Towards Perpetual Sensor Networks via Deploying Multiple Mobile Wireless Chargers

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***Abstract*—In this paper, we study the use of multiple mobile charging vehicles to charge sensors in a large-scale wireless sensor network for a given monitoring period, where sensors can be charged by the vehicles with wireless power transfer. Since each sensor may experience multiple charges to avoid its energy expiration for the period, we first consider a charging problem of scheduling the multiple mobile vehicles to collaboratively charge sensors so that none of the sensors will run out of its energy and the sum of traveling distance (referred to as the service cost) of these vehicles can be minimized. Due to NP-hardness of the problem, we then propose a novel approximation algorithm for it, assuming that sensor energy consumption rates do not change over time. Otherwise, we devise a heuristic algorithm through minor modifications to the approximation algorithm. We finally evaluate the performance of the proposed algorithms via simulations. Experimental results show that the proposed algorithms are very promising, which can reduce upto 45% of the service cost in comparison with the service cost delivered by a greedy algorithm.**

I. Introduction

Wireless sensor networks (WSNs) have played an important role in many surveillance and monitoring applications including environmental sensing, target tracking, structural health monitoring, etc [17]. As conventional sensors are powered by batteries, their limited battery capacity obstructs the large-scale deployments of WSNs. Although there have been many energy saving approaches developed in the past decade to minimize sensor energy consumptions or balance energy expenditures among sensors [2], the lifetime of WSNs remains a main performance bottleneck in real deployments of WSNs, since wireless data transmission consumes substantial energy.

To mitigate the limited energy problem, researchers proposed to enable sensors to harvest ambient energy from their surroundings such as solar energy, vibration energy, wind energy, etc [11]. However, the temporally-spatially varying nature of renewable energy resources makes the prediction of sensor energy harvesting rates very difficult. For instance, it is shown that the difference of energy generating rates in sunny, cloudy and shadowy days can be upto three orders of magnitude in a solar harvesting system [8]. Moreover, the harvesting energy sources are intermittent and not always available. Such unpredictability and intermittency pose enormous challenges in the efficient usage of harvested energy for a myriad of monitoring or surveillance tasks.

The recent breakthrough in the wireless power transfer technique based on strongly coupled magnetic resonances has drawn plenty of attentions [6]. Kurs *et al.* demonstrated that it is possible to achieve an approximate 40% efficiency of wireless power transfer for powering a $60W$ light bulb from a distance of two meters without any wire lines and plugs [6]. Armed with this advanced technology, sensor can be charged at steady and high charging rates. On the other hand, another breakthrough in the ultra-fast charging battery materials further fuels the feasibility of the wireless power transfer technique. Scientists from MIT implemented a ultra-fast charging in material $LiFePO_4$, which can be charged at a rate as high as $400\ Coulombs$ per second [4]. The time of fully-charging a battery thus can be shortened into a few seconds. Therefore, wireless power charging is a promising technique to prolong the lifetime of WSNs.

In this paper, we employ multiple mobile wireless chargers to replenish sensors in a large-scale WSN with wireless power transfer for a monitoring period T, so that none of the sensors will run out of its energy. We consider a flexible sensor energy charging paradigm that sensors are charged by the mobile chargers in an on-demand fashion and their data routing is decoupled from the energy replenishment. As each sensor in the network must be charged many times to avoid its energy expiration during T, the challenges for scheduling the mobile chargers are: (1) when should we activate a charging task to dispatch the mobile chargers to replenish sensor energy? (2) which sensors are to be included in each charging task? (3) given a set of to-be-charged sensors, which sensors should be charged by which mobile charger? We tackle these challenges by formulating a novel optimization problem and devising efficient scheduling algorithms for it.

The main contributions of this paper can be summarized as follows.

- Under the on-demand energy replenishment paradigm, we first formulate a novel service cost minimization problem of finding a series of charging schedulings for the mobile chargers to maintain the perpetual operations of sensors during period T, such that the total traveling distance of the mobile chargers is minimized. This objective is critical for the WSN maintenance cost reduction.
- Due to NP-hardness of the problem, we then propose an approximation algorithm with a provable approximation ratio if energy consumption rates of sensors

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are fixed. Otherwise, we provide a novel heuristic solution through minor modifications to the approximate solution.

- We finally conduct extensive experiments by simulations to evaluate the algorithm performance. Experimental results demonstrate that the proposed algorithms are very promising, which can reduce upto 45% of the service cost by a greedy algorithm.

The rest of this paper is organized as follows. Section II reviews related work. Section III introduces preliminaries. Section IV devises an algorithm for a q-rooted TSP problem, which will be served as a subroutine of the proposed algorithms. Sections V and VI propose approximation and heuristic algorithms for the problem under fixed and variable energy consumption rates, respectively. Section VII evaluates the algorithm performance, and Section VIII concludes the paper.

II. Related work

The wireless power transfer technology based on strongly magnetic resonances has drawn a lot of attention and researchers adopted the wireless energy replenishment to prolong the lifetime of WSNs [10], [13], [14], [15], [20], [3], [16], [9], [7], [12]. Most of them jointly considered data flow routing and sensor energy replenishment. For instance, Shi *et al.* [10] proposed to replenish sensor energy in a WSN by employing a wireless charging vehicle to periodically visit each sensor. They formulated a problem of maximizing the ratio of the vacation time of the charging vehicle to the renewable energy cycle time, by considering both data flow routing and the charging time of each sensor, assuming that the data generating rate of each sensor is unchanged. They later extended their work to two more general settings that the charging vehicle can charge multiple sensors simultaneously [13], or the vehicle can replenish sensor energy and collect sensor data at the meantime [14], [15]. Zhao *et al.* [20] also provided a joint design of data gathering and energy replenishment by exploiting sink mobility. To this end, for every fixed interval, they first chose a set of to-be-charged sensors, and then delivered a data gathering solution, such that the network utility is maximized while maintaining perpetual network operations. They extended their study by taking the energy consumption of data sensing and reception into consideration [3].

The joint consideration of data flow routing and energy replenishment in aforementioned works suffers from three drawbacks in real WSN deployments due to unrealistic assumptions. The first one is that most of them assumed that the data generating rate of each sensor is fixed in the entire network lifetime. However, sensor data rates usually are application-dependent, and are very likely to experience significant changes over time. For instance, in a flood detection WSN, high data sampling rates of sensors are required to better monitor water levels at different observation locations when there is a storm. The second one is that they also assumed the data flow conservation at each sensor node, that is, the amount of data that each sensor sends out is equal to the amount of data the sensor received plus the data it generates. This assumption ignores data aggregation at intermediate nodes, while data aggregation is an efficient technique to reduce data traffic volume, as sensing data of different sensors usually are temporally-spatially correlated [5]. The last one is the assumption of all reliable communications between sensors, which contrasts the well-known fact that wireless communication is notoriously unreliable [19]. Retransmissions at some sensors may result in substantial energy consumptions on them. Unlike existing studies, in this paper we advocate that the sensor energy replenishment should be decoupled from the design of data flow routing protocols. The benefits by doing so include: (1) the devised charging solutions can be applicable to sensor networks for various purposes, including periodic monitoring, event detections, surveillance coverage, etc. (2) sensors can be free from the high computational and communicational overhead introduced by complex energy management algorithms. The producing of sensors can be simplified and their prices thus can be reduced, which is critical for WSNs requiring deploying hundreds even thousands of sensors. (3) there is no need that we charge each sensor periodically. Only is there such a need from sensors that will run out of their energy soon, we then dispatch charging vehicles to replenish energy to them, thereby significantly saving WSN operational costs.

There are also recent works that adopted the on-demand sensor energy replenishment. Xu *et al.* [16] considered the problem of scheduling k mobile chargers to replenish a set of to-be-charged sensors, such that the maximum time spent among the k chargers is minimized, for which they proposed constant approximation algorithms. Ren *et al.* [9] recently studied the employment of a single mobile charger to charge on-demand sensors under the travel distance constraint. Liang *et al.* [7] proposed an approximation algorithm for minimizing the number of mobile vehicles needed for charging a set of to-be-charged sensors, subject to the energy capacity constraint on each mobile vehicle. Wang *et al.* [12] developed a hybrid approach for scheduling multiple mobile chargers to charge sensors: active and passive energy replenishment. Orthogonal to these works, we consider minimizing the total traveling distance of multiple mobile chargers to maintain the sensor network perpetual operations for a period T, which is crucial to reduce the network operational cost.

III. Preliminaries

A. Network model

We consider a wireless sensor network consisting of n sensors deployed in a two-dimensional space. Let V be the set of sensors. There is one stationary base station in the network and all sensing data from sensors will be relayed to the base station directly or through multihop relays. Each sensor $v_i \in V$ is powered by a rechargeable battery with energy capacity B_i. Assume that the entire network monitoring period is T (T typically is long). Since each sensor consumes its energy on data sensing, processing, transmission and reception, it is required to be charged periodically to avoid its energy depletion.

