

In this section, we first derive the solution of the generic functional optimization problem defined above and prove some of its structural properties. We then consider in greater detail two special extreme cases, namely, when either $P_{d0}(v)=0$ or $P_{d1}(v)=0$ for all $v\in V$. The section is finally concluded with a discussion of some practical considerations and implications of the optimization solution properties.

A. Optimization analysis and solution

We begin by observing that expression (3) is linear in τ_s; hence, its maximum will always be attained at either $\tau_s=0$ or $\tau_s=1$. Accordingly, we consider these two options separately, beginning with $\tau_s=0$.

If $\tau_s=0$, then the probability of successful retransmission is given by (2). We tackle the maximization of (2) by considering it as the limit of a discrete problem. Accordingly, we again divide the space V into a lattice of small cubes of size Δv as above; thus, the probability of successful retransmission is given by (1). We now calculate the partial derivative of (1) with respect to a single variable $\tau(v_0)$, corresponding to a particular location v_0:

$$\frac{\partial P_{suc}}{\partial\tau(v_0)} = P_s(v_0)P_{d2}(v_0)\rho(v_0)\Delta v\cdot \prod_{v'\neq v_0}\left[1-P_s(v')\tau(v')\left(P_{d1}(v')+P_{d2}(v')\right)\rho(v')\Delta v\right] + \sum_{v''\neq v_0} P_s(v'')\tau(v'')P_{d2}(v'')\rho(v'')\Delta v\cdot \left[-P_s(v_0)\left(P_{d1}(v_0)+P_{d2}(v_0)\right)\rho(v_0)\Delta v\right]\cdot \prod_{v'\neq v_0,v''}\left[1-P_s(v')\tau(v')\left(P_{d1}(v')+P_{d2}(v')\right)\rho(v')\Delta v\right] \quad (4)$$

We observe that the derivative does not depend on $\tau(v_0)$ itself; therefore, if the sign of (4) at v_0 is positive, then P_{suc} will be maximized by setting $\tau(v_0)=1$; conversely, if it is negative, the optimal setting will be $\tau(v_0)=0$.

In the limit of $\Delta v\to 0$, the derivative expression (4) becomes

$$\lim_{\Delta v\to 0}\frac{1}{\Delta v}\frac{\partial P_{suc}}{\partial\tau(v_0)} = P_s(v_0)\rho(v_0)\left[P_{d2}(v_0) - \left(P_{d1}(v_0)+P_{d2}(v_0)\right)\int_{v\in V} P_s(v)\tau(v)P_{d2}(v)\rho(v)dv\right]\cdot \exp\left\{-\int_{v\in V} P_s(v)\tau(v)\left(P_{d1}(v)+P_{d2}(v)\right)\rho(v)dv\right\}, \quad (5)$$

and its sign is therefore determined by the sign of the part in brackets on the right-hand side of (5), namely $\left[P_{d2}(v_0) - \left(P_{d1}(v_0)+P_{d2}(v_0)\right)\int_v P_s(v)\tau(v)P_{d2}(v)\rho(v)dv\right]$.

Repeating the process for every location $v_0\in V$, we obtain a set of conditions for $\tau(v)$ that must be satisfied simultaneously in all locations in the network. This is formally stated in the following lemma.

Lemma 1. *If the function $\tau(v)$ satisfies*
$$\begin{cases}\tau(v)=1 & \textit{if } \frac{P_{d2}(v)}{P_{d1}(v)+P_{d2}(v)} > \int_{v\in V} P_s(v)\tau(v)P_{d2}(v)\rho(v)dv\\ \tau(v)=0 & \textit{if } \frac{P_{d2}(v)}{P_{d1}(v)+P_{d2}(v)} < \int_{v\in V} P_s(v)\tau(v)P_{d2}(v)\rho(v)dv\end{cases}$$
then it maximizes P_{suc} for $\tau_s=0$ (as given by (2)*).*

Unfortunately, Lemma 1 does not lead to an explicit solution of $\tau(v)$, since the condition is stated in a recursive fashion. The following theorem provides the final link to that end.

