Performance Analysis of Distributed Raptor Codes in Wireless Sensor Networks

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***Abstract*—In this paper, we propose a distributed network coding (DNC) scheme based on the Raptor codes for wireless sensor networks (WSNs), where a group of sensor nodes, acting as source nodes, communicate with a single sink through some other sensor nodes, serving as relay nodes, in a multi-hop fashion. At the sink, a graph-based Raptor code is formed on the fly. After receiving a sufficient number of encoded packets, the sink begins to decode. The main contributions of this paper are the derivation of a bit error rate (BER) lower bound for the LT-based DNC scheme over Rayleigh fading channels under maximum-likelihood (ML) decoding, and the derivations of upper and lower BER bounds for the proposed Raptor-based DNC scheme on the basis of the derived BER bound of LT codes.**

***Index Terms*—Distributed network coding, wireless sensor networks, Raptor codes, Quasi-static Rayleigh fading channel, upper and lower bounds.**

I. Introduction

WIRELESS systems, such as last mile, sensor or community networks, are likely to be a major form of future communications. Next generation wireless communication networks will go beyond point-to-point or point-to-multipoint paradigms of existing cellular networks. They will be based on complex interactions, where individual communication nodes cooperate with one another in order to improve the performance of their own communication and that of the entire network. Cooperative communications [1], [2], based on the use of relay nodes, have emerged as a promising approach to increase spectral and power efficiency, communication coverage and reliability.

As a special channel coding strategy developed for cooperative communication networks, distributed coding techniques attracted significant research interest recently. The distributed code construction concept has been applied to conventional channel coding to form, for example, distributed turbo codes [3], distributed space-time codes [4] and distributed low-density parity-check (LDPC) codes [5]. These developments show that the distributed coding schemes can improve the transmission reliability over point-to-point wireless communication channels.

The distributed coding schemes discussed above are mainly developed for small-scale unicast relay networks, in which messages are sent from a single source to a single sink through single/multiple relay nodes. In large-scale wireless sensor networks (WSNs), a huge number of sensor nodes are deployed to gather information from the surrounding environment. The information gathered from these sensor nodes, which are referred to as source nodes, are often delivered to a common sink through other sensor nodes, serving as relay nodes. As the source and relay nodes are in different spatial locations, their signals can be combined at the destination to achieve spatial diversity. Clearly, if a sink receives replicas of the transmitted data via multiple relay nodes, its data will have higher spacial diversity and hence better performance [6]. The standardization activities in IEEE 802.11 WiFi [7], 802.15 Zigbee [8], 802.16j/m WiMAX [9] attest to the vital role that multiple relays will be playing in future broadband communication networks.

Compared with a traditional sensor network in which a predefined route is first established and the relay nodes along the route simply store and forward the received data, it has been shown that cooperative communication offers a significant advantage in improving communication reliability, increasing spectral efficiency and reducing energy consumption [1], [2]. A widely used cooperative communication technique is network coding [10].

When network coding is employed, the relay nodes encode the packets received from multiple source nodes. Coding operations enable the relay nodes to compress the data, and whenever possible, to reduce the number of transmissions and bandwidth consumption. Prior work shows that network coding achieves the multi-cast network capacity by transmitting linear combinations of received data [10], [11], [12].

Due to the broadcast nature of wireless channels, network coding appears to be a natural fit for cooperative communication networks where multiple source nodes communicate with a common sink through multiple relay nodes. There are a large number of published papers in this area, e.g., [13], [14], [15], [16], [17], [18], [19]. An important realization of network coding in wireless sensor networks is a scheme referred to as adaptive network coded cooperation (ANCC), which exploits a

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combined network-channel coding gain [18]. In that approach, the authors proposed a distributed coding scheme in which a real-time network topology is matched with the graphs of LDPC and other graph codes. In this way, the network code design problem is simplified to the design of a graph code. This scheme is further considered in [20], where extrinsic information transfer (EXIT) charts are employed to design irregular low-density generator matrix (LDGM) codes [21] for a WSN over Rayleigh fading channels.

In the aforementioned works, source nodes are assumed to be able to communicate directly with their common sink. However, in some WSN applications, where direct links are not always available, communications of data occur in a multi-hop way. Furthermore, reliable transmission mechanisms, such as Automatic Repeat reQuest (ARQ), are often neglected in the literature. As a result, network coding is only performed at the physical layer.

Fountain codes [22], which are also referred to as rateless codes, are a class of reliable transmission codes by using continuous transmission schemes. A class of practical Fountain codes is Luby-transform (LT) codes [23]. LT codes are capable of recovering the original information from any set of output symbols whose size is close to the optimal value [24]. To apply the rateless codes to the design of reliable transmission strategies, an LT-based distributed network coding (DNC) scheme for WSNs is proposed in [25]. In that work, the authors considered multiple source nodes communicating with a common sink via multi-hop paths and derived upper bounds for the LT-based schemes over Rayleigh fading channels.

In this paper, we derive a lower bound for the LT-based DNC scheme over Rayleigh fading channels. We also propose a DNC scheme for WSNs based on Raptor codes [24], in which a conventional outer code is concatenated with an inner LT code to reduce the error floor. The main challenge of introducing Raptor codes into WSNs is to operate precoding in a decentralized manner [26]. In [27], a random-walk based decentralized fountain codes algorithm is proposed. In that approach, the precoding nodes are selected among the relay nodes with a special calculated probability and each selected precoding node performs precoding according to a special defined degree distribution. In [28], a distributed rateless code for WSNs is proposed. In that approach, the precoding nodes are selected among the source nodes. It is realized by following steps: First, each source node broadcasts its symbol to the source nodes as well as to the relay nodes. Then, each source node generates a random number uniformly distributed between 0 and 1. By comparing the generated value with a pre-defined value, the source nodes are divided into two groups, one group acts as source nodes only, the other one acts as both source nodes and precoding nodes. Finally, the precoding nodes perform precoding according to a pre-defined degree distribution.

In this paper, a WSN with multiple source nodes, multiple relay nodes and a single sink is considered. All the source nodes send data packets to the relay nodes by using a random access MAC (Media Access Control) protocol. All the relay nodes which participate in the process of transmission formed two relay groups, namely, a precoding relay group and an LT-coding relay group. The groups are formed in a decentralized way. There is no need of a central coordinator or elaborate inter-node collaboration to select precoding relay nodes or LT-coding relay nodes. But a central coordinator is needed to control the operations of the precoding relay nodes. Each relay node in the LT-coding relay group selects packets randomly from its buffer and then performs a linear network coding and a graph code is formed on the fly at the sink.

We derive analytical upper and lower bounds for the bit error rate (BER) of the proposed Raptor-based DNC scheme over Rayleigh fading channels under maximum-likelihood (ML) decoding [29], [30]. These derived bounds are shown to be asymptotically tight with the increasing of the expanding coefficient. As the derived analytical bounds are tight for large SNRs, our ML analysis is of interest and may be used to optimize degree distributions for the proposed scheme [31, Ch. 8, p. 283]. The analytical bounds can also be used to compare the performance of Raptor codes with different parameters.

The remainder of the paper is organized as follows. In Section II, a brief review of the LT codes and Raptor codes is given. Section III introduces the system model. In Section IV, the lower bound of the LT-based DNC scheme over Rayleigh fading channels is developed under the assumption of ML decoding. Based on the developed BER bounds of LT-based DNC scheme, we derive the upper and lower BER bounds of the Raptor-based DNC scheme over Rayleigh fading channels under ML decoding. The numerical and simulation results are given in Section V. Conclusions are drawn in Section VI.