In this paper we consider using q wireless mobile chargers to replenish the sensors in the network, where mobile charger l is located at depot r_l, $1 \leq l \leq q$. Let $R = \{r_1, r_2, \cdots, r_q\}$ be the set of depot locations of these q mobile chargers. To determine charging trajectories of the q mobile chargers, we define a weighted undirected graph $G = (V \cup R, E; w)$, where for any two distinct nodes (sensors or depots) u and v in $V \cup R$, there is an edge $e = (u, v) \in E$ between them with their Euclidean distance $w(e)$ as its edge weight. Each time mobile charger l is dispatched to charge some sensors, it always starts from and ends at its depot r_l for recharging its electricity or refuelling its petrol. In other words, each charging tour of mobile charger l in G is a *closed tour* including depot r_l. For any closed tour C in G, denote by $w(C)$ the weighted sum of the edges in C, i.e. $w(C) = \sum_{e \in E(C)} w(e)$. We assume that each mobile charger has enough energy to replenish all sensors if needed in each charging tour. We consider a point-to-point charging, i.e. to efficiently charge a sensor, some mobile charger must be in the vicinity of the sensor and the sensor will be charged to its fully capacity. Once a sensor is fully charged, its lifetime can last from several weeks to months, depending on its working load such as sensing rates, the volume of data relayed, etc. Meanwhile, the q mobile chargers can collaboratively finish a charging task within a few hours (e.g. sensor batteries can be made with ultra-fast charging battery materials [4]). We thus assume that the time spent by the q mobile chargers per charging task, including the time for charging sensors and the time on their traveling, is several orders of magnitude less than the lifetime of a fully-charged sensor. Therefore, we ignore the time spent per charging task. The similar assumption has also been adopted in [18] and [20]. We further assume that the energy consumption of a sensor during its charging tour is negligible, too.

B. Notations and Notions

A charging scheduling of q mobile chargers is to dispatch each of the q mobile chargers from its depot to collaboratively visit a set of to-be-charged sensors in the current round, and each charger will return to its depot after finishing its charging tour. Assume that at time t_j, let closed tours $C_{j,1}, C_{j,2}, \cdots, C_{j,q}$ be the charging tours of the q mobile chargers, where tour $C_{j,l}$ by mobile charger l contains its depot r_l and $1 \le l \le q$. Let $\mathcal{C}_j = \{C_{j,1}, C_{j,2}, \cdots, C_{j,q}\}$. Notice that it is likely that some tours $C_{j,l}$ may contain none of the sensors, and if so, $V(C_{j,l}) = \{r_l\}$ and $w(C_{j,l}) = 0$. We represent each charging scheduling by a 2-tuple $(\mathcal{C}_j, t_j)$, where all sensors in tour $C_{j,l} \in \mathcal{C}_j$ will be charged to their full energy capacities by mobile charger l, all the q mobile chargers are dispatched at time t_j, and $0 < t_j < T$. Denote by $V(C_{j,l})$ and $V(\mathcal{C}_j)$ the set of nodes in $C_{j,l}$ and $\mathcal{C}_j$, respectively. Then, $V(\mathcal{C}_j) = \cup_{l=1}^{q} V(C_{j,l})$.

The charging cycle of a sensor $v_i \in V$ is the duration between its two consecutive chargings, and its *maximum charging cycle* τ_i is the maximum duration between its two consecutive chargings so that it will not run out of energy. Since different WSNs adopt different sensing and routing protocols, different sensors have different energy consumption rates and different maximum charging cycles. If the energy consumption rate of each sensor $v_i \in V$ does not vary for period T, denote by ρ_i and τ_i its energy consumption rate and maximum charging cycle, then $\tau_i = \frac{B_i}{\rho_i}$, where B_i is its energy capacity. Note that sensors with shorter charging cycles need to be charged more frequently than sensors with longer charging cycles. Since each time the q mobile chargers are dispatched to charge a set of sensors, they will consume their electricity or petrol, thereby incurring a service cost. We thus define *the service cost* of the q mobile chargers as the sum of their travel distances for charging sensors for a period T.

C. Problem definitions

We note that not every sensor must be replenished in each charging round, as energy consumption rates of different sensors may significantly vary. For instance, since the sensors near to the base station have to relay data for other remote sensors, their energy consumption rates are much higher than that of the others. Therefore, we can see that a naive strategy of charging all sensors per round will significantly increase the service cost. Also, as the distance between some sensor and its nearest depot in a large-scale sensor network can be far away from each other, it is crucial to schedule the q mobile chargers by taking both the maximum charging cycles and the geographical locations of the sensors into account.

Assume that the location coordinates (x_i, y_i) of each sensor $v_i \in V$ are given. Given a metric complete graph $G = (V \cup R, E)$ with q mobile chargers located at q depots in R, a distance function $w : E \mapsto \mathbb{R}^+$, a monitoring period T, and a maximum charging cycle function $\tau : V \mapsto \mathbb{R}^+$, *the service cost minimization problem with fixed maximum charging cycles* is to find a series of charging schedulings $(\mathcal{C}_1, t_1), \cdots, (\mathcal{C}_p, t_p)$ of the q mobile chargers such that the total length of all closed tours (or the service cost), $\sum_{j=1}^{p} w(\mathcal{C}_j) = \sum_{j=1}^{p} \sum_{l=1}^{q} w(C_{j,l})$, is minimized, subject to that, for each sensor $v_i \in V$, (i) the time gap between its any two consecutive charging schedulings $(\mathcal{C}_{j_1}, t_{j_1})$ and $(\mathcal{C}_{j_2}, t_{j_2})$ is no more than its maximum charging cycle τ_i, i.e. $|t_{j_2} - t_{j_1}| \le \tau_i$, $v_i \in \mathcal{C}_{j_1}$ and $v_i \in \mathcal{C}_{j_2}$; and (ii) the duration from its last charging to the end of period T is also no more than τ_i, where $\mathcal{C}_j = \{C_{j,1}, C_{j,2}, \cdots, C_{j,q}\}$, $C_{j,l}$ is the charging tour of mobile charger l located at depot r_l, $1 \le l \le q$, and $0 \le t_1 < \cdots < t_p < T$. This problem is NP-hard since it can be reduced from the classical NP-hard Traveling Salesman Problem (TSP), omitted.

So far we have assumed that the maximum charging cycle of each sensor $v_i \in V$ in the entire period T is fixed. However, in reality, it may experience significant changes over time. For this general setting, we define *the service cost minimization problem with variable maximum charging cycles* as follow. Given a wireless sensor network, a period T, a set R of q depots with q mobile chargers at which the mobile chargers will be refueled/charged and starting from and ending its charging tours, the maximum charger cycle $\tau_i(t)$ of each sensor v_i varying with time t with $0 < t \le T$, the problem is to find a series of charging schedulings of the q mobile chargers such that the service cost by them is minimized, subject to that none of the sensors runs out of energy within period T.

We finally consider a *q-rooted TSP problem* as follows, which will be used as a subroutine. Assume that there is a set of to-be-charged-sensors $V^c \subseteq V$ at some time point. Given a subgraph $G^c = (V^c \cup R, E^c; w)$ of G with $|R| = q \ge 1$ and q mobile chargers, the problem is to find q closed tours $C_1, C_2, \cdots, C_q$ in G^c, such that the total length of the q tours, $\sum_{l=1}^{q} w(C_l)$, is minimized, subject to that these q tours cover all sensors in V^c, i.e. $V^c \subseteq \bigcup_{l=1}^{q} V(C_l)$, and each of the q tours contains a distinct depot in R. The q-rooted TSP problem is NP-hard, since the classical TSP problem is a special case of it when $q = 1$.

IV. Algorithm for the q-rooted TSP problem

In this section, we propose a 2-approximation algorithm for the q-rooted TSP problem, which will serve as a subroutine of the approximation algorithm for the service cost minimization problem. The basic idea of the algorithm for the q-rooted TSP problem is that we first find q-rooted trees with the minimum total cost, and we then show that the total cost of the q-rooted trees is a lower bound on the optimal cost of the q-rooted TSP problem. We finally transform each of the trees into a closed tour with the cost of each tour being no more than twice the cost of its corresponding tree.

Before we proceed, we consider *the q-rooted minimum spanning forest problem*: given a graph $G^c = (V^c \cup R, E^c; w)$, $q = |R|$, and $w : E^c \mapsto \mathcal{R}^+$, the q-rooted Minimum Spanning Forest (q-rooted MSF) problem is to find q disjoint trees $T_1, T_2, \cdots, T_q$ spanning all nodes in V^c with each tree containing a distinct depot in R, such that the total cost of the q trees, $\sum_{l=1}^{q} w(T_l)$, is minimized.