Theorem 1. *Expression* (2) *is maximized by a function $\tau(v)$ that satisfies*

$$\begin{cases}\tau(v)=1 & \textit{if } \frac{P_{d2}(v)}{P_{d1}(v)+P_{d2}(v)} > T \textit{ or}\\ & \quad\frac{P_{d2}(v)}{P_{d1}(v)+P_{d2}(v)} = T \textit{ and } v\in R_T;\\ \tau(v)=0 & \textit{if } \frac{P_{d2}(v)}{P_{d1}(v)+P_{d2}(v)} < T \textit{ or}\\ & \quad\frac{P_{d2}(v)}{P_{d1}(v)+P_{d2}(v)} = T \textit{ and } v\notin R_T.\end{cases} \quad (6)$$

where the threshold T and the region $R_T\subseteq\left\{v|\frac{P_{d2}(v)}{P_{d1}(v)+P_{d2}(v)}=T\right\}$ solve the integral equation

$$T = \int_{\left\{v|\frac{P_{d2}(v)}{P_{d1}(v)+P_{d2}(v)}>T\right\}\bigcup R_T} P_s(v)P_{d2}(v)\rho(v)dv. \quad (7)$$

Proof: If $\tau(v)$ satisfies the conditions in (6), then, by the definition of T and R_T, it is easily confirmed that it satisfies the conditions of Lemma 1.

†The wildcard notation P_{sd*} refers to the probabilities P_{sd0}, P_{sd1}, and P_{sd2} considered together as a group.

It remains to show that the integral equation (7) has a solution. Indeed, we observe that the integral $I(t) \triangleq \int_{\left\{v|\frac{P_{d2}(v)}{P_{d1}(v)+P_{d2}(v)}>t\right\}} P_s(v)P_{d2}(v)\rho(v)dv$ is positive and monotonously decreasing in t (since a larger t implies a smaller integration domain). Consequently, there exists a unique finite T that is the supremum of $\{t|t \leq I(t)\}$ (or, equivalently, the infimum of $\{t|t \geq I(t)\}$). It is possible that $I(t)$ is discontinuous at $t = T$, such that $I(T^+) = I(T) < T$ and $I(T^-) > T$. Accordingly, we define $\Delta I \triangleq I(T^-) - I(T) = \int_{\left\{v|\frac{P_{d2}(v)}{P_{d1}(v)+P_{d2}(v)}=T\right\}} P_s(v)P_{d2}(v)\rho(v)dv$. If $\Delta I = 0$ (no discontinuity), then, obviously, equation (7) is satisfied for any R_T. Otherwise, since the integrand $P_s(v)P_{d2}(v)\rho(v)$ is bounded, there must exist a region $R_T \subseteq \left\{v|\frac{P_{d2}(v)}{P_{d1}(v)+P_{d2}(v)} = T\right\}$ such that $\int_{R_T} P_s(v)P_{d2}(v)\rho(v)dv = T - I(T)$, and that will therefore be a solution of (7). ■

We point out that equation (7) can be solved numerically by a search for the solution of the equation $I(t) - t = 0$ (e.g. using Newton's method), followed by a separate search for the region R_T if the first search fails due to a discontinuity of $I(t)$. Moreover, we underscore that such a discontinuity is not possible under many common propagation models, where the set $\left\{v|\frac{P_{d2}(v)}{P_{d1}(v)+P_{d2}(v)} = T\right\}$ has measure zero for any T, and, therefore, the extra complication of finding the region R_T does not arise. Finally, we emphasize that Theorem 1 does not claim that the optimal strategy $\tau(v)$ is *unique*, as the target integral is not impacted by any variation of the solution over any set of measure 0.