II. Preliminary

A. LT Codes

LT codes are the first invented universal fountain codes. Universal fountain codes refer to a class of codes whose original symbols can be recovered from any set of output symbols, whose size is close to optimal, with high probability. Let $\mathbf{G}$ be the generator matrix of LT codes. The degree distribution of $\mathbf{G}$ can be described by $\mu(x) = \sum_{d=1}^{k} \mu_d x^d$, where μ_d denotes the probability that d of the k input symbols are chosen. The encoding process is carried out by two phases:

- Choose a degree d from the distribution $\mu(x)$ with probability μ_d.
- Choose d input symbols uniformly from the k input symbols, and operate XOR on the d input symbols to form an output symbol.

The process of construction and transmission of output symbol is repeated until a sufficient number of output symbols are obtained at the receiver. Assume that the total number of the output symbols is $Q = \eta k$, where η is called the expanding coefficient.

B. Raptor Codes

Raptor codes are the first class of fountain codes with linear time encoding and decoding. The key idea of Raptor coding is concatenating a traditional error correcting code with an LT code to relax the condition that all input symbols need to be recovered in LT codes. The encoding process is carried out by two phases:

- Encode k input symbols with an (n, k) error correcting block code to form n intermediate symbols.

- Encode the n intermediate symbols with an LT code. Each output symbol is generated by randomly choosing a degree d from the degree distribution $\mu(x)$, choosing d distinct input symbols uniformly at random, and operating XOR on them.

Thus, a Raptor code can be specified by parameters $(k, C, \mu(x))$, where C is the (n, k) error correcting block code, called the pre-code, and $\mu(x)$ is the generator polynomial of the degree distribution of the LT code.

Corresponding to the encoding process, the decoding process also can be represented by two phases:

- Recover required fraction of intermediate symbols from the received symbols at the sink.
- Recover all input symbols from the fixed fraction of intermediate symbols, which is recovered at the first phase.

Raptor codes have a significant theoretical and practical improvement over LT codes. Raptor codes require $O(1)$ time to generate an encoding symbol.

III. System Model

We consider a WSN in which the sensor nodes are scattered in a sensor field as shown in Fig. 1. Each of these scattered sensor nodes has the capability to collect data and route data back to the sink. We assume that there is a single sink in the sensor network. We further assume that K_L out of the sensor nodes have data to transmit. These K_L sensor nodes are referred to as source nodes. The rest sensor nodes have no data to transmit and act as relay nodes.

We consider a random access MAC protocol, e.g., Carrier Sense Multiple Access-Collision Avoidance (CSMA-CA), which is widely used in WSNs, such as ZigBee networks. Further, we only consider 3-hop transmissions. We assume that all the sensor nodes in the network know primarily the degree distributions of the designed distributed Raptor codes.

The relay nodes can be classified into two groups: a precoding relay group $\mathbb{R}_1$, and an LT-coding relay group $\mathbb{R}_2$. Depending on the header information of received packets, a relay node determines which group it belongs to. We assume that we have a pre-knowledge of the total number of sensor nodes which may act as relay nodes in the precoding relay group and denote this number by K_P. The pre-knowledge can be obtained through a pre-test of the sensor network or simply by estimating the number of the relay nodes which are within the transmission range of the source nodes. More detailed description of the classification of the two groups will be given later in this section. Fig. 1 shows the model of the investigated system. Let S_i, $R^1_{j'}$, R^2_j and D represent the ith source node, the j'th relay node in relay group $\mathbb{R}_1$, the jth relay node in relay group $\mathbb{R}_2$ and the sink, respectively, where $i \in \{1, 2, \ldots, K_L\}$, $j' \in \{1, 2, \ldots, K_P\}$ and $j \in \{1, 2, \ldots, N\}$.

We assume that the channels between the source and relay nodes in the precoding relay group, between the relay nodes in the two relay groups, and between the relay nodes in the LT-coding relay group and the sink are quasi-static Rayleigh fading channels. We denote by $\hbar^{os}_{i,j'}$ the fading coefficient for the link between the ith source node S_i and the j'th relay node $R^1_{j'}$ in relay group $\mathbb{R}_1$. Let $\hbar^{to}_{j',j}$ and $\hbar^{Dt}_j$ be the channel fading coefficients for the links between $R^1_{j'}$ and the jth relay node R^2_j in relay group $\mathbb{R}_2$, and between R^2_j and the sink D, respectively. The fading coefficients $\hbar^{os}_{i,j'}$, $\hbar^{to}_{j',j}$ and $\hbar^{Dt}_j$ remain constant over the length of one data packet, but change independently from packet to packet. These fading coefficients have unit mean square values [32], i.e., $E\{\mid \hbar^{os}_{i,j'} \mid^2\} = 1$, $E\{\mid \hbar^{to}_{j',j} \mid^2\} = 1$ and $E\{\mid \hbar^{Dt}_j \mid^2\} = 1$. Here $E\{\cdot\}$ denotes the expectation.

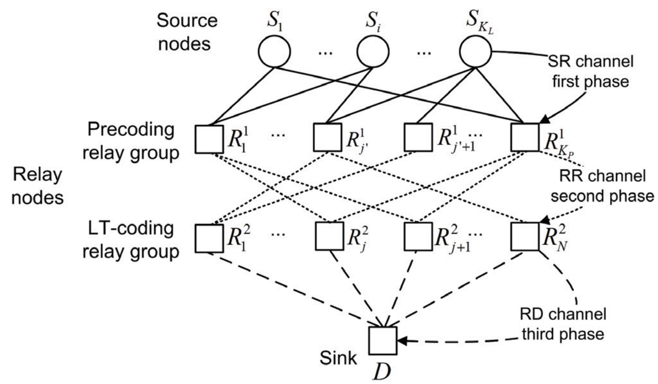

Fig. 1. A network graph used to describe the system model.

In our analysis we assume that all the communication channels, from the source nodes to the relay nodes in precoding relay group $\mathbb{R}_1$, from the relay nodes in $\mathbb{R}_1$ to the relay nodes in LT-coding relay group $\mathbb{R}_2$, and from the relay nodes in $\mathbb{R}_2$ to the sink, are spatially independent and we allow the relay nodes to operate with different transmit powers. For simplicity, we adopt the binary phase shift keying (BPSK) [33] modulation and assume perfect channel state information (CSI) at the receivers.

We assume that each symbol in the information sequence of each source node is generated according to a statistically independent and identically distributed (i.i.d) probability distribution function. Binary symbols are considered in this paper. The data delivery from the source nodes to the sink is carried out in four phases.