For the q-rooted MSF problem, we develop an exact algorithm as follows. We start by constructing an auxiliary graph $G_r = (V^c \cup \{r\}, E_r; w_r)$ from $G^c = (V^c \cup R, E^c; w)$ by contracting the q depots in R into a single root r: (i) remove the q depots in R and introduce a new node r; (ii) for each $r_l \in R$, introduce an edge $(v, r) \in E_r$ for each edge $(v, r_l) \in E^c$, where $v \in V^c$; (iii) $w_r(v, r) = \min_l\{w(v, r_l)\}$. We then find an MST T of G_r. We finally break T into q disjoint trees $T_1, T_2, \cdots, T_q$ by un-contracting the roots in R. This un-contraction means that an edge (v, r) is mapped to an edge (v, r_l), where $w_r(v, r) = w(v, r_l)$. Note that each tree T_l roots at depot r_l. The detailed algorithm is presented in `Algorithm` 1.

Algorithm 1 `q-rooted MSF`

Input: $G^c = (V^c \cup R, E^c; w)$, $w : E^c \mapsto \mathbb{R}^+$, and $q = |R|$.
Output: a solution for the q-rooted MSF problem
1: Construct a graph $G_r = (V^c \cup \{r\}, E_r; w_r)$ from G^c by contracting the q depots in R into a single root r;
2: Find an MST T in G_r;
3: Decompose the MST T into q disjoint rooted trees $T_1, T_2, \cdots, T_q$ by un-contracting depots in R.

Lemma 1: There is an exact algorithm for the q-rooted MSF problem, which takes time $O(n^2)$, where $n = |V^c \cup R|$.

Proof: Assume that trees $T_1^*, T_2^*, \cdots, T_q^*$ form an optimal solution to the q-rooted MSF problem. We show that the trees $T_1, T_2, \cdots, T_q$ delivered by `Algorithm` 1 are optimal. On one hand, since the q trees $T_1, T_2, \cdots, T_q$ form a feasible solution, then $\sum_{l=1}^{q} w(T_l^*) \leq \sum_{l=1}^{q} w(T_l)$. On the other hand, as each tree T_l^* contains a depot $r_l \in R$, we can construct a spanning tree T' in graph G_r by contracting the q depots into a single root r, and $w(T') = \sum_{l=1}^{q} w(T_l^*)$. As the MST T is the minimum one, we have $w(T) \leq w(T')$. Since $\sum_{l=1}^{q} w(T_l) = w(T)$, $\sum_{l=1}^{q} w(T_l) = w(T) \leq w(T') \leq \sum_{l=1}^{q} w(T_l^*)$. Therefore, $\sum_{l=1}^{q} w(T_l) = \sum_{l=1}^{q} w(T_l^*)$, i.e. the found trees $T_1, T_2, \cdots, T_q$ form an optimal solution to the problem. The time complexity of `Algorithm` 1 is analyzed as follows. Constructing graph G_r takes time $O(E^c) = O(n^2)$. Finding the MST T in G_r takes $O(n^2)$ time, while un-contracting the MST T also takes time $O(E^c) = O(n^2)$. `Algorithm` 1 thus runs in $O(n^2)$ time. ∎

With the help of the exact algorithm for the q-rooted MSF problem, we devise a 2-approximation algorithm for the q-rooted TSP problem in `Algorithm` 2.

Algorithm 2 `q-rooted TSP`

Input: $G^c = (V^c \cup R, E^c; w)$, $w : E^c \mapsto \mathbb{R}^+$, and $q = |R|$.
Output: A solution $\mathcal{C}$ for the q-rooted TSP problem
1: Find q optimal trees $T_1, T_2, \cdots, T_q$ for the q-rooted MSF problem in G^c by calling `Algorithm` 1;
2: For each tree T_l, double the edges in T_l, find a Eulerian tour C_l', and obtain a less cost closed tour C_l by short-cutting repeated nodes in C_l'. Let $\mathcal{C} = \{C_1, C_2, \cdots, C_q\}$.

We show that `Algorithm` 2 delivers a 2-approximate solution by the following theorem.

Theorem 1: There is a 2-approximation algorithm for the q-rooted TSP problem, which takes time $O(|V^c \cup R|^2)$.

Proof: Assume that closed tours $C_1^*, C_2^*, \cdots, C_q^*$ form the optimal solution to the q-rooted TSP problem in G^c. For each tour C_l^*, we can obtain a tree T_l' by removing any edge in C_l^*. Then, $w(T_l') \leq w(C_l^*)$, where $1 \leq l \leq q$. It is obvious that trees $T_1', T_2', \cdots, T_q'$ form a feasible solution to the q-rooted MSF problem. As trees $T_1, T_2, \cdots, T_q$ form the optimal solution by Lemma 1, $\sum_{l=1}^{q} w(T_l) \leq \sum_{l=1}^{q} w(T_l') \leq \sum_{l=1}^{q} w(C_l^*)$. Also, we can see that the total cost of each found tour C_l is no more than twice the total cost of tree T_l, i.e. $w(C_l) \leq 2w(T_l)$. Therefore, $\sum_{l=1}^{q} w(C_l) \leq \sum_{l=1}^{q} 2w(T_l) \leq 2\sum_{l=1}^{q} w(C_l^*)$. The time complexity analysis is straightforward, omitted. ∎

V. Approximation Algorithm with Fixed Maximum Charging Cycles

In this section, we devise an approximation algorithm for the service cost minimization problem in a rechargeable sensor network with fixed maximum charging cycles.

A. Overview of the approximation algorithm

Given a maximum charging cycle function: $\tau : V \mapsto R^+$ and a monitoring period T, if there is a series of mobile charger schedulings for T such that no sensor expires, then we say that these schedulings form *a feasible solution* to the service cost minimization problem, i.e. for each sensor $v_i \in V$, the maximum duration between its any two consecutive chargings is no more than τ_i. *A series of feasible charging schedulings* of the q mobile chargers is *an optimal solution* if the service cost of the solution is the minimum one.

The basic idea behind the proposed algorithm is to construct another charging cycle function $\tau'(\cdot)$ for the sensors based on the maximum charging cycle function $\tau(\cdot)$, by exploring the combinatorial property of the problem.

We construct a very special charging cycle function $\tau'(\cdot)$ that charging cycles of the n sensors will form a geometric sequence as follows. Let $\tau_1, \tau_2, \cdots, \tau_n$ be the maximum charging cycles of sensors $v_1, v_2, \ldots, v_n$ in the network. Assume that $\tau_1 \leq \tau_2 \leq \cdots \leq \tau_n$. Let $\tau_1', \tau_2', \cdots, \tau_n'$ be the charging cycles of the sensors and $\tau'_i \le \tau'_j$ if $\tau_i \le \tau_j$. We construct $\tau'(\cdot)$ as follows. We partition the set V of the sensors into $K+1$ disjoint subsets $V_0, V_1, \cdots, V_K$, where $K = \lfloor \log_2 \frac{\tau_n}{\tau_1} \rfloor$, and sensor $v_i \in V$ with maximum charging cycle τ_i is contained in V_k if $2^k \tau_1 \le \tau_i < 2^{k+1}\tau_1$. Then, $k = \lfloor \log_2 \frac{\tau_i}{\tau_1} \rfloor$. Let $\tau'_i = 2^k \tau_1$. We assign each sensor in V_k with the identical charging cycle $2^k \tau'_1 = 2^k \tau_1$, $0 \le k \le K$. Consequently, the charging cycles of sensors in $V_0, V_1, \cdots, V_K$ are $\tau_1, 2\tau_1, \cdots, 2^K \tau_1$, respectively. We can see that the assigned charging cycle τ'_i of sensor v_i is no less than the half its maximum charging cycle τ_i, since

$$\tau'_i \;=\; 2^{\lfloor \log \frac{\tau_i}{\tau_1} \rfloor} \tau_1 \;>\; 2^{\log \frac{\tau_i}{\tau_1} - 1} \tau_1 \;=\; \frac{\tau_i}{2}, \quad \forall v_i \in V. \tag{1}$$

B. Approximation algorithm

Given the charging cycle function $\tau'(\cdot)$, we can see that τ'_j is divisible by τ'_i for any two sensors v_i and v_j if $\tau_i \le \tau_j$ and $1 \le i < j \le n$. For simplicity, assume that the monitoring period T is divisible by the maximum assigned charging cycle τ'_n, let $T = 2m\tau'_n = 2m2^K \tau_1$, where m is a positive integer. The solution delivered by the proposed algorithm consists of a series of schedulings of the q mobile chargers. Specifically, we first find a sequence of schedulings of the q mobile chargers for a period τ'_n. Then, we repeat the found schedulings for the next time period of τ'_n, and so on. We repeat these scheduling sequence for the period T no more than $\lfloor T/\tau'_n \rfloor - 1 = 2m-1$ times.

In the following, we construct a series of schedulings for a period $\tau'_n = 2^K \tau_1$. Recall that we have partitioned the sensor set V into $K+1$ disjoint subsets $V_0, V_1, \ldots, V_K$, and the charging cycle of each sensor in V_k is $2^k \tau_1$ with $0 \le k \le K$, We further partition the period τ'_n into 2^K *equal time intervals* with each interval lasting τ_1, and label them from the left to right as the 1st, 2nd, ..., and the 2^Kth time interval. Clearly, all sensors in V_0 must be charged at each of these 2^K time intervals; all sensors in V_1 must be charged at every second time interval; and all sensors in V_k ($V_k \ne \emptyset$) must be charged at every 2^k time interval, $0 \le k \le K$. Thus, there are 2^K charging schedulings of the q mobile chargers and each charging scheduling is dispatched at each time interval. Let $\mathcal{C}_j = \{C_{j,1}, C_{j,2}, \cdots, C_{j,q}\}$ be the set of closed tours of the q mobile chargers at time interval j, where $1 \le j \le 2^K$. Assume that each sensor is fully charged at time $t = 0$. The charging schedulings of the q mobile chargers for a period τ'_n are as follows.