The above analysis has considered the maximization of P_{suc} for $\tau_s = 0$. We now consider the option of $\tau_s = 1$. The same technique of calculating the partial derivative of the success probability with respect to $\tau(v_0)$ yields, in this case,

$$\lim_{\Delta v \to 0} \frac{1}{\Delta v}\frac{\partial P_{suc}}{\partial \tau(v_0)} = P_s(v_0)\rho(v_0) \cdot \Bigg\{ \Bigg[P_{d2}(v_0) - (P_{d1}(v_0) + P_{d2}(v_0)) \int_{v\in V} P_s(v)\tau(v)P_{d2}(v)\rho(v)dv \Bigg] P_{sd0} - (P_{d1}(v_0) + P_{d2}(v_0))\, P_{sd2} \Bigg\} \cdot \exp\left\{ -\int_{v\in V} P_s(v)\tau(v)\left(P_{d1}(v) + P_{d2}(v)\right)\rho(v)dv \right\}, \quad (8)$$

which in turn leads to the following result.

Theorem 2. *If $\tau_s = 1$, then expression* (3) *is maximized by a function $\tau(v)$ that satisfies*

$$\begin{cases} \tau(v) = 1 & \text{if } \frac{P_{d2}(v)}{P_{d1}(v)+P_{d2}(v)} > T \text{ or } \\ & \frac{P_{d2}(v)}{P_{d1}(v)+P_{d2}(v)} = T \text{ and } v \in R_T; \\ \tau(v) = 0 & \text{if } \frac{P_{d2}(v)}{P_{d1}(v)+P_{d2}(v)} < T \text{ or } \\ & \frac{P_{d2}(v)}{P_{d1}(v)+P_{d2}(v)} = T \text{ and } v \notin R_T. \end{cases} \quad (9)$$

where the threshold T and region $R_T \subseteq \left\{v|\frac{P_{d2}(v)}{P_{d1}(v)+P_{d2}(v)} = T\right\}$ are determined by the solution of the integral equation

$$T = \frac{P_{sd2}}{P_{sd0}} + \int_{\left\{v|\frac{P_{d2}(v)}{P_{d1}(v)+P_{d2}(v)}>T\right\}\bigcup R_T} P_s(v)P_{d2}(v)\rho(v)dv. \quad (10)$$

The proof of Theorem 2 is similar to Theorem 1, based on the sign of the right-hand side of (8) rather than (5).

To summarize, the uncoordinated cooperative retransmission strategy that maximizes the success probability (3) is given either by $\tau_s = 0$ and $\tau(v)$ that satisfies the conditions of Theorem 1, or by $\tau_s = 1$ and $\tau(v)$ derived from the conditions of Theorem 2. There is no apparent general way to state in general which of these two options will achieve the higher success probability; thus, the optimal strategy calculation for any given network instance should simply evaluate both and choose the better one under the respective parameter settings.

B. Special cases

In this subsection we consider two special cases of the above analysis that are useful in practice and allow the results to be presented in a simpler form.

The first special case is when $P_{sd0} = 0$ and $P_{d0}(v) = 0$ for all $v \in V$; in other words, reception failures are never caused by signal blockage, thus any concurrent transmissions will always interfere with each other. This leads to a strict requirement that exactly one node (either a cooperative neighbor or the source) must make the retransmission.

The second special case is when $P_{sd1} = 0$ and $P_{d1}(v) = 0$ for all $v \in V$; thus, there is no concept of an "interfering link" – if a transmission from some location is not strong enough to be successful in the absence of interference, then it will itself not cause interference to others. This relaxes the success requirement and allows any number of retransmissions from nodes without a link to the destination, as long as only one transmission is made over a strong link.

From a practical perspective, these special cases correspond to different possible causes of reception failures in mobile networks. The first special case corresponds to situations where reception failures are predominantly due to signal distortions caused by, e.g., multipath fading or Doppler shifts, while the second special case describes situations where physical obstacles between a transmitter and a receiver that (temporarily) block the propagation path are the main cause of failures.

1) $P_{sd0} = 0$ and $P_{d0}(v) = 0$ (all-interfering case): The assumption $P_{d0}(v) = 0$ is equivalent to $P_{d1}(v) + P_{d2}(v) = 1$ for all $v \in V$. This immediately simplifies the statement of Theorem 1; specifically, all the conditions and the integral equation in that theorem that involve $\frac{P_{d2}(v)}{P_{d1}(v)+P_{d2}(v)}$ reduce to be expressed via $P_{d2}(v)$ itself, i.e. the optimal cooperation range becomes defined simply by a threshold of the probability of having a strong link to the destination.