- Broadcast phase: The information sequence of each source node is segmented into a number of equal length data segments at first. Then a header and a cycle redundance check (CRC) code are added to each data segment to form a data packet. The header consists of the identity of the node, the number of hops of the transmitted packets, etc. The number of hops for the packets transmitted from source nodes is set to 1.
- Precoding phase: When a relay node receives a packet, it decodes the packet and checks the correctness of the decoded packet by using CRC. If the packet is correct, the relay node then checks the number of hops of the received packet. If the number is equal to 1, the node determines that it belongs to the precoding relay group. We assume that there exists a central coordinator controlling the operations of the relay nodes in $\mathbb{R}_1$. At first, each relay node in $\mathbb{R}_1$ is allocated a number p with probability Ω_p according to a pre-defined degree

distribution[1] $\Omega(x) = \sum_{p=1}^{n} \Omega_p x^p$. Denote by $\mathbf{m}_i \in \text{GF}(2)^{\epsilon\times 1}$, $i \in \{1, 2, \cdots, K_L\}$, the data packet of the ith source node. The received data packet from S_i at $R^1_{j'}$, ($R^1_{j'} \in \mathbb{R}_1$), can be written as $\mathbf{y}^{os}_{i,j'} = \hbar^{os}_{i,j'}\mathbf{x}_i + \mathbf{n}^{o}_{i,j'}$. Here $\mathbf{x}_i = (-1)^{\mathbf{m}_i} \in \{\pm 1\}^{\epsilon}$ is the transmitted data packet from S_i, $\mathbf{n}^{o}_{i,j'}$ is the additive white Gaussian noise vector of size $\epsilon \times 1$ at $R^1_{j'}$, where ϵ denotes the length of the data packet. If the decoded packet is correct, the node then puts the correctly decoded data segments together with its connection information into its buffer. After collecting enough number of data segments, i.e., the number of data segments in a relay node's buffer is larger than its allocated number p, the relay node selects a number p of data segments from its buffer to perform network coding by using linear combinations in the field of GF(2) [29]. Since $p \ll K_L$, it is with very high probability that there are p correctly decoded data segments in the buffer. Thus, we can always assume that this network coding process is achievable. The network-coded data are then augmented with a header and a CRC code to form a network-coded data packet. The header contains the connection information and the number of hops of the packet. The connection information includes the identities of the source nodes which are used to form the network coded packet and the identity of the relay node from which the coded packet is broadcast. The number of hops for the packets transmitted from the relay nodes in precoding group $\mathbb{R}_1$ is set to 2. The network-coded data packet is then broadcast at each relay node in $\mathbb{R}_1$ to the network with a probability P_P, where $P_P = \frac{K}{K_P}$. Here $K = \frac{K_L}{r}$ denote the total number of the selected relay nodes in $\mathbb{R}_1$ and r is the desired code rate of the pre-code.

- LT-coding phase: If a relay node receives a packet with the number of hops of 2, then it determines that it belongs to the LT-coding relay group[2]. In the LT-coding phase, each relay node in $\mathbb{R}_2$ randomly selects a number of q with probability μ_q according to a pre-stored degree distribution[3] $\mu(x) = \sum_q \mu_q x^q$. Each relay node in $\mathbb{R}_2$ receives packets from the relay nodes in $\mathbb{R}_1$. The received data packet from $R^1_{j'}$ at R^2_j can be represented as $\mathbf{y}^{to}_{j',j} = \hbar^{to}_{j',j}\mathbf{x}^{o}_{j'} + \mathbf{n}^{t}_{j',j}$. Here $\mathbf{x}^{o}_{j'} = (-1)^{\mathbf{s}_{j'}} \in \{\pm 1\}^{\epsilon}$ is the modulated data packet transmitted from $R^1_{j'}$, where $\mathbf{s}_{j'} \in GF(2)^{\epsilon\times 1}$ is the network-coded data packet at $R^1_{j'}$. And $\mathbf{n}^{t}_{j',j}$ is the additive white Gaussian noise vector of size $\epsilon \times 1$ at R^2_j. Each relay node in $\mathbb{R}_2$ first decodes the received data packets from the relay nodes in $\mathbb{R}_1$ and checks the correctness using CRC code, then puts the correctly decoded data segments together with their connection information into its buffer. After collecting enough number of data segments, i.e., the number of data segments in the relay node's buffer is larger than its previously selected number q, each relay node in $\mathbb{R}_2$ randomly selects a number q of data segments from its buffer to perform network coding by using linear combinations in the field of GF(2). Since $q \ll K$, we can always assume that this network coding process is possible to achieve. The formed network coded data segment is then appended with a CRC code and a header. The header contains the connection information and the number of hops of the packet. The connection information includes the identities of the source nodes and the relay nodes in $\mathbb{R}_1$ which are used to form the network coded data segment, it also includes the identity of the relay node in $\mathbb{R}_2$ from which the packet is transmitted. The number of hops for the packets transmitted from any nodes in $\mathbb{R}_2$ is set to 3. The network-coded packet is then broadcast to the sink.
- Data recovery phase: The sink collects enough packets transmitted from the relay nodes in the LT-coding group, then begins to decode. The sink node knows whether the packets are from LT coding group by checking the number of hops of the received packets. If the number of hops equal to 3, then it is from the LT-coding group. The received data packet from the jth relay node in $\mathbb{R}_2$ at the sink D can be expressed as $\mathbf{y}^{Dt}_j = \hbar^{Dt}_j\mathbf{x}^t_j + \mathbf{n}^D_j$, where $\mathbf{x}^t_j \in \{\pm 1\}^{\epsilon}$ is the modulated data packet transmitted from R^2_j, and $\mathbf{n}^D_j$ is the additive white Gaussian noise vector of size $\epsilon \times 1$. After receiving Q data packets from the relay nodes in relay group $\mathbb{R}_2$, where $Q \geq K$, the sink begins to decode by using the ML algorithm, based on the soft information of the received data packets.

[1]In the precoding phase, LDPC codes are used. Each relay node, which belongs to the precoding relay group, performs its next operation under the control of the central coordinator. $\Omega(x)$ is the generator polynomial of the degree distribution of the LDPC codes [24], [34]. The information of the degree distribution $\Omega(x)$ is previously stored in the central coordinator.

[2]Note that the relay node belonging to the LT-coding relay group may also belong to the precoding relay group. But in different phases, it performs its operation according to different degree distributions.

[3]In the LT-coding phase, LT codes are used. Each relay node, which belongs to the LT-coding relay group, performs its next operation according to the degree distribution of the LT codes. $\mu(x)$ is the generator polynomial of the degree distribution of the LT codes [23], [24]. The information of the degree distribution $\mu(x)$ is previously stored in the sensor nodes.

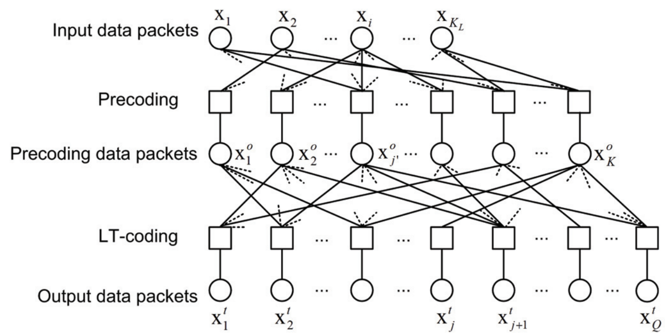

Fig. 2. A Tanner graph of Raptor code derived from the system model.

Since a small header, which contains the connection information of the sensor nodes, is added in each packet when each relay node forwards data, the sink knows how the checks are formed and can accordingly replicate the code on graph. The length of each header is much shorter than that of the data packet, therefore, the throughput loss due to the header can be neglected.

At the sink, we can map the network topology described above to a Tanner graph associated with the Raptor codes, as shown in Fig. 2. The formed code graph consists of K_L input data packets, corresponding to the K_L source nodes, K

precoding data packets, corresponding to the K relay nodes which are selected to transmit network-coded packets in $\mathbb{R}_1$, and Q output data packets. Here Q represents the total number of transmissions from all the relay nodes in $\mathbb{R}_2$ at a time instant.

If the data packets cannot be recovered completely at the sink, the sink simply sends a data request to the $(Q+1)$th[4] relay node in relay group $\mathbb{R}_2$ to ask for another network-coded data packet, and then carries out decoding. This process runs on until all the information are recovered. Since the relay nodes are asked for data transmission one by one, redundant transmissions are avoided.