The 1st group of q closed tours $\mathcal{C}_1 = \{C_{1,1}, \cdots, C_{1,q}\}$ contain all sensors in V_0 and the depot set R. The q mobile chargers are dispatched at time τ_1.

The 2nd group of q closed tours $\mathcal{C}_2 = \{C_{2,1}, \cdots, C_{2,q}\}$ contain all sensors in $V_0 \cup V_1$ and the depot set R. The q mobile chargers are dispatched at time $2\tau_1$.

$\vdots$

The jth group of q closed tours $\mathcal{C}_j = \{C_{j,1}, \cdots, C_{j,q}\}$ contain all sensors in $\cup_{(j \mod 2^k)=0} V_k$ where $0 \le k \le K'$ and $K' = \lfloor \log_2 j \rfloor$, and the depot set R with $1 \le j \le 2^K$. The q mobile chargers are dispatched at time $j\tau_1$.

$\vdots$

The 2^Kth group of q tours $\mathcal{C}_{2^K} = \{C_{2^K,1}, \cdots, C_{2^K,q}\}$ contain all sensors in $\cup_{i=0}^{K} V_i = V$ and the depot set R. The q mobile chargers are dispatched at time $2^K \tau_1$.

Thus, the series of charging schedulings for a period τ'_n is $(\mathcal{C}_1, \tau_1), \ldots, (\mathcal{C}_j, j\tau_1), \ldots, (\mathcal{C}_{2^K}, 2^K \tau_1)$, where 2-tuple $(\mathcal{C}_j, j\tau_1)$ represents that the q mobile chargers are dispatched at time $j\tau_1$ and the set of to-be-charged sensors is $\cup_{C_{j,i} \in \mathcal{C}_j} V(C_{j,i}) = \cup_{(j \mod 2^k)=0} V_k \cup R$, where $0 \le k \le K'$, $K' = \lfloor \log_2 j \rfloor$, and $1 \le j \le 2^K$. Then, the charging schedulings for period $T = 2m\tau'_n$ are

$$\begin{array}{lll}
(\mathcal{C}_1, \tau_1), & \cdots, (\mathcal{C}_{2^K-1}, (2^K-1)\tau_1), & (\mathcal{C}_{2^K}, 2^K \tau_1) \\
(\mathcal{C}_1, \tau'_n + \tau_1), & \cdots, (\mathcal{C}_{2^K-1}, \tau'_n + (2^K-1)\tau_1), & (\mathcal{C}_{2^K}, \tau'_n + 2^K \tau_1), \\
\vdots & & \\
\multicolumn{3}{l}{(\mathcal{C}_1, (2m-1)\tau'_n + \tau_1), \cdots, (\mathcal{C}_{2^K-1}, (2m-1)\tau'_n + (2^K-1)\tau_1).}
\end{array}$$

Note that we do not perform a charging scheduling at time $T = 2m\tau'_n$ as there is no such need. Given a set $V(\mathcal{C}_j)$ of to-be-charged sensors and the depot set R, we find the charging tours for the q mobile chargers by calling `Algorithm` 2. The proposed algorithm is described in `Algorithm` 3.

Algorithm 3 `MinTotalDistance`

Input: $G = (V \cup R, E; w)$, maximum charging cycles $\tau \mapsto \mathbb{R}^+$, q mobile chargers, and a monitoring period T.
Output: A series of charging schedulings $\mathcal{C}$ for period T

Let $\tau_1, \tau_2, \cdots, \tau_n$ be the sorted maximum charging cycles of sensors $v_1, v_2, \ldots, v_n$ in ascending order;
For each sensor v_i, let $\tau'_i = 2^{\lfloor \log_2 \frac{\tau_i}{\tau_1} \rfloor}$;
Partition sensors in V into $K+1$ disjoint subsets $V_0, V_1, \cdots, V_K$, where sensor $v_i \in V_k$ if $2^k \tau_1 = 2^{\lfloor \log_2 \frac{\tau_i}{\tau_1} \rfloor} \tau_1$, $0 \le k \le K$, and $K = \lfloor \log_2 \frac{\tau_n}{\tau_1} \rfloor$;
$\mathcal{C} \leftarrow \emptyset$; /* the solution */
/* Construct schedulings $(\mathcal{C}_1, \tau_1), \cdots, (\mathcal{C}_{2^K}, 2^K \tau_1)$ */
for $j \leftarrow 1$ to 2^K **do**
    /* Construct the node set $V(\mathcal{C}_j)$ of the q closed tours in $\mathcal{C}_j$ at time $t_j = j\tau_1$ */
    $V(\mathcal{C}_j) \leftarrow R$; /* depot set */
    for $k \leftarrow 0$ to $\lfloor \log_2 j \rfloor$ **do**
        if $(j \mod 2^k) = 0$ **then**
            $V(\mathcal{C}_j) \leftarrow V(\mathcal{C}_j) \cup V_k$
        end if
    end for
    Find q tours $\mathcal{C}_j = \{C_{j,1}, \cdots, C_{j,q}\}$ in the induced subgraph $G[V(\mathcal{C}_j)]$ by applying `Algorithm` 2;
    $\mathcal{C} \leftarrow \mathcal{C} \cup \{(\mathcal{C}_j, t_j)\}$;
end for
for $m' \leftarrow 2$ to $\lfloor T/\tau'_n \rfloor$ **do**
    for $j \leftarrow 1$ to 2^K **do**
        $\mathcal{C} = \mathcal{C} \cup \{(\mathcal{C}_j, m' \cdot \tau'_n + t_j)\}$
    end for
end for

C. Algorithm analysis

We now show that `Algorithm` 3 delivers an approximate solution. We also analyze its time complexity.

Lemma 2: `Algorithm` 3 delivers a feasible solution.

Proof: It is obvious that the solution delivered by `Algorithm` 3 is feasible, as the charging cycle τ'_i of each sensor $v_i \in V$ in the solution is no more than its maximum charging cycle τ_i, i.e. $\tau_i' \leq \tau_i$. Thus, no sensors will die in the period T, the claim then follows. ■

The following lemma provides an lower bound on the optimal service cost, which is the cornerstone for bounding the service cost of the solution delivered by `Algorithm` 3.

Lemma 3: Given the sensor set partitioning $V_0, V_1, \ldots, V_K$ based on the maximum charging cycles of sensors, each sensor in V_k is assigned with a same charging cycle $2^k\tau_1$, $0 \leq k \leq K$. Let OPT be the service cost of an optimal solution to the service cost minimization problem. Denote by $\mathcal{D}_k^* = \{D_{k,1}^*, D_{k,2}^*, \cdots, D_{k,q}^*\}$ the optimal q closed tours for the q-rooted TSP problem in the induced graph $G[R \cup V_0 \cup V_1 \cup \cdots \cup V_k]$, then $w(\mathcal{D}_k^*) = \sum_{l=1}^{q} w(D_{k,l}^*) \leq \frac{OPT}{m \cdot 2^{K-k}}$, assuming that $T = 2m\tau_n'$ and $0 \leq k \leq K$.

Proof: To show that $w(\mathcal{D}_k^*) \leq \frac{OPT}{m \cdot 2^{K-k}}$, we partition the entire period $T = 2m\tau_n' = 2m \cdot 2^K\tau_1$ into $m \cdot 2^{K-k}$ equal time intervals with each lasting time $t_k = 2^{k+1}\tau_1$. These intervals are $(0,\ t_k], (t_k,\ 2t_k], \cdots, ((j-1)t_k,\ jt_k], \cdots, ((m2^{K-k} - 1)t_k,\ m2^{K-k}t_k]$, where the jth time interval is the interval $((j-1) \cdot t_k, j \cdot t_k]$, $1 \leq j \leq m \cdot 2^{K-k}$. Note that $m2^{K-k}t_k = m2^{K-k}2^{k+1}\tau_1 = T$.