Furthermore, in this case a choice of $\tau_s = 1$ immediately implies that P_{suc} (expression (3)) is maximized with $\tau(v) = 0$ everywhere; indeed, a concurrent transmission by any node other than the source will cause a certain collision. Hence, in this case, we have $P_{suc} = P_{sd2}$. Thus, the optimal cooperative retransmission strategy is defined by Theorem 1 if it results in $P_{suc} > P_{sd2}$; otherwise, the optimal strategy is to have a retransmission by the original source only.

2) $P_{sd1} = 0$ and $P_{d1}(v) = 0$ (non-interfering case): We observe that, when $P_{d1}(v) = 0$ for all $v \in V$, $\frac{P_{d2}(v)}{P_{d1}(v)+P_{d2}(v)}$ degenerates to a constant value of 1 everywhere and therefore the threshold condition of theorems 1 and 2 can no longer help determine the optimal cooperation region. In order to solve the optimization problem in this case, we turn back to the success probability expression (3) and note that, with $P_{d1}(v) = 0$, both integrals (outside and inside the exponent) coincide. In other words, noting that $P_{sd2} = 1 - P_{sd0}$, P_{suc} can be rewritten as

$$P_{suc} = \tau_s P_{sd2}(1-\lambda)\exp(-\lambda) + \lambda \exp(-\lambda), \quad (11)$$

where $\lambda \triangleq \int_{v \in V} P_s(v)\tau(v)P_{d2}(v)\rho(v)dv$; thus, the entire dependence of the success probability on $\tau(v)$ is captured through λ. Thus, the functional optimization reduces to a simple optimization problem of two variables, with the constraints

$$0 \leq \tau_s \leq 1 \quad (12)$$

$$0 \leq \lambda \leq \lambda^{\max} \triangleq \int_{v \in V} P_s(v)P_{d2}(v)\rho(v)dv, \quad (13)$$

where $\lambda^{\max}$ is the largest possible value of λ that can be obtained by setting $\tau(v) = 1$ everywhere. Once the optimal λ is found, a degree of freedom remains for the choice of any particular function $\tau(v)$ that corresponds to that value of λ.

C. Discussion and practical implications

The analysis presented in this section provides a generic method for computing the optimal cooperation strategy for nodes at any location in the network. Given any specific model of wireless propagation, the method can be applied constructively (by numerically solving the respective integral equation) to determine the optimal cooperation region, expressed in terms of a threshold of the ratio $\frac{P_{d2}(v)}{P_{d1}(v)+P_{d2}(v)}$. If the functions $P_{d^*}(v)$ depend only on the distance to the destination, this threshold can be directly converted to a simple distance threshold, and the optimal cooperation region in that case becomes a circle or a sphere centred around the destination node. We emphasize that the solution of the respective integral equation in order to determine the cooperation region need not be undertaken by every node after every overheard packet; rather, it only needs to be computed once in advance, based on the *a priori* density and propagation model, and then applied to subsequent transmissions independently. The question of how the *a priori* network density is determined or estimated is application-dependent, and is beyond the scope of this paper.

The retransmission strategy should, of course, only apply to a packet whose original transmission by the sender was unsuccessful. This means that, to avoid spurious retransmissions, all nodes in the optimal cooperation region must overhear the receiver's acknowledgment of successful packets (this is immaterial for nodes outside the cooperation region). This will indeed be the case with high probability, in light of Theorems 1 and 2 and the fact that the feedback channel is generally even more reliable than the data channel, since acknowledgments are short and less prone to channel outages than data packets. For this reason, spurious retransmissions of successful packets will be rare; their occurrence can be reduced further using negative acknowledgments or explicit retransmission requests, which are beyond the scope of the paper.