IV. Upper and Lower Bounds on the Bit Error Probability of Raptor Codes

In this section, we first derive the lower BER bound of the LT-based DNC scheme with ML decoding over Rayleigh fading channels, which will be used for the derivation of the lower BER bound of the Raptor-based DNC scheme. Then we develop both upper and lower BER bounds on the bit error probability of the proposed finite-length Raptor-based DNC scheme under ML decoding over Rayleigh fading channels.

The data packets from all the source nodes are arranged into a matrix $\mathbf{\Lambda} = [\mathbf{m}_1, \mathbf{m}_2, \cdots, \mathbf{m}_{K_L}] \in \mathrm{GF}(2)^{\epsilon \times K_L}$. All the data packets formed at all the relay nodes in $\mathbb{R}_1$ during the precoding phase are grouped into a matrix $\mathbf{\Lambda}_s = [\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_K] \in \mathrm{GF}(2)^{\epsilon \times K}$, where $\mathbf{s}_{j'} \in \mathrm{GF}(2)^{\epsilon \times 1}$, $j' \in \{1, 2, \cdots, K\}$, is the network-coded data packet of the j'th selected relay node in $\mathbb{R}_1$. The output data packets of the relay nodes in $\mathbb{R}_1$ and $\mathbb{R}_2$ are formed as described in Section III, according to the pre-determined degree distributions $\Omega(x)$ and $\mu(x)$, respectively.

The network-coded data packets formed at $\mathbb{R}_2$ in the LT-coding phase are then transmitted over quasi-static Rayleigh fading channels to the sink D. Let Q denote the number of data packets collected at the sink. Then Q can be expressed as $Q = \eta K = \frac{\eta}{r} K_L$, where the expanding coefficient η is a real number equal to or greater than one.

The output from the Q relay nodes in $\mathbb{R}_2$ can be represented by $\mathbf{V} = \mathbf{\Lambda}_s \mathbf{G} = \mathbf{\Lambda} \mathbf{G}_L \mathbf{G}$, where $\mathbf{V} \in \mathrm{GF}(2)^{\epsilon \times Q}$. $\mathbf{G}_L$ is a randomly generated $K_L \times K$ binary matrix formed at $\mathbb{R}_1$, which is associated with the graph code used in the precoding phase of the Raptor code. $\mathbf{G}$ is a randomly generated $K \times Q$ binary matrix formed at $\mathbb{R}_2$, which is associated with the LT code. The jth column of $\mathbf{V}$ is the transmitted data packet from the jth relay node in $\mathbb{R}_2$. Since the binary information sequences of the source nodes are assumed to be i.i.d, we only consider an arbitrary row of $\mathbf{\Lambda}$ in the analysis and denoted by $\mathbf{m}$. Let $\mathbf{v}$ be the corresponding row of $\mathbf{V}$ when $\mathbf{m}$ is chosen as the input, i.e., $\mathbf{v} = \mathbf{m} \mathbf{G}_L \mathbf{G}$.

The parity check matrix of the generator matrix $\mathbf{G}_L$ is denoted by $\mathbf{H}$. The (i, j')th element of $\mathbf{G}_L(i, j')$ is equal to one if the ith source node's data is used to generate the output data at the j'th selected relay node in $\mathbb{R}_1$, otherwise it is zero. In a similar way, $\mathbf{G}(j', j) = 1$, if the data of the j'th selected relay node in $\mathbb{R}_1$ is used to generate the output data of the jth relay node in $\mathbb{R}_2$, otherwise it is zero. Since the both applied codes are linear, their performance is independent of the transmitted information sequence. Thus, we can assume that the all zero sequence is transmitted, i.e., $\mathbf{m} = \mathbf{0}$. The error information sequence is denoted by $\mathbf{e}$.

A. Lower Bound on the Bit Error Probability of LT Codes

Remark: The lower bound on the bit error probability for LT codes over fast Rayleigh fading channels with perfect CSI[5] is given by the probability that only one symbol in the decoded sequence is not recovered correctly [24].

The information sequence error probability can be computed as

$$P_e = P_r \left(\bigcup_{\mathbf{e}: w(\mathbf{e}) \neq 0} \hat{\mathbf{m}} = \mathbf{e} \right), \tag{1}$$

where $\hat{\mathbf{m}}$ is the estimated information sequence, $w(\cdot)$ denotes the Hamming weight.

The lower bound on the bit error probability for LT codes over fast Rayleigh fading channels with perfect CSI is given by the probability that only one symbol in the decoded sequence is not recovered correctly [24]. The weight-1 information sequence error probability can be computed as

$$\begin{aligned} & P_r \left(\bigcup_{\mathbf{e}: w(\mathbf{e}) = 1} \hat{\mathbf{m}} = \mathbf{e} \right) \\ = & \sum_{\mathbf{e}: w(\mathbf{e}) = 1} P_r(\hat{\mathbf{m}} = \mathbf{e}) \\ = & P_r(w(\mathbf{e}) = 1 \text{ and the error } \mathbf{e} \text{ is undetectable}) \\ & + P_r(w(\mathbf{e}) = 1 \text{ and the error } \mathbf{e} \text{ is detectable}). \end{aligned} \tag{2}$$

The probability in (2) for the event that the error $\mathbf{e}$ is undetectable when $w(\mathbf{e}) = 1$, can be calculated by the joint probability $P_r(\mathbf{eG} = \mathbf{0}, w(\mathbf{e}) = 1)$. It is due to the fact that $\mathbf{G}$ may contain a row with all zero elements as the matrix $\mathbf{G}$ is randomly generated. It can be expressed as

$$\begin{aligned} & P_r(\mathbf{eG} = \mathbf{0}, w(\mathbf{e}) = 1) \\ = & \left(\sum_{w(\mathbf{c})} \mu_{w(\mathbf{c})} P_r(\mathbf{e} \otimes \mathbf{c} = \mathbf{0}, w(\mathbf{e}) = 1 | w(\mathbf{c})) \right)^Q, \end{aligned} \tag{3}$$

where $\otimes$ represents vector multiplication over GF(2), $\mathbf{c}$ is a column vector of the matrix $\mathbf{G}$, $\mu_{w(\mathbf{c})}$ is the probability that $\mathbf{c}$ has weight $w(\mathbf{c})$.

The probability $P_r(\mathbf{e} \otimes \mathbf{c} = 0, w(\mathbf{e}) = 1 | w(\mathbf{c}))$ in (3) is equal to the probability of the event that the element at a special position of $\mathbf{c}$, which is corresponding to the position

[4] After receiving the request from the sink, each relay node in the LT-coding relay group transmits a new formed data packet to the sink with the probability of $\frac{1}{N}$, where N is the number of relay nodes in the LT-coding relay group. We assume that this number can be estimated. As a result, every time the sink sends a transmit request to the relay nodes, only one relay node responds to the request. The responding relay node is regarded as the $(Q+1)$th node.