Assume that there is an optimal solution consisting of charging schedulings $(\mathcal{C}_1^*, t_1^*), \cdots, (\mathcal{C}_p^*, t_p^*)$ with $0 < t_1^* \leq \ldots \leq t_p^* < T$. Recall that OPT is the sum of lengths of the p charging schedulings (or the service cost of the q mobile chargers), $OPT = \sum_{s=1}^{p} w(\mathcal{C}_s^*) = \sum_{s=1}^{p}\sum_{l=1}^{q} w(C_{s,l}^*)$. We further partition the p charging schedulings into $m2^{K-k}$ disjoint groups according to their dispatching times, where charging scheduling $\mathcal{C}_s^*$ is in group j if its dispatching time t_s^* is within time interval j, i.e. $t_s^* \in ((j-1)t_k,\ jt_k]$, where $1 \leq s \leq p$ and $1 \leq j \leq m2^{K-k}$. Denote by $\mathcal{G}_j$ and $w(\mathcal{G}_j)$ the set of charging schedulings in group j and the cost sum of charging schedulings in $\mathcal{G}_j$, respectively, i.e. $w(\mathcal{G}_j) = \sum_{\mathcal{C}_s^* \in \mathcal{G}_j} w(\mathcal{C}_s^*)$, $1 \leq j \leq m2^{K-k}$. Then, $\sum_{j=1}^{m2^{K-k}} w(\mathcal{G}_j) = OPT$. Among the $m2^{K-k}$ groups, there must be a group $\mathcal{G}_j$ whose service cost $w(\mathcal{G}_j)$ is no more than $\frac{1}{m2^{K-k}}$ of the optimal cost OPT, i.e.

$$w(\mathcal{G}_j) \leq \frac{OPT}{m2^{K-k}}. \tag{2}$$

We claim that each sensor in $\bigcup_{i=0}^{k} V_i$ must be charged at least once by the charging schedulings in $\mathcal{G}_j$ by contradiction. Assume that there is a sensor $v_i \in \bigcup_{i=0}^{k} V_i$ which will not be charged by any charging scheduling in $\mathcal{G}_j$. Following the node set partitioning, the maximum charging cycle τ_i of v_i must be less than $2 \cdot 2^k\tau_1 = 2^{k+1}\tau_1$ by inequality (1), i.e. $\tau_i < 2^{k+1}\tau_1$. On the other hand, since v_i will not be charged by any charging scheduling in group $\mathcal{G}_j$ while it is still survived, this implies that its maximum charging cycle must be no less than t_k, i.e. $\tau_i \geq t_k = 2^{k+1} \cdot \tau_1$, this results in a contradiction. Thus, v_i must be charged by at least one charging scheduling in $\mathcal{G}_j$.

We now construct feasible solution $\mathcal{C}_k = \{C_{k,1}, C_{k,2}, \cdots, C_{k,q}\}$ to the q-rooted TSP problem in the induced subgraph $G[R \cup V_0 \cup \cdots \cup V_k]$ based on the charging schedulings in $\mathcal{G}_j$, such that the service cost $w(\mathcal{C}_k)$ is no more than $w(\mathcal{G}_j)$. Since each closed tour in $\mathcal{G}_j$ contains a depot $r_l \in R$, we partition the closed tours in $\mathcal{G}_j$ by the depot that each tour contains. To this end, we partition tours in $\mathcal{G}_j$ into q disjoint subgroups $\mathcal{G}_{j,1}, \mathcal{G}_{j,2}, \cdots, \mathcal{G}_{j,q}$, where subgroup $\mathcal{G}_{j,l}$ includes all closed tours in $\mathcal{G}_j$ that contains depot r_l, $1 \leq l \leq q$. For each subgroup $\mathcal{G}_{j,l}$, since each tour contains depot r_l, the union of all close tours in $\mathcal{G}_{j,l}$ forms a connected Eulerian graph. Then, we can derive a Eulerian circuit $C_{k,l}'$ from this Eulerian graph and $w(C_{k,l}') = w(\mathcal{G}_{j,l})$. We further obtain a closed tour $C_{k,l}$ including only nodes in $R \cup V_0 \cup \cdots \cup V_k$ once from $C_{k,l}'$, by the removal of the nodes not in $R \cup V_0 \cup \cdots \cup V_k$ and the nodes with multiple appearances, and performing path short-cutting. Since edge weights satisfy the triangle inequality, we have

$$w(C_{k,l}) \leq w(C_{k,l}') \leq w(\mathcal{G}_{j,l}), \quad 1 \leq l \leq q. \tag{3}$$

As each sensor in $\bigcup_{i=0}^{k} V_i$ is be charged at least once by the charging schedulings in $\mathcal{G}_j$, and tour $C_{k,l}$ contains depot r_l, we have $\bigcup_{i=0}^{k} V_i \subseteq \bigcup_{l=1}^{q} V(C_{k,l})$. Then, tours in $\mathcal{C}_k$ form a feasible solution to the q-rooted TSP problem in the induced graph $G[R \cup V_0 \cup V_1 \cup \cdots \cup V_k]$. Let $\mathcal{D}_k^* = \{D_{k,1}^*, D_{k,2}^*, \cdots, D_{k,q}^*\}$ be the optimal q closed tours. Then,

$$\sum_{l=1}^{q} w(D_{k,l}^*) \leq \sum_{l=1}^{q} w(C_{k,l}). \tag{4}$$

Combining (2), (3), and (4), the lemma then follows. ■

Following Lemmas 2 and 3, we show the approximation ratio of `Algorithm` 3 by the following theorem.

Theorem 2: There is a $2(K+2)$-approximation algorithm for the service cost minimization problem with fixed maximum charging cycles, which takes time $O(\frac{\tau_{max}}{\tau_{min}}n^2 + \frac{T}{\tau_{min}}n)$, where $K = \lfloor \log_2 \frac{\tau_{max}}{\tau_{max}} \rfloor$, $\tau_{max} = \max_{i=1}^{n}\{\tau_i\}$ and $\tau_{min} = \min_{i=1}^{n}\{\tau_i\}$.

Proof: By Lemma 2, `Algorithm` 3 delivers a feasible solution. The rest is to analyze its approximation ratio.

Recall that the charging schedulings delivered by `Algorithm` 3 for the period $T = 2m\tau_n'$ are: $(\mathcal{C}_1, \tau_1), \cdots, (\mathcal{C}_{2^K}, 2^K\tau_1)$, $(\mathcal{C}_1, \tau_n' + \tau_1), \cdots, (\mathcal{C}_{2^K}, \tau_n' + 2^K\tau_1)$, $\cdots$, $(\mathcal{C}_1, (2m-1)\tau_n' + \tau_1), \cdots, (\mathcal{C}_{2^K-1}, (2m-1)\tau_n' + (2^K-1)\tau_1)$. The total service cost during T then is

$$(2m-1)\sum_{j=1}^{2^K} w(\mathcal{C}_j) + \sum_{j=1}^{2^K-1} w(\mathcal{C}_j) \leq 2m\sum_{j=1}^{2^K} w(\mathcal{C}_j). \tag{5}$$

Let $\mathcal{C}(\tau_n') = \{(\mathcal{C}_1, \tau_1'), (\mathcal{C}_2, \tau_2'), \ldots, (\mathcal{C}_{2^K}, \tau_n')\}$. From the construction of $\mathcal{C}(\tau_n')$, we can see that there are 2^{K-1} identical charging schedulings in $\mathcal{C}(\tau_n')$ with each only containing the nodes in $R \cup V_0$, denote by $\mathcal{D}_0$ the charging scheduling and $w(\mathcal{D}_0)$ the service cost of the scheduling. In general, there are 2^{K-1-k} identical charging schedulings in $\mathcal{C}(\tau_n')$ with each only containing the nodes in $R \cup V_0 \cup V_1 \ldots \cup V_k$, denote by $\mathcal{D}_k$ the charging scheduling and $w(\mathcal{D}_k)$ the cost of the scheduling with $0 \leq k \leq K-1$. Finally, there is one charging scheduling in $\mathcal{C}(\tau_n')$ containing the nodes in $R \cup V_0 \cup \cdots \cup V_K = R \cup V$, denote by $\mathcal{D}_K$ the charging scheduling and $w(\mathcal{D}_K)$ the scheduling cost. We then rewrite the upper bound on the service cost in Inequality (5) as

$$2m\sum_{j=1}^{2^K} w(\mathcal{C}_j) = 2m(\sum_{k=0}^{K-1} 2^{K-1-k} w(\mathcal{D}_k) + w(\mathcal{D}_K)). \quad (6)$$

Denote by $\mathcal{D}_k^* = \{D_{k,1}^*, D_{k,2}^*, \cdots, D_{k,q}^*\}$ the set of the optimal q closed tours for the q-rooted TSP problem in the induced graph $G[R \cup V_0 \cup \cdots \cup V_k]$. Since $\mathcal{D}_k$ is a 2-approximate solution by Theorem 1, $w(\mathcal{D}_k) \leq 2w(\mathcal{D}_k^*)$, $0 \leq k \leq K$. Also, by Lemma 3, $w(\mathcal{D}_k^*) \leq \frac{OPT}{m2^{K-k}}$. We have

$$\begin{aligned} &2m(\sum_{k=0}^{K-1} 2^{K-1-k} w(\mathcal{D}_k) + w(\mathcal{D}_K)) \\ &\leq 4m(\sum_{k=0}^{K-1} 2^{K-1-k} \frac{OPT}{m2^{K-k}} + \frac{OPT}{m}) = 2(K+2)OPT. \quad (7) \end{aligned}$$