We have illustrated the optimization technique for uncoordinated cooperative retransmissions by considering the metric of the success probability of a single retransmission attempt. Clearly, the retransmission success probability can be improved further by allowing nodes to make several retransmissions in succession before declaring a failure (up to a preset limit that may depend on the application's reliability or delay constraints). It is interesting to note that, with multiple successive retransmissions, the optimal cooperation strategy will no longer involve only the *a priori* information about the environment; rather, nodes will be able to revise their beliefs about the distribution of the network state (i.e. the number and locations of peers who may have overheard the packet, and the states of their channels to the destination) *a posteriori* and vary the optimal cooperation region for each subsequent retransmission attempt. The process of computing an optimal sequence of retransmission probabilities through belief updates about the network state has been introduced in [17] for a fixed-topology network, and its adaptation to a highly dynamic network context is left for future work.

IV. Numerical Evaluation

In this section, we demonstrate the performance of the uncoordinated strategy and its dependence on various network parameters numerically. For our simulations, we use the log-normal shadowing model, such that the long-term (*a priori*) probability of a successful transmission between a pair of nodes is described as follows:

$$\log P_r = \log P_t - L - \alpha \log d + \psi, \quad (14)$$

where P_r is the received power, P_t is the transmitted power, L is the power loss at a unit distance from the transmitter, d is the distance between the link endpoints, α is the path loss exponent, and ψ is the random shadowing (Gaussian-distributed with a zero mean). Henceforth, we assume $\alpha = 2.7$ and $\sigma_\psi = 11.8dB$; these values are chosen to represent a typical city environment [18]. We consider a transmission to be successful if P_r is greater than some sensitivity threshold $P_{\min}$. Thus, the *a priori* successful transmission probability of a link (i.e. $P_s(v)$ at a distance d from the source or $P_{d2}(v)$ at a distance d to the destination) is $\Pr\{\psi \geq \log P_{\min} - \log P_t + L + \alpha \log d\}$; that is, governed by the Gaussian distribution of ψ. Hence, there is a one-to-one correspondence between P_t and the probability of a successful transmission over a unit distance, which we henceforth denote by $P_{unit\_distance}$; e.g., this probability is 0.5 for $\log P_t = \log P_{\min} + L$. We assume below that P_t and $P_{\min}$ are identical for all nodes in the network, and that the nodes are randomly distributed on a two-dimensional plane with constant density $\rho(v) = \rho$ (i.e. a point Poisson process).

We choose to base the following evaluations on the assumption that any unsuccessful cooperative retransmission always causes an interference, or, in other words, on the first special case discussed in section III-B, where $P_{d1}(v) = 1 - P_{d2}(v)$. The reason for focusing on this case is to choose the most "hostile" possible scenario for the uncoordinated cooperation, such that any outcome counted as a success in this model will also be a success in a real system (even under a SINR-based physical interference model). Furthermore, we focus our attention on the more interesting case where $\tau_s = 0$ in the optimal retransmission strategy; thus, the probability of the direct link to recover immediately after the failed original transmission (P_{sd2}) is not high enough to use $\tau_s = 1$, i.e. a retransmission by the original source without cooperation. Again, this will

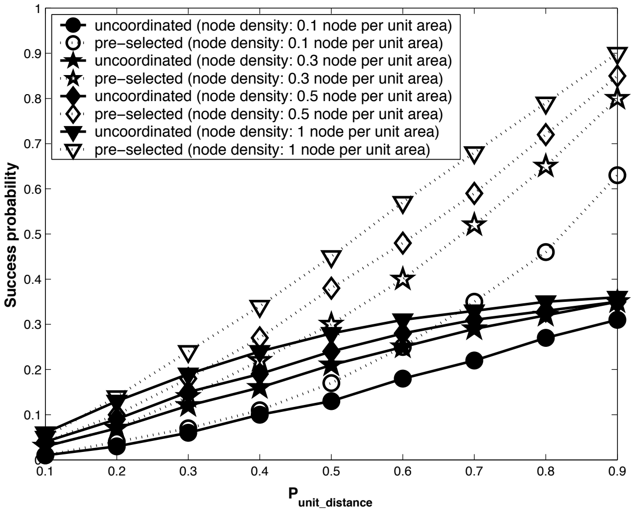


Fig. 2. Success probability vs. $P_{unit\_distance}$, for fixed density.