[5] At the sink, the received data packets form a data matrix $\mathbf{V}' = [\mathbf{y}_1^{Dt}, \ldots, \mathbf{y}_j^{Dt}, \ldots, \mathbf{y}_Q^{Dt}]$. The decoding process at the sink is in a line-by-line order. And due to the fact that the fading coefficients remain constant over the length of one data packet but change independently among different packets, the decoding process at the sink can be seen as over a fast Rayleigh fading channel but with perfect CSI.

where the nonzero element occurs in the error sequence $\mathbf{e}$, is zero. Therefore

$$P_r\left(\mathbf{e}\otimes\mathbf{c}=0, w(\mathbf{e})=1|w(\mathbf{c})\right)=\frac{\binom{K-1}{w(\mathbf{c})}}{\binom{K}{w(\mathbf{c})}}=\varsigma_1^{w(\mathbf{c})}. \tag{4}$$

We now focus on the second term $P_r(w(\mathbf{e}) = 1$ and the error $\mathbf{e}$ is detectable$)$ in (2). Note that the probability of the event that the error $\mathbf{e}$ is detectable when $w(\mathbf{e})=1$, is equal to the joint probability $P_r(\mathbf{v}\to\mathbf{v}')$ given $w(\mathbf{e})=1$. Here $\mathbf{v}=\mathbf{mG}$ is the transmitted codeword and $\mathbf{v}'=\mathbf{m}'\mathbf{G}$ is the decoded codeword, with $\mathbf{v}\neq\mathbf{v}'$.

For the Rayleigh fading channel, $P_r(\mathbf{v}\to\mathbf{v}')=\mathbf{E}_{\hbar}\{P_r(\mathbf{v}\to\mathbf{v}'|\hbar)\}$, where $\mathbf{E}_{\hbar}$ is the expectation over the fading coefficient vector $\hbar=[\hbar_1,\hbar_2,\cdots,\hbar_Q]$. Here $\hbar_j$, $j\in\{1,2,\cdots,Q\}$, is the channel coefficient for the wireless channel from the jth relay node in $\mathbb{R}_2$ to the sink. $\hbar_j$ remains constant within one transmitted data packet, but changes packet by packet.

The conditional probability $P_r(\mathbf{v}\to\mathbf{v}'|\hbar)$ is the probability that the decoded codeword is equal to $\mathbf{v}'$ given the channel fading coefficient $\hbar$:

$$P_r(\mathbf{v}\to\mathbf{v}'|\hbar)=Q\left(\sqrt{\frac{1}{2N_0}\sum_{j\in\phi}\hbar_j^2\ 2E_s^jd_j^2}\right), \tag{5}$$

where $d_j^2\triangleq|y_j'-y_j|^2/2E_s^j$ is the normalized squared Euclidean distance between two modulation symbols y_j' and y_j. E_s^j is the transmitted symbol energy for the jth relay node in $\mathbb{R}_2$ and ϕ is the set of all j for which $y_j'\neq y_j$ given $w(\mathbf{e})=1$. Here $\mathbf{y}=[y_1,\cdots,y_Q]$ is the modulated symbol sequence corresponding to the codeword $\mathbf{v}$. $\mathbf{y}'$ is the modulated symbol sequence corresponding to $\mathbf{v}'$.

With the alternative expression of the Q-function [35]

$$Q(y)=\frac{1}{\pi}\int_0^{\pi/2}\exp\left(-\frac{y^2}{2\sin^2(\theta)}\right)d\theta, \tag{6}$$

the error probability for the fast Rayleigh fading channel can be expressed as

$$\begin{aligned}
& P_r(\mathbf{v}\to\mathbf{v}') \\
= \ & \mathbf{E}_{\hbar}\{P_r(\mathbf{v}\to\mathbf{v}'|\hbar)\} \\
= \ & \mathbf{E}_{\hbar}\left\{\frac{1}{\pi}\int_0^{\pi/2}\exp\left(-\frac{\frac{1}{2N_0}\sum_{j\in\phi}\hbar_j^2 2E_s^jd_j^2}{2\sin^2(\theta)}\right)d\theta\right\} \\
= \ & \frac{1}{\pi}\int_0^{\pi/2}\mathbf{E}_{\hbar}\left\{\prod_{j\in\phi}\exp\left(-\frac{\frac{1}{2N_0}\hbar_j^2 2E_s^jd_j^2}{2\sin^2(\theta)}\right)\right\}d\theta \\
= \ & \frac{1}{\pi}\int_0^{\pi/2}\prod_{j\in\phi}\mathbf{E}_{\hbar_j}\left\{\exp\left(-\frac{\hbar_j^2E_s^jd_j^2}{2N_0\sin^2(\theta)}\right)\right\}d\theta.
\end{aligned} \tag{7}$$

The fading coefficient $\hbar_j$ follows the Rayleigh distribution, and the probability density function (PDF) for Rayleigh distribution is $P_r(\hbar_j)=\frac{\hbar_j}{\sigma^2}\exp\left(-\frac{\hbar_j^2}{2\sigma^2}\right)$, where σ is the parameter for Rayleigh distribution, and $\sigma^2=\frac{1}{2}$. Thus, the expectation function $\mathbf{E}_{\hbar_j}\left\{\exp\left(-\frac{\hbar_j^2E_s^jd_j^2}{2N_0\sin^2(\theta)}\right)\right\}$ in (7) can be calculated by

$$\begin{aligned}
& \mathbf{E}_{\hbar_j}\left\{\exp\left(-\frac{\hbar_j^2E_s^jd_j^2}{2N_0\sin^2(\theta)}\right)\right\} \\
= \ & \int_0^{+\infty}\exp\left(-\frac{\hbar_j^2E_s^jd_j^2}{2N_0\sin^2(\theta)}\right)\frac{\hbar_j}{\sigma^2}\exp\left(-\frac{\hbar_j^2}{2\sigma^2}\right)d\hbar_j \\
= \ & \frac{1}{1+\frac{E_s^jd_j^2}{2N_0\sin^2(\theta)}}.
\end{aligned} \tag{8}$$

Therefore, we have

$$P_r(\mathbf{v}\to\mathbf{v}')=\frac{1}{\pi}\int_0^{\pi/2}\prod_{j\in\phi}\frac{1}{1+\frac{E_s^jd_j^2}{2N_0\sin^2(\theta)}}d\theta. \tag{9}$$

Note that E_s^j reflects the fact that different relay nodes may have different power. Assuming that the BPSK modulation is used, then, for $y_j'\neq y_j$, we have that $d_j^2=2$ [33]. Thus, the right side of the equation in (9) becomes $\frac{1}{\pi}\int_0^{\pi/2}\prod_{j\in\phi}\frac{1}{1+\frac{E_s^j}{N_0\sin^2(\theta)}}d\theta$.

Denote by $\gamma_1^{w(\mathbf{c})}$ the probability that the erroneous codeword $\mathbf{v}'$ has a symbol value equal to 1 given that $\mathbf{e}$ has weight $w(\mathbf{e})=1$ and $\mathbf{c}$ has weight $w(\mathbf{c})$. Then we have

$$\begin{aligned}
\gamma_1^{w(\mathbf{c})} &= 1-P_r(\mathbf{e}\otimes\mathbf{c}=0, w(\mathbf{e})=1|w(\mathbf{c})) \\
&= 1-\frac{\binom{K-1}{w(\mathbf{c})}}{\binom{K}{w(\mathbf{c})}}.
\end{aligned} \tag{10}$$

The probability that the erroneous codeword $\mathbf{v}'$ has a symbol value equal to 1 given that $\mathbf{e}$ has weight $w(\mathbf{e})=1$ is denoted by β_1, which is given by

$$\begin{aligned}
\beta_1 &= \sum_{w(\mathbf{c})}\mu_{w(\mathbf{c})}\gamma_1^{w(\mathbf{c})} \\
&= \sum_{w(\mathbf{c})}\left(1-\frac{\binom{K-1}{w(\mathbf{c})}}{\binom{K}{w(\mathbf{c})}}\right) \\
&= \sum_{w(\mathbf{c})}\mu_{w(\mathbf{c})}(1-\varsigma_1^{w(\mathbf{c})}).
\end{aligned} \tag{11}$$