We analyze the time complexity of `Algorithm` 3 as follows. Partitioning the sensor set V into $K+1$ disjoint subsets $V_0, V_1, \ldots, V_K$ takes $O(n)$ time, based on the assigned charging cycles for the n sensors. We then analyze the running time of constructing 2^K charging schedulings $\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_{2^K}$ for the mobile chargers for a period of τ_n' as follows. Given the sensor set $C(V_j)$ for any j with $1 \leq j \leq 2^K$, it takes $O(|V(C_j)|^2)$ time to find a 2-approximate solution to the q-rooted TSP problem by Theorem 1 in a complete graph $G[V(C_j)]$ induced by the nodes in $V(C_j)$ while $V(C_j) \subseteq V$. Thus, the time of constructing the 2^K charging schedulings is $O(2^K n^2) = O(\frac{\tau_{max}}{\tau_{min}} n^2)$, where $\tau_{max} = \max_{i=1}^n \{\tau_i\}$ and $\tau_{min} = \min_{i=1}^n \{\tau_i\}$. Having these 2^K closed tours, the rest of closed tours can be obtained by repeatedly copying these 2^K close tours but assigning them with different dispatching times. Thus, the time complexity of the solution $\mathcal{C}$ is $O(\frac{\tau_{max}}{\tau_{min}} n^2 + \frac{T}{\tau_n'} n) = O(\frac{\tau_{max}}{\tau_{min}} n^2 + \frac{T}{\tau_{min}} n)$ time. ∎

VI. Heuristic Algorithm with Variable Maximum Charging Cycles

So far we have developed an approximation algorithm for the service cost minimization problem, assuming that maximum charging cycles of sensors are fixed. In this section we devise a novel heuristic algorithm for it without this assumption as follows.

A. Dynamic maximum charging cycles of sensors

Within the period T, the energy consumption rates of sensors may dynamically change over time, which results in the changes of sensor maximum charging cycles eventually. To respond to the variation, each sensor monitors its own energy information including residual energy and energy consumption rate periodically (e.g. every a few hours). Based on the energy information, it can predicts its residual lifetime, using a light-weight prediction technique as follows. Let $\hat{\rho}_i(t+1)$ be the predicted energy consumption rate of sensor v_i for time $t+1$, $\hat{\rho}_i(t+1) = \gamma \cdot \rho_i(t) + (1-\gamma) \cdot \hat{\rho}_i(t)$, where γ is a constant with $0 < \gamma < 1$, $\hat{\rho}_i(t)$ and $\rho_i(t)$ are the predicted and monitored energy consumption rates of sensor v_i at time t, respectively. Assume that the residual energy of sensor v_i at time t is $re_i(t)$, we estimate its residual lifetime $l_i(t)$ and maximum charging cycle $\tau_i(t)$ by $l_i(t) = \frac{re_i(t)}{\hat{\rho}_i(t+1)}$ and $\hat{\tau}_i(t) = \frac{B_i}{\hat{\rho}_i(t+1)}$, respectively.

The base station maintains the updated energy information of each sensor. We assume that there is a variation threshold of maximum charging cycle at each sensor, if the variation is under the pre-defined threshold, nothing is to be done. Otherwise, the sensor sends an updating request to the base station and the base station takes proper actions.

B. Heuristic Algorithm

Assume that the base station receives maximum charging cycle updatings from some sensors at time t, which implies that the charging schedulings based on previous maximum charging cycles may not be applicable any more, otherwise these sensors will deplete their energy prior to their next charging cycles. For example, assume a sensor has changed its maximum charging cycle from a longer one to a shorter one, it might be dead if the sensor still is charged with the previous longer charging cycle since the sensor now can only last its operations for a shorter cycle once it is fully charged.

The basic idea of the heuristic algorithm is as follows. When the base station receives maximum charging cycle updatings, it checks whether the previous series of schedulings still are applicable for these updated maximum charging cycles. If so, nothing needs to be done. Otherwise, the algorithm re-computes a new series of schedulings by first applying the approximation algorithm based on updated maximum charging cycles, followed by minor modifications to the solution delivered by the approximation algorithm.

Assume that the previous maximum charging cycle of sensor v_i is $\hat{\tau}_i(t-1)$ and it was charged at a charging cycle $\hat{\tau}_i'(t-1)$ in the previous series of schedulings, where $\hat{\tau}_i'(t-1) \leq \hat{\tau}_i(t-1)$. At time t, the base station receives the maximum charging cycle updating of sensor v_i, which changes from $\hat{\tau}_i(t-1)$ to $\hat{\tau}_i(t)$. The base station then checks the feasibility of the previous schedulings as follows. If $\hat{\tau}_i'(t-1) \leq \hat{\tau}_i(t) < 2\hat{\tau}_i'(t-1)$, the previous schedulings are still feasible as sensor v_i will be charged with a charging cycle $\hat{\tau}_i'(t-1)$ no more than its current maximum charging cycle $\hat{\tau}_i(t)$. Otherwise ($\hat{\tau}_i(t) < \hat{\tau}_i'(t-1)$ or $\hat{\tau}_i(t) \geq 2\hat{\tau}_i'(t-1)$), we re-compute a new series of schedulings based on updated maximum charging cycles since the previous schedulings are not feasible any more (i.e. $\hat{\tau}_i(t) < \hat{\tau}_i'(t-1)$), or though the schedulings still are feasible, they are not optimal in terms of the service cost (i.e. $\hat{\tau}_i(t) \geq 2\hat{\tau}_i'(t-1)$). In the following, we re-compute a new series of schedulings.

To re-compute a new series of schedulings, we first invoke the proposed approximation algorithm based on the updated maximum charging cycles. Assume that the updated maximum charging cycles of the n sensors are $\hat{\tau}_1(t), \hat{\tau}_2(t), \cdots, \hat{\tau}_n(t)$ and their residual lifetimes are $\hat{l}_1(t), \hat{l}_2(t), \cdots, \hat{l}_n(t)$, where $0 < t < T$. We further assume that the solution delivered by the approximation algorithm based on the updated maximum charging cycles is:

$$\begin{aligned} &(\mathcal{C}_1, t + \hat{\tau}_1(t)), \quad (\mathcal{C}_2, t + 2\hat{\tau}_1(t)), \quad \cdots, \quad (\mathcal{C}_{2^K}, t + 2^K\hat{\tau}_1(t)), \\ &(\mathcal{C}_1, t + \hat{\tau}_n'(t) + \hat{\tau}_1(t)), (\mathcal{C}_2, t + \hat{\tau}_n'(t) + 2\hat{\tau}_1(t)), \cdots, (\mathcal{C}_{2^K}, t + \hat{\tau}_n'(t) + 2^K\hat{\tau}_1(t)), \\ &\vdots \\ &(\mathcal{C}_1, t + x\hat{\tau}_n'(t) + \hat{\tau}_1(t)), \cdots, (\mathcal{C}_y, t + x\hat{\tau}_n'(t) + y\hat{\tau}_1(t), \end{aligned}$$

where $t + x\hat{\tau}_n'(t) + y\hat{\tau}_1(t) < T$, $t + x\hat{\tau}_n'(t) + (y+1)\hat{\tau}_1(t) \geq T$, and x and y are positive integers. The most updated charging cycles of the n sensors in the solution are $\hat{\tau}'_1(t), \hat{\tau}'_2(t), \cdots, \hat{\tau}'_n(t)$, where $\hat{\tau}'_i(t) = 2^{\lfloor \log_2 \frac{\hat{\tau}_i(t)}{\hat{\tau}_1(t)} \rfloor} \hat{\tau}_1(t)$.

Note that the solution delivered may not be feasible, as different sensors may have different residual energy, which violates the condition of applying the approximation algorithm, that is, all sensors must be fully charged initially. The residual energy in some sensor v_i may not support its operation until its next charging time $t + \hat{\tau}'_i(t)$, i.e. $\hat{l}_i(t) < \hat{\tau}'_i(t)$. Denote by V^a the set of sensors with $\hat{l}_i(t) < \hat{\tau}'_i(t)$. We then schedule the mobile chargers to replenish sensors in V^a to avoid their energy expiration, through adding a new charging scheduling $(\mathcal{C}'_0, t)$ and modifying the first 2^K schedulings from $(\mathcal{C}_1, t + \hat{\tau}_1(t)), (\mathcal{C}_2, t + 2\hat{\tau}_1(t)), \cdots, (\mathcal{C}_{2^K}, t + 2^K \hat{\tau}_1(t))$ to $(\mathcal{C}'_1, t + \hat{\tau}_1(t)), (\mathcal{C}'_2, t + 2\hat{\tau}_1(t)), \cdots, (\mathcal{C}'_{2^K}, t + 2^K \hat{\tau}_1(t))$. Therefore, the solution delivered by the algorithm is:

$$
\begin{array}{l}
(\mathcal{C}'_0, t), \ (\mathcal{C}'_1, t + \hat{\tau}_1(t)), \quad (\mathcal{C}'_2, t + 2\hat{\tau}_1(t)), \quad \cdots, \ (\mathcal{C}'_{2^K}, t + 2^K \hat{\tau}_1(t)), \\
\quad (\mathcal{C}_1, t + \hat{\tau}'_n(t) + \hat{\tau}_1(t)), (\mathcal{C}_2, t + \hat{\tau}'_n(t) + 2\hat{\tau}_1(t)), \cdots, (\mathcal{C}_{2^K}, t + \hat{\tau}'_n(t) + 2^K \hat{\tau}_1(t)), \\
\quad \vdots \\
\quad (\mathcal{C}_1, t + x\hat{\tau}'_n(t) + \hat{\tau}_1(t)), \cdots, (\mathcal{C}_y, t + x\hat{\tau}'_n(t) + y\hat{\tau}_1(t).
\end{array}
$$

The rest is to construct the first $2^K + 1$ charging schedulings.