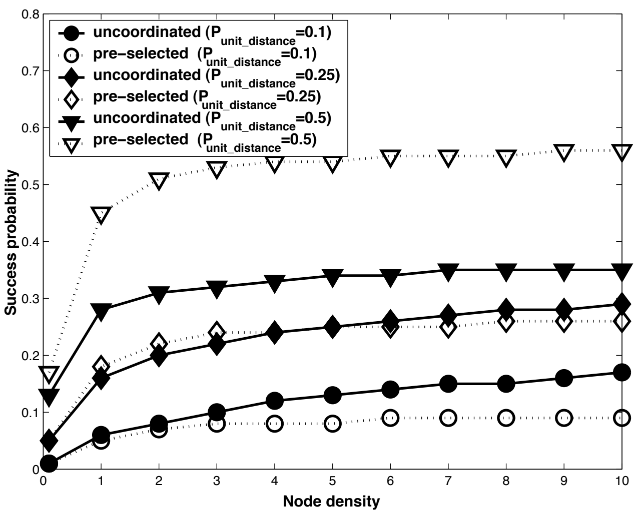


Fig. 3. Success probability vs. normalized node density, for fixed $P_{unit\_distance}$.

lead to a lower bound on the success probability that can be achieved in a real setting.

The results are presented in figures 2-4. Figure 2 shows the success probability of the optimal uncoordinated retransmission strategy as a function of $P_{unit\_distance}$ (equivalently, transmission power), for several values of density ρ, while Figure 3 plots the success probability versus ρ, for several values of $P_{unit\_distance}$. The values in both figures are normalized to a unit distance defined as the distance between the source and destination. Finally, in Figure 4, the success probability is shown as a function of the physical distance between source and destination, with a fixed transmission power and for several values of physical node density.

For comparison purposes, we also obtain the expected success probability achievable by an optimally-located single cooperative relay selected *a priori*, which is simply the maximum $P_s P_{d2}$ among all nodes in the network (obtained as an average of the maximum $P_s P_{d2}$ value in a large sample of random networks of density ρ); this is labeled as "pre-selected" in the figures.

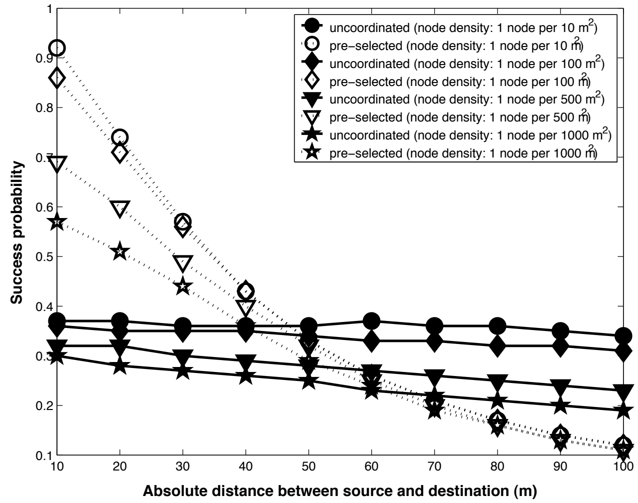


Fig. 4. Success probability vs. source-destination distance for fixed transmission power (set for transmission success probability 0.5 at distance 30m).

We observe from figures 2-3 that, when $P_{unit\_distance}$ is large, uncoordinated retransmission performs worse than a pre-selected single relay. This is expected, since a single well-placed relay will have a very good probability of overhearing and successfully retransmitting a packet to the destination, while the performance of an uncoordinated strategy is limited due to collisions. Indeed, consider the second special case discussed in section III-B. For that case, the uncoordinated success probability is bounded by $\max_\lambda \lambda e^{-\lambda} = \frac{1}{e} \approx 0.368$ (see (11)), and, clearly, this bound applies in our setting as well, which has stricter conditions for a successful retransmission. Incidentally, we note that this is the same bound as for the slotted ALOHA throughput [19], which features a similar uncoordinated access to a shared resource with collisions.