The joint probability $P_r(\mathbf{v}\to\mathbf{v}')$ given that $w(\mathbf{e})=1$, can be represented by the joint probability $P_r(\mathbf{eG}\neq\mathbf{0}, w(\mathbf{e})=1)$. Thus, the probability of the event that the error $\mathbf{e}$ is detectable when $w(\mathbf{e})=1$, can be written as

$$\begin{aligned}
& P_r(\mathbf{eG}\neq\mathbf{0}, w(\mathbf{e})=1) \\
= \ & \sum_{k=1}^{Q}\frac{1}{\pi}\binom{Q}{k}\beta_1^k(1-\beta_1)^{Q-k} \\
& \cdot\int_0^{\pi/2}\left(\frac{1}{1+\frac{E_s^k}{N_0\sin^2(\theta)}}\right)^k d\theta.
\end{aligned} \tag{12}$$

We now note that both (3) and (12) depend on $\mathbf{e}$ only through $w(\mathbf{e})=1$. Also, there are 2^K-1 nonzero error sequences $\mathbf{e}$, of which $\binom{K}{1}$ have weight 1. Taking these facts into account, we have that the information bit error probability of the LT code which can be summarized as:

Lemma: Consider an LT code with parameters $\mu(x)$, K, Q and expanding coefficient η. Its lower BER bound under

ML decoding over a fast Rayleigh fading channel is $\xi_L > \max\{0, \xi_L^{LT}\}$, where ξ_L^{LT} is given by

$$\xi_L^{LT} = \frac{1}{K}\binom{K}{1}\Bigg[\left(\sum_{w(\mathbf{c})}\mu_{w(\mathbf{c})}\varsigma_1^{w(\mathbf{c})}\right)^Q + \sum_{k=1}^{Q}\frac{1}{\pi}\binom{Q}{k}\beta_1^k(1-\beta_1)^{Q-k} \cdot \int_0^{\pi/2}\left(\frac{1}{1+\frac{E_s^k}{N_0\sin^2(\theta)}}\right)^k d\theta\Bigg],$$

where

$$\varsigma_1^{w(\mathbf{c})} = \frac{\binom{K-1}{w(\mathbf{c})}}{\binom{K}{w(\mathbf{c})}},$$

$$\beta_1 = \sum_{w(\mathbf{c})}\mu_{w(\mathbf{c})}(1-\varsigma_1^{w(\mathbf{c})}),$$

and $w(\cdot)$ denotes the Hamming weight, $\mathbf{c}$ is a column vector of the matrix $\mathbf{G}$, $\mu_{w(\mathbf{c})}$ is the probability that $\mathbf{c}$ has weight $w(\mathbf{c})$.

B. Upper Bound on the Bit Error Probability of Raptor Codes

The probability of decoding error for the Raptor codes is calculated by the probability that the information cannot be recovered correctly either by the LT decoder or by the precoding decoder. We assume that after the LT decoder, the error sequence is $\mathbf{e}$. The probability that the precoding decoder fails to recover the information can be computed as

$$P_r(\mathbf{e}\mathbf{H}^T = \mathbf{0}) = \sum_{t=1}^{K} P_r(\mathbf{e}\mathbf{H}^T = \mathbf{0}, w(\mathbf{e}) = t), \quad (13)$$

where $\mathbf{H}^T$ is the transpose matrix of the parity check matrix $\mathbf{H}$.

The term in the right side of (13) can be expressed as

$$P_r(\mathbf{e}\mathbf{H}^T = \mathbf{0}, w(\mathbf{e}) = t) = \left(\sum_{w(\mathbf{h})}\Omega_{w(\mathbf{h})}P_r((\mathbf{e}\otimes\mathbf{h}) = 0, w(\mathbf{e}) = t|w(\mathbf{h}))\right)^{K-K_L} \quad (14)$$

where $\mathbf{h}$ is a column vector of the matrix $\mathbf{H}^T$, $\Omega_{w(\mathbf{h})}$ is the probability that $\mathbf{h}$ has weight $w(\mathbf{h})$.

When $t = 1$, the probability $P_r(\mathbf{e}\otimes\mathbf{h} = 0|w(\mathbf{h}), w(\mathbf{e}) = t))$ in (14) is equal to the probability that all the elements in a special row of $\mathbf{H}^T$ are zeros, and the special row corresponds to the position where the nonzero value occurs in the error sequence $\mathbf{e}$. When $t > 1$, let $\zeta = \{i_1, i_2, \cdots, i_t\}$ be the set of size $|\zeta| = w(\mathbf{e}) = t \leq K$, whose elements are the positions where all the nonzero values exist in $\mathbf{e}$. Let $\mathbf{h}(\zeta)$ be the corresponding subvector of $\mathbf{h}$. Then $P_r(\mathbf{e}\otimes\mathbf{h} = 0|w(\mathbf{h}), w(\mathbf{e}) = t))$ is equal to the probability that $\mathbf{h}(\zeta)$ contains an even number of ones. Therefore

$$P_r(\mathbf{e}\otimes\mathbf{h} = 0, w(\mathbf{e}) = t|w(\mathbf{h})) = \frac{\sum_{\alpha=even,0\leq\alpha\leq t}\binom{t}{\alpha}\binom{K-t}{w(\mathbf{h})-\alpha}}{\binom{K}{w(\mathbf{h})}} = \varsigma_t^{R,w(\mathbf{h})}. \quad (15)$$

Note that (13) depends on $\mathbf{e}$ only through $w(\mathbf{e}) = t$. Also, there are $2^K - 1$ nonzero error sequences $\mathbf{e}$, of which $\binom{K}{t}$ have weight t. Taking these factors into account, we have that the information bit error probability of the Raptor code can be upper bounded by $P_b < \min\{1, \xi_U^{Raptor}\}$, where

$$\xi_U^{Raptor} = \sum_{t=1}^{K}\frac{t}{K}\binom{K}{t}\Big[\xi_U^t(1-\xi_U)^{K-t} \cdot P_r(\mathbf{e}\mathbf{H}^T = \mathbf{0}, w(\mathbf{e}) = t)\Big]. \quad (16)$$

The scaling factor t/K in (16) is the number of erroneous bits per information sequence in a weight-t information error sequence. Therefore, we can get the BER upper bound of Raptor codes which can be described as:

Theorem 1: Consider a Raptor code with parameters $\Omega(x)$, $\mu(x)$, K_L, K, Q, r and expanding coefficient η. Its upper BER bound under ML decoding over a fast Rayleigh fading channel is $P_b < \min\{1, \xi_U^{Raptor}\}$, where ξ_U^{Raptor} is given by

$$\xi_U^{Raptor} = \sum_{t=1}^{K}\frac{t}{K}\binom{K}{t}\xi_U^t(1-\xi_U)^{K-t} \cdot \left(\sum_{w(\mathbf{h})}\Omega_{w(\mathbf{h})}\varsigma_t^{R,w(\mathbf{h})}\right)^{K-K_L},$$

where

$$\varsigma_t^{R,w(\mathbf{h})} = \frac{\sum_{\alpha=even,0\leq\alpha\leq t}\binom{t}{\alpha}\binom{K-t}{w(\mathbf{h})-\alpha}}{\binom{K}{w(\mathbf{h})}},$$

where $\mathbf{h}$ is a column vector of the matrix $\mathbf{H}^T$, $\mathbf{H}^T$ is the transpose matrix of the parity check matrix $\mathbf{H}$, and $\Omega_{w(\mathbf{h})}$ is the probability that $\mathbf{h}$ has weight $w(\mathbf{h})$.