Let $V^a_t = \{v_i | v_i \in V^a \ \& \ \hat{l}_i(t) < \hat{\tau}_1(t)\}$, which implies that the residual lifetime of each sensor in V^a_t is less than $\hat{\tau}_1(t)$ and $V^a_t \subseteq V^a$. We construct a scheduling $(\mathcal{C}_0, t)$, in which all sensors in V^a_t will be charged at time t. We then, like the node set partition in the approximation algorithm, partition the set $V^a \setminus V^a_t$ into $K + 1$ disjoint sets $V^a_0, V^a_1, \cdots, V^a_K$ according to their residual lifetimes, where $K = \lfloor \log_2 \frac{\hat{\tau}_n(t)}{\hat{\tau}_1(t)} \rfloor$ and a sensor $v_i \in V^a \setminus V^a_t$ is contained in V^a_k if $2^k \hat{\tau}_1(t) \le \hat{l}_i(t) < 2^{k+1} \hat{\tau}_1(t)$. Note that the residual lifetime $\hat{l}_i(t)$ of each sensor v_i in V^a_k at time t is no less than $2^k \hat{\tau}_1(t)$ but no greater than its charging cycle $\hat{\tau}'_i(t)$, i.e. $2^k \hat{\tau}_1(t) \le \hat{l}_i(t) < \hat{\tau}'_i(t)$. To avoid the energy expiration of sensor v_i, we can add it into any one of the schedulings: $\{(\mathcal{C}_0, t), (\mathcal{C}_1, t + \hat{\tau}_1(t)), (\mathcal{C}_2, t + 2\hat{\tau}_1(t)), \cdots, (\mathcal{C}_{2^k}, t + 2^k \hat{\tau}_1(t))\}$. However, to minimize the service cost, we add sensor v_i into a nearest scheduling $\mathcal{C}_j$, i.e. the nearest distance between sensor v_i and nodes in scheduling $\mathcal{C}_j$ is no less than the nearest distance between it and nodes in any other scheduling $\mathcal{C}_l$, where $0 \le j, l \le 2^k$. The detailed construction of the $2^K + 1$ schedulings is as follows.

We construct the $2^K + 1$ schedulings by iteratively invoking `Algorithm 1` for the q-rooted minimum spanning forest problem. Denote by $V(\mathcal{C}_j^{(k)})$ and $V(\mathcal{C}_j^{(k+1)})$ the constructed node sets of scheduling $\mathcal{C}'_j$ before and after iteration k, respectively, where $0 \le k \le K$. Note that $\mathcal{C}_j^{(k)} = \{C_{j,1}^{(k)}, \cdots, C_{j,q}^{(k)}\}$ and $V(\mathcal{C}_j^{(k)}) = \bigcup_{l=1}^{q} V(C_{j,l}^{(k)})$. After $K + 1$ iterations, we let $V(\mathcal{C}'_j) = V(\mathcal{C}_j^{(K+1)})$. We finally obtain scheduling $\mathcal{C}'_j$ by applying `Algorithm 2` for the q-rooted TSP problem in the induced graph $G[V(\mathcal{C}'_j)]$. Consequently, each sensor in $V^a_t \cup V^a_0 \cup \cdots \cup V^a_K = V^a$ will be charged in time. Initially, let $V(\mathcal{C}_0^{(0)}) = V^a_t \cup R$ and $V(\mathcal{C}_j^{(0)}) = V(\mathcal{C}_j)$, where $1 \le j \le 2^K$. At iteration k ($0 \le k \le K$), we first construct an auxiliary graph $G^{(k)} = (V^a_k \cup R^{(k)}, E^{(k)}; w^{(k)})$ based on node sets V^a_k and $V(\mathcal{C}_0^{(k)}), V(\mathcal{C}_1^{(k)}), \cdots, V(\mathcal{C}_{2^k}^{(k)})$, where there is a root $r_j^{(k)}$ in $R^{(k)}$ representing node set $V(\mathcal{C}_j^{(k)})$, $0 \le j \le 2^k$, and $E^{(k)} = V^a_k \times V^a_k \cup V^a_k \times R^{(k)}$. Then, $|R^{(k)}| = 2^k + 1$. For each edge $(u, v) \in V^a_k \times V^a_k$, $w^{(k)}(u, v)$ is the Euclidean distance between nodes u and v. For each edge $(u, r_j^{(k)}) \in V^a_k \times R^{(k)}$, $w^{(k)}(u, r_j^{(k)})$ is the nearest Euclidean distance between node u and nodes in $V(\mathcal{C}_j^{(k)})$. We then obtain $2^k + 1$ minimum cost rooted trees $T_0^{(k)}, T_1^{(k)}, \cdots, T_{2^k}^{(k)}$, by invoking `Algorithm 1` on $G^{(k)}$, where tree $T_j^{(k)}$ contains root $r_j^{(k)}$ and $0 \le j \le 2^k$. Note that each sensor in V^a_k is contained in a tree $T_j^{(k)}$ and $V^a_k = V(T_0^{(k)}) \cup V(T_1^{(k)}) \cup \cdots \cup V(T_{2^k}^{(k)}) - R^{(k)}$. Then, the sensors in tree $T_j^{(k)}$ will be charged in scheduling $(\mathcal{C}'_j, t + j\hat{\tau}_1(t))$. To this end, we let $V(\mathcal{C}_j^{(k+1)}) = V(\mathcal{C}_j^{(k)}) \cup V(T_j^{(k)}) - \{r_j^{(k)}\}$ if $0 \le j \le 2^k$, otherwise ($2^k + 1 \le j \le 2^K$), $V(\mathcal{C}_j^{(k+1)}) = V(\mathcal{C}_j^{(k)})$. We refer to this heuristic algorithm as ***MinTotalDistance-var***.

Theorem 3: There is a heuristic algorithm for the service cost minimization problem with variable maximum charging cycles, which takes $O(\frac{\tau_{max}}{\tau_{min}} n^2 + \frac{T}{\tau_{min}} n + \frac{\tau_{max}^2}{\tau_{min}^2})$ time, where $n = |V|$, $\tau_{max} = \max_{i=1}^{n}\{\tau_i\}$, and $\tau_{min} = \min_{i=1}^{n}\{\tau_i\}$.

Proof: It is obvious that the heuristic algorithm delivers a feasible solution. The time complexity analysis is similar to the one in Theorem 3, omitted. ■

VII. Performance Evaluation

In this section, we evaluate the performance of the proposed algorithms through experimental simulations. We also study the impact of important parameters on the algorithm performance, including the network size, sensor charging cycle distributions, and the ratio of the maximum charging cycle to the minimum charging cycle.

A. Experimental environment

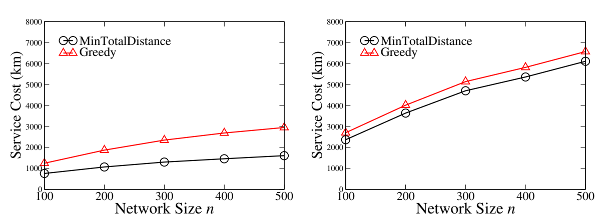
We consider a WSN consisting of from 100 to 500 sensors in a $1,000m \times 1,000m$ square area, in which sensors are randomly deployed. The base station is located at the center of the square. There are 5 depots in the WSN ($q = 5$) and at each depot there is a mobile charger. To reduce the total traveling distance of the q mobile chargers, one depot is co-located with the base station, as the most energy-consuming sensors in a WSN are usually close to the base station for relaying data for other remote sensors. The rest of $q - 1$ depots are randomly distributed in the area. The entire monitoring period is $T = 1,000$ and period T is partitioned into equal time slots with each lasting ΔT (ΔT typically is short, e.g. $\Delta T = 10$). We assume that the maximum charging cycle $\tau_i(t)$ of each sensor $v_i \in V$ does not change within each time slot ΔT. Even it does change, the difference can be neglected.