On the other hand, in the region of low $P_{unit\_distance}$, the uncoordinated strategy performs almost as well, or even better than a pre-selected single relay, especially when the node density is high. This effect is best observed in the curves of $P_{unit\_distance} = 0.25$ in Figure 3. We explain this effect as follows. For a single relay, regardless of the node density, the success probability cannot exceed that of an ideally placed relay mid-way between the source and destination, which is the square of the success probability at half a unit distance. On the other hand, the uncoordinated strategy exploits the high node density by defining a small cooperation region around the destination. Even if the probability of a strong link over a unit distance is low, the probability of a node in the cooperation region to overhear the packet will be reasonably high, thanks to the sheer number of nodes in the region; that node can then successfully retransmit it to the destination over a short distance. In fact, it can be shown that, for any fixed $P_{unit\_distance}$ (no matter how small), the probability of successful reception for the optimal uncoordinated strategy will tend to $\frac{1}{e}$ as $\rho \to \infty$; thus, uncoordinated retransmission maintains a reasonable success probability even under very bad wireless conditions, by taking advantage of the very large number of independent potential relays.

The same effect is even more pronounced in Figure 4, which is based on practical values of node density and transmission power. Here, each curve corresponds to an average density, ranging from 1 node per $10m^2$ (a dense network, e.g. a small room of people with wireless devices) to 1 node per $1000m^2$ (a sparse network in an open field or large hall). We set the transmission power to correspond to a transmission success probability of 0.5 at a distance of 30m, and vary the distance d_{sd} between the communicating endpoints (source and destination) between 10m and 100m. Clearly, as d_{sd} grows, the link quality deteriorates and the performance of both the coordinated single relay and our uncoordinated retransmission strategy degrades. However, we clearly observe that the uncoordinated success probability degrades far more gracefully, and considerably outperforms coordinated relaying when the distance is large. The reason is that, intuitively, with increasing d_{sd}, the expected number of nodes in the general area between the source and destination increases as well. This leads to a greater optimal cooperation region where $\tau(v) = 1$, with a corresponding increase in the expected number of nodes participating in the retransmission.

V. Conclusion

We have studied uncoordinated cooperative retransmission in highly dynamic wireless networks, where nodes independently decide whether to retransmit an overheard packet without any prior agreement with other neighbors, thereby risking collision among multiple such retransmissions but eliminating the cooperation overhead if the transmission results in a successful reception by the destination. We modeled the respective tradeoff as a functional optimization problem, seeking the retransmission probability as a function of location to maximize the probability of successful reception, and provided a generic solution of the optimal cooperation region depending only on the *a priori* node density and the wireless signal propagation model. Our numerical evaluation demonstrated that the strategy performs especially well in scenarios with low channel quality (i.e. low transmission power or high level of noise) and high node density, in which case it even outperforms traditional (coordinated) relaying methods.

In this paper, we have introduced a general framework for uncoordinated cooperation, and presented a generic analysis of optimizing the successful delivery rate of a single packet via a simple strategy, involving merely a single immediate retransmission. Such a strategy can be directly applicable in application scenarios with infrequent packet transmissions, such as environmental monitoring. Further work is required to alleviate some of our simplifying assumptions and integrate the approach into a broader MAC protocol design. First, we considered the optimization problem for a single packet in isolation, and the analysis should be extended to allow for multiple original packets transmitted in the network at the same time. In addition, the retransmission success probability can be improved further by allowing nodes to randomize the timing of their retransmission, or make several cooperative retransmissions in succession before declaring a failure; however, this improvement increases the delay overhead per packet, and therefore needs to be considered carefully in the relevant application context. The extension of our analysis to cover these additional possibilities is left for future work.

References

[1] L. Lai, K. Liu, and H. El Gamal. The three-node wireless network: Achievable rates and cooperation strategies. *IEEE Trans. Inf. Theory*, 52(3):805–828, March 2006.

[2] A. Sendonaris, E. Erkip, and B. Aazhang. User cooperation diversity (parts I,II). *IEEE Trans. Commun.*, 51(11):1927–1948, November 2003.