ξ_U is the upper BER bound of the LT codes over the special fast Rayleigh fading channel under ML decoding [25]. It can be written as

$$\xi_U < \min\Bigg\{1, \sum_{\kappa=1}^{K}\frac{\kappa}{K}\binom{K}{\kappa}\Bigg[\left(\sum_{w(\mathbf{c})}\mu_{w(\mathbf{c})}\varsigma_\kappa^{w(\mathbf{c})}\right)^Q + \sum_{k=1}^{Q}\frac{1}{\pi}\binom{Q}{k}\beta_\kappa^k(1-\beta_\kappa)^{Q-k} \cdot \int_0^{\pi/2}\left(\frac{1}{1+\frac{E_s^k}{N_0\sin^2(\theta)}}\right)^k d\theta\Bigg]\Bigg\},$$

where

$$\varsigma_\kappa^{w(\mathbf{c})} = \frac{\sum_{\alpha=even,\alpha\leq\kappa}\binom{\kappa}{\alpha}\binom{K-\kappa}{w(\mathbf{c})-\alpha}}{\binom{K}{w(\mathbf{c})}},$$

$$\beta_\kappa = \sum_{w(\mathbf{c})}\mu_{w(\mathbf{c})}(1-\varsigma_\kappa^{w(\mathbf{c})}).$$

C. Lower Bound on the Bit Error Probability of Raptor Codes

The error probability of Raptor codes can be lower bounded by following the Bonferroni inequality [36]. The probability can be computed as

$$\begin{aligned} P_b &= \sum_{t=1}^{K} \frac{t}{K}\binom{K}{t}\xi_L^t(1-\xi_L)^{K-t}P_r(\mathbf{eH}^T=\mathbf{0}, w(\mathbf{e})=t) \\ &\quad -\frac{1}{2}\sum_{t=1}^{K}\frac{t}{K}\binom{K}{t}\xi_U^t(1-\xi_U)^{K-t} \\ &\quad \cdot P_r(\mathbf{eH}^T=\mathbf{0}, \mathbf{e}'\mathbf{H}^T=\mathbf{0}, w(\mathbf{e})=t), \end{aligned} \tag{17}$$

where $\mathbf{e}'$ is another nonzero error sequence different from the error sequence $\mathbf{e}$.

Let $z \in GF(2)^K$ and $I_z = \{i_{z_1}, i_{z_2}, \ldots, i_{z_\tau}\}$ be the set of indices such that $z(t) = 1$ for $t \in I_z$, otherwise $z(t) = 0$. Define three binary vectors z_0, z_1 and z_2 as $z_0(t) = 1$ if and only if $t \in I_{z_0} := I_e \cap I_{e'}$, $z_1(t) = 1$ if and only if $t \in I_{z_1} := I_e \backslash I_{e'}$ and $z_2(t) = 1$ if and only if $t \in I_{z_2} := I_{e'} \backslash I_e$. Let τ_0, τ_1 and τ_2 be the size of the sets I_{z_0}, I_{z_1} and I_{z_2}, respectively. Then, $P_r(\mathbf{eH}^T = \mathbf{0}, \mathbf{e}'\mathbf{H}^T = \mathbf{0}, w(\mathbf{e}) = t)$ in (17) can be expressed as

$$\begin{aligned} &P_r(\mathbf{eH}^T=\mathbf{0}, \mathbf{e}'\mathbf{H}^T=\mathbf{0}, w(\mathbf{e})=t) \\ &= \sum_{\tau_0=0}^{t}\sum_{\tau_1=t-\tau_0}\sum_{\tau_2=0}^{K-\tau_0}\frac{\binom{t}{\tau_0}}{2^t}\frac{\binom{K-\tau_0}{t-\tau_0}}{2^{K-\tau_0}}\frac{\binom{K-\tau_0}{\tau_2}}{2^{K-\tau_0}}\Bigg(\sum_{w(\mathbf{h})}\Omega_{w(\mathbf{h})} \\ &\quad \cdot P_r(\mathbf{e}\otimes\mathbf{h}=\mathbf{0}, \mathbf{e}'\otimes\mathbf{h}=\mathbf{0}, w(\mathbf{e})=t|\tau_0,\tau_1,\tau_2)\Bigg)^{K-K_L} \end{aligned} \tag{18}$$

Corresponding to the three sets I_{z_0}, I_{z_1} and I_{z_2}, each column of the matrix $\mathbf{H}^T$, $\mathbf{h}$, can be divided into three parts, $\mathbf{h}_{\tau_0}$, $\mathbf{h}_{\tau_1}$ and $\mathbf{h}_{\tau_2}$. Let $\mathbf{h}_{\tau_0}$ be the subvector of $\mathbf{h}$ such that all the elements of this subvector are selected from $\mathbf{h}$ according to the indices in set I_{z_0}. The length of $\mathbf{h}_{\tau_0}$ is τ_0. The same operation is applied to the formation of $\mathbf{h}_{\tau_1}$ and $\mathbf{h}_{\tau_2}$, in which the elements are selected according to the indices in set I_{z_1} and I_{z_2}, and have length τ_1 and τ_2, respectively. Therefore, the conditional probability of the last product term in (22) can be computed as

$$\begin{aligned} &P_r(\mathbf{e}\otimes\mathbf{h}=\mathbf{0}, \mathbf{e}'\otimes\mathbf{h}=\mathbf{0}, w(\mathbf{e})=t|\tau_0,\tau_1,\tau_2) \\ &= P_r(\mathbf{e}_{\tau_0}\otimes\mathbf{h}_{\tau_0}=\mathbf{0})P_r(\mathbf{e}_{\tau_1}\otimes\mathbf{h}_{\tau_1}=\mathbf{0}) \\ &\quad \cdot P_r(\mathbf{e}'_{\tau_2}\otimes\mathbf{h}_{\tau_2}=\mathbf{0}) + P_r(\mathbf{e}_{\tau_0}\otimes\mathbf{h}_{\tau_0}=\mathbf{1}) \\ &\quad \cdot P_r(\mathbf{e}_{\tau_1}\otimes\mathbf{h}_{\tau_1}=\mathbf{1})P_r(\mathbf{e}'_{\tau_2}\otimes\mathbf{h}_{\tau_2}=\mathbf{1}) \\ &= \sum_{w(\mathbf{h}_{\tau_0})=0}^{\min\{\tau_0,w(\mathbf{h})\}}\sum_{w(\mathbf{h}_{\tau_1})=0}^{\min\{\tau_1,w(\mathbf{h})-w(\mathbf{h}_{\tau_0})\}} \\ &\quad \sum_{w(\mathbf{h}_{\tau_2})=\min\{\tau_2,w(\mathbf{h})-w(\mathbf{h}_{\tau_0})-w(\mathbf{h}_{\tau_1})\}} \\ &\quad \frac{\binom{\tau_0}{w(\mathbf{h}_{\tau_0})}}{2^{\tau_0}}\frac{\binom{\tau_1}{w(\mathbf{h}_{\tau_1})}}{2^{\tau_1}}\frac{\binom{\tau_2}{w(\mathbf{h}_{\tau_2})}}{2^{\tau_2}} \\ &\quad \cdot\Big[A(w(\mathbf{h}_{\tau_0}),\tau_0)A(w(\mathbf{h}_{\tau_1}),\tau_1)A(w(\mathbf{h}_{\tau_2}),\tau_2) \\ &\quad +\overline{A}(w(\mathbf{h}_{\tau_0}),\tau_0)\overline{A}(w(\mathbf{h}_{\tau_1}),\tau_1)\overline{A}(w(\mathbf{h}_{\tau_2}),\tau_2)\Big], \end{aligned} \tag{19}$$