We adopt two distributions of sensor charging cycles: the *linear distribution* and the *random distribution*. In the linear distribution, the charging cycle τ_i of each sensor $v_i \in V$ is chosen from an interval $[\bar{\tau}_i - \sigma, \bar{\tau}_i + \sigma]$, where $\bar{\tau}_i$ and σ are the average and variance of its charging cycle, respectively, and $\sigma = 2$ in the default setting. The average charging cycle $\bar{\tau}_i$ of sensor v_i is proportional to its distance to the base station, i.e. the sensors nearest to the base station have the minimum average charging cycle τ_{min} while the farthest sensors have the maximum average charging cycle τ_{max}. While in the random distribution, its charging cycle τ_i is randomly chosen from the interval $[\tau_{min}, \tau_{max}]$, where $\tau_{min} = 1$ and $\tau_{max} = 50$ in the default setting. The linear distribution characterizes the sensor energy consumptions in most WSN applications, in which data transmission takes a large portion of sensor energy consumption. Since sensors near to the base station have to relay sensing data from remote sensors, they consume their energy much quicker and result in a shorter charging cycle. Furthermore, by adjusting the ratio $\frac{\tau_{max}}{\tau_{min}}$, this model can also incorporate data aggregation at intermediate sensors, i.e. a smaller value of $\frac{\tau_{max}}{\tau_{min}}$ indicates a higher extent of data aggregation. On the other hand, the random distribution captures the sensor energy consumptions in multimedia sensor networks, where camera sensors can consume substantial energy on image processing [1]. Thus, the maximum charging cycle of one sensor in such a network is not closely related to its distance to the base station.

To compare the performance of the proposed algorithms, we also implement a greedy charging algorithm. In the greedy algorithm, each sensor sends a charging request to the base station when it will deplete its energy soon. Once receiving a request, the base station commands the q mobile chargers to charge those sensors whose estimated residual lifetimes are less than a given threshold Δl with $\Delta l = \tau_{min} = 1$. Each value in figures is the average of the results by applying each mentioned algorithm to 100 different network topologies with the same network size.

B. Performance with fixed maximum charging cycles

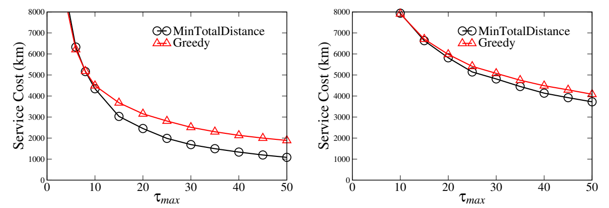
In this subsection, we evaluate the performance of algorithm `MinTotalDistance` against the greedy algorithm, assuming that maximum charging cycles within T are fixed. We first study the performance of the algorithms by varying network size n from 100 to 500 under the two different charging cycles distributions. Fig. 1 (a) shows that the service cost delivered by algorithm `MinTotalDistance` is 55% to 60% of the service cost by the greedy algorithm under the linear distribution. In contrast, Fig. 1 (b) demonstrates that the service cost by the former algorithm is 87% to 93% of the service cost by the latter one. The rationale behind is that, under the linear distribution, the sensors with short charging cycles are near the base station, while under the random distribution, the sensors with short charging cycles are randomly located in the network, these q mobile chargers must travel much longer to replenish them.



(a) linear distribution (b) random distribution

Fig. 1. Performance of algorithms `MinTotalDistance` and `Greedy` by varying network size when $\tau_{min} = 1$ and $\tau_{max} = 50$.

We then examine the performance of the two algorithms by varying τ_{max} from 1 to 50 while fixing $\tau_{min} = 1$. Fig. 2 (a) clearly present that the service cost by algorithm



(a) linear distribution (b) random distribution

Fig. 2. Performance of algorithms `MinTotalDistance` and `Greedy` by varying the maximum charging cycle τ_{max} when $\tau_{min} = 1$ and $n = 200$.

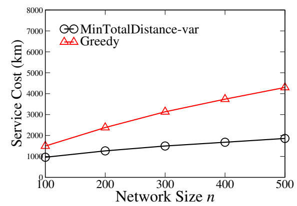


Fig. 3. Performance of algorithms `MinTotalDistance-var` and `Greedy` by varying network size when $\tau_{min} = 1$, $\tau_{max} = 50$, $\Delta T = 10$ and $\sigma = 2$.

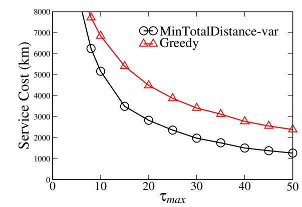


Fig. 4. Performance of algorithms `MinTotalDistance-var` and `Greedy` by varying τ_{max} when $n = 200$, $\tau_{min} = 1$, $\Delta T = 10$ and $\sigma = 2$.

`MinTotalDistance` is almost identical to the one by the greedy algorithm when the maximum charging cycle τ_{max} is no greater than 10. However, the service cost by the former algorithm significantly outperforms that by the latter one when τ_{max} is larger than 10, and the gap between them becomes bigger with the increase of τ_{max}. This is because that when the value of τ_{max} is small (e.g. $\tau_{max} = 5$), most sensors in the network have short charging cycles and the mobile chargers have to visit most area in the network to charge them. On the other hand, more sensors have longer charging cycles when τ_{max} becomes bigger. The proposed algorithm `MinTotalDistance` takes both locations and charging cycles of sensors into consideration while the greedy algorithm only considers charging each sensor as less frequently as possible (i.e. replenish each sensor almost at its maximum charging cycle). We also note that the approximation ratio $2(K+2)$ delivered by algorithm `MinTotalDistance` is small when τ_{max} is not large, where $K = \lfloor \log_2 \frac{\tau_{max}}{\tau_{min}} \rfloor$, which indicates that the service cost by algorithm `MinTotalDistance` is fractional of the optimal. Fig. 2 (b) also show that the performance of both algorithms are only marginally different under the random distribution. Therefore, in the rest we only evaluate their performance under the linear distribution.

C. Performance with variable maximum charging cycles

In this subsection, we first investigate the performance of the proposed algorithm `MinTotalDistance-var` against the greedy algorithm with variable maximum charging cycles. Fig. 3 and Fig. 4 illustrate the performance of both algorithms under the linear distribution, by varying network size n and the maximum charging cycle τ_{max}, from which we can see that algorithm `MinTotalDistance-var` is still competitive as it did under fixed maximum charging cycles.

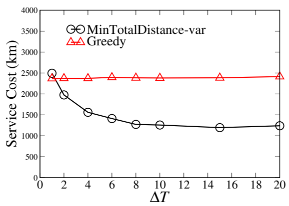


Fig. 5. Performance of algorithms `MinTotalDistance-var` and `Greedy` by varying ΔT when $n = 200$, $\tau_{min} = 1$, $\tau_{max} = 50$, and $\sigma = 2$.

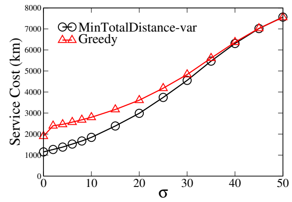


Fig. 6. Performance of algorithms `MinTotalDistance-var` and `Greedy` by varying σ when $n = 200$, $\tau_{min} = 1$, $\tau_{max} = 50$, and $\Delta T = 10$.

We then study the stability of charging cycles on the algorithm performance by varying parameter ΔT from 1 (i.e. extremely unstable) to 20 (i.e. rather stable). Fig. 5 shows that the service cost by algorithm `MinTotalDistance-var` is almost identical to the one by the greedy algorithm when the charging cycles are extremely unstable ($\Delta T = 1$) and their service costs decrease with the increase of the stability of the sensor charging cycles (a larger ΔT). We also note that algorithm `MinTotalDistance-var` outperforms the greedy algorithm significantly even when charging cycles are only fixed in a short time slot ΔT (e.g. $\Delta T = 4 \ll \tau_{max} = 50$), which indicates that algorithm `MinTotalDistance-var` can quickly adapt to charging cycle changes.

We finally investigate the impact of the variance σ of charging cycles on the algorithm performance by varying it from 0 to 50. Fig. 6 present that the service costs delivered by both algorithms increase with a larger value of variance σ. In particular, the service cost by algorithm `MinTotalDistance-var` becomes as high as that by the greedy algorithm when σ is quite large (e.g. $\sigma = 50$). The rationale behind this is that when σ is large, it is more likely that the sensors far from the base station can have short charging cycles, as the charging cycle of sensor v_i is chosen from a large interval $[\bar{\tau}_i - \sigma, \bar{\tau}_i + \sigma]$, thereby leading to the increase of the service cost of the q mobile chargers.

VIII. Conclusions

In this paper, we studied the use of multiple mobile chargers to charge sensors in a wireless sensor network in a flexible on-demand manner, such that none of the sensors runs out of its energy for a monitoring period T. To this end, we first formulated a novel service cost minimization problem, by finding a series of charging schedulings of these mobile chargers to maintain the perpetual operations of sensors so that the total traveling distance of these chargers during T is minimized. As this optimization problem is NP-hard, we then devised an approximation algorithm for it with a provable approximation ratio if maximum charging cycles of sensors are fixed. Otherwise, we developed a novel heuristic solution through minor modifications to the approximate solution. We finally conducted extensive experiments through simulations to evaluate the performance of the proposed algorithms and experimental results show that the proposed algorithms are very promising.

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