[3] J.N. Laneman, D.N.C. Tse, and G.W. Wornell. Cooperative diversity in wireless networks: Efficient protocols and outage behavior. *IEEE Trans. Inf. Theory*, 50(12):3062–3080, December 2004.

[4] M. Zorzi and R.R. Rao. Geographic random forwarding (GeRaF) for ad hoc and sensor networks: Energy and latency performance. *IEEE Trans. Mobile Comput.*, 2(4):349–365, October 2003.

[5] S. Biswas and R. Morris. ExOR: Opportunistic multi-hop routing for wireless networks. In *Proc. ACM SIGCOMM*, Philadelphia, PA, August 2005.

[6] A.S. Ibrahim, A.K. Sadek, W. Su, and K.J.R. Liu. Cooperative communications with relay selection: when to cooperate and whom to cooperate with? *IEEE Trans. Wireless Commun.*, 7(7):2814–2827, July 2008.

[7] L. Sun, T. Zhang, L. Lu, and H. Niu. Cooperative communications with relay selection in wireless sensor networks. *IEEE Trans. Consum. Electron.*, 55(2):513–517, May 2009.

[8] J. Vicario, A. Bel, J. Lopez-Salcedo, and G. Seco. Opportunistic relay selection with outdated CSI: Outage probability and diversity analysis. *IEEE Trans. Wireless Commun.*, 8(6):2872–2876, June 2009.

[9] P.T. Eugster, R. Guerraoui, S.B. Handurukande, P. Kouznetsov, and A.M. Kermarrec. Lightweight probabilistic broadcast. *ACM Trans. Computer Systems*, 21(4):341–374, November 2003.

[10] L. Pelusi, A. Passarella, and M. Conti. Opportunistic networking: Data forwarding in disconnected mobile ad hoc networks. *IEEE Commun. Mag.*, 44(11):134–141, November 2006.

[11] A. Ribeiro, N.D. Sidiropoulos, and G.B. Giannakis. Optimal distributed stochastic routing algorithms for wireless multihop networks. *IEEE Trans. Wireless Commun.*, 7(11):4261–4272, November 2008.

[12] M. Grossglauser and D.N.C. Tse. Mobility increases the capacity of ad hoc wireless networks. *IEEE/ACM Trans. Networking*, 10(4):477–486, August 2002.

[13] P. Nain, D. Towsley, B. Liu, and Z. Liu. Properties of random direction models. In *Proc. IEEE Infocom*, Miami, FL, March 2005.

[14] Y. Shi, Y.T. Hou, J. Liu, and S. Kompella. How to correctly use the protocol interference model for multi-hop wireless networks. In *Proc. ACM MobiHoc*, New Orleans, LA, May 2009.

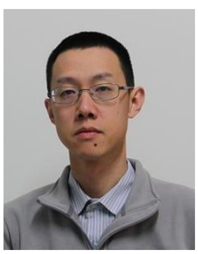
[15] X. Che, X. Liu, X. Ju, and H. Zhang. Adaptive instantiation of the protocol interference model in mission-critical wireless networks. In *Proc. IEEE SECON*, Boston, MA, June 2010.

[16] T. Yang, G. Mao, and W. Zhang. Connectivity of wireless CSMA multi-hop networks. In *Proc. IEEE International Conference on Communications (ICC)*, Kyoto, Japan, June 2011.

[17] L. Xiong, L. Libman, and G. Mao. Distributed strategies for minimum-latency cooperative retransmission in wireless networks. In *Proc. IEEE LCN*, Zurich, Switzerland, October 2009.

[18] S.Y. Seidel, T.S. Rappaport, S. Jain, M. Lord, and R. Singh. Path loss, scattering, and multipath delay statistics in four European cities for digital cellular and microcellular radiotelephone. *IEEE Trans. Veh. Technol.*, 40(4):721–730, November 1991.

[19] D. Bertsekas and R. Gallager. *Data Networks*. Prentice-Hall, Englewood Cliffs, NJ, 2nd edition, 1992.



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