where $\mathbf{e}_{\tau_0}$ and $\mathbf{e}_{\tau_1}$ are subvectors of $\mathbf{e}$ with length τ_0 and τ_1, respectively. The subvector $\mathbf{e}_{\tau_0}$ is formed by selecting elements from $\mathbf{e}$ according to the indices in set I_{z_0}, and the elements in $\mathbf{e}_{\tau_1}$ are selected by the same way according to the indices in set I_{z_1}. All the elements in $\mathbf{e}_{\tau_0}$ and $\mathbf{e}_{\tau_1}$ are in one-to-one correspondence with that of $\mathbf{h}_{\tau_0}$ and $\mathbf{h}_{\tau_1}$, respectively. $\mathbf{e}'_{\tau_2}$ is the subvector of $\mathbf{e}'$ with length τ_2, and the elements of $\mathbf{e}'_{\tau_2}$ are selected from $\mathbf{e}'$ according to the indices in set I_{z_2}. The same as the case of $\mathbf{e}_{\tau_0}$ and $\mathbf{e}_{\tau_1}$, all the elements in $\mathbf{e}'_{\tau_2}$ are in one-to-one correspondence with that of $\mathbf{h}_{\tau_2}$. $w(\mathbf{h}_{\tau_p})$ is the Hamming weight of $\mathbf{h}_{\tau_p}$. Denote by $A(w(\mathbf{h}_{\tau_p}), \tau_p)$ the probability that $z_p \otimes \mathbf{h}_{\tau_p} = 0$, and $\overline{A}(w(\mathbf{h}_{\tau_p}), \tau_p)$ the probability that $z_p \otimes \mathbf{h}_{\tau_p} = 1$. $A(w(\mathbf{h}_{\tau_p}), \tau_p)$ and $\overline{A}(w(\mathbf{h}_{\tau_p}), \tau_p)$ can be computed respectively as

$$\begin{aligned} &A(w(\mathbf{h}_{\tau_p}),\tau_p) \\ &= P_r\left(z_p\otimes\mathbf{h}_{\tau_p}=0|w(\mathbf{h}),w(\mathbf{h}_{\tau_2}),\tau_p\right) \\ &= \frac{\sum_{\alpha=even,0\le\alpha\le\min\{\tau_p,w(\mathbf{h}_{\tau_p})\}}\binom{\tau_p}{\alpha}\binom{K-\tau_p}{w(\mathbf{h})-\alpha}}{\binom{K}{w(\mathbf{h})}}, \\ &\overline{A}(w(\mathbf{h}_{\tau_p}),\tau_p) \\ &= P_r\left(z_p\otimes\mathbf{h}_{\tau_p}=1|w(\mathbf{h}),w(\mathbf{h}_{\tau_2}),\tau_p\right) \\ &= \frac{\sum_{\alpha=odd,\alpha\le\min\{\tau_p,w(\mathbf{h}_{\tau_p})\}}\binom{\tau_p}{\alpha}\binom{K-\tau_p}{w(\mathbf{h})-\alpha}}{\binom{K}{w(\mathbf{h})}}. \end{aligned} \tag{20}$$

We note that (17) depends on $\mathbf{e}$ only through $w(\mathbf{e}) = t$. Also, there are $2^K - 1$ nonzero error sequences $\mathbf{e}$, of which $\binom{K}{t}$ have weight t. Taking these factors into account, we have that the bit error probability of the Raptor code can be lower bounded by $P_b > \max\{0, \xi_L^{Raptor}\}$, where

$$\begin{aligned} \xi_L^{Raptor} &= \sum_{t=1}^{K}\frac{t}{K}\binom{K}{t}\xi_L^t(1-\xi_L)^{K-t} \\ &\quad \cdot P_r\left(w(\mathbf{eH}^T)=0, w(\mathbf{e})=t\right) \\ &\quad -\frac{1}{2}\sum_{t=1}^{K}\frac{t}{K}\binom{K}{t}\xi_U^t(1-\xi_U)^{K-t} \\ &\quad \cdot P_r(\mathbf{eH}^T=\mathbf{0}, \mathbf{e}'\mathbf{H}^T=\mathbf{0}, w(\mathbf{e})=t). \end{aligned} \tag{21}$$

The scaling factor t/K in (21) is the number of erroneous information bits per information symbol in a weight-t information error sequence. Therefore, we can get the BER lower bound on the bit error probability of Raptor codes which can be described as:

Theorem 2: Consider a Raptor code with parameters $\Omega(x)$, $\mu(x)$, K_L, K, Q, r and expanding coefficient η. Its lower BER bound under ML decoding over a fast Rayleigh fading channel is $P_b > \max\{0, \xi_L^{Raptor}\}$, where ξ_L^{Raptor} is given by

$$\begin{aligned} &\xi_L^{Raptor} \\ &= \sum_{t=1}^{K}\frac{t}{K}\binom{K}{t}\xi_L^t(1-\xi_L)^{K-t}\left(\sum_{w(\mathbf{h})}\Omega_{w(\mathbf{h})}\varsigma_t^{R,w(\mathbf{h})}\right)^{K-K_L} \\ &\quad -\frac{1}{2}\sum_{t=1}^{K}\frac{t}{K}\binom{K}{t}\xi_U^t(1-\xi_U)^{K-t}\sum_{\tau_0=0}^{t}\sum_{\tau_1=t-\tau_0}\sum_{\tau_2=0}^{K-\tau_0} \\ &\quad \frac{\binom{t}{\tau_0}}{2^t}\frac{\binom{K-\tau_0}{\tau_1}}{2^{K-\tau_0}}\frac{\binom{K-\tau_0}{\tau_2}}{2^{K-\tau_0}}\left(\sum_{w(\mathbf{h})}\Omega_{w(\mathbf{h})}\zeta(\tau_0,\tau_1,\tau_2,t)\right)^{K-K_L}, \end{aligned}$$

where

$$\varsigma_t^{R,w(\mathbf{h})} = \frac{\sum_{\alpha=even,0\le\alpha\le t}\binom{t}{\alpha}\binom{K-t}{w(\mathbf{h})-\alpha}}{\binom{K}{w(\mathbf{h})}},$$

$$\zeta(\tau_0,\tau_1,\tau_2,t) = \sum_{w(\mathbf{h}_{\tau_0})=0}^{\min\{\tau_0,w(\mathbf{h})\}} \sum_{w(\mathbf{h}_{\tau_1})=0}^{\min\{\tau_1,w(\mathbf{h})-w(\mathbf{h}_{\tau_0})\}} \sum_{w(\mathbf{h}_{\tau_2})=\min\{\tau_2,w(\mathbf{h})-w(\mathbf{h}_{\tau_0})-w(\mathbf{h}_{\tau_1})\}} \frac{\binom{\tau_0}{w(\mathbf{h}_{\tau_0})}}{2^{\tau_0}} \frac{\binom{\tau_1}{w(\mathbf{h}_{\tau_1})}}{2^{\tau_1}} \frac{\binom{\tau_2}{w(\mathbf{h}_{\tau_2})}}{2^{\tau_2}} \cdot\Big[A(w(\mathbf{h}_{\tau_0}),\tau_0)A(w(\mathbf{h}_{\tau_1}),\tau_1)A(w(\mathbf{h}_{\tau_2}),\tau_2) + \overline{A}(w(\mathbf{h}_{\tau_0}),\tau_0)\overline{A}(w(\mathbf{h}_{\tau_1}),\tau_1)\overline{A}(w(\mathbf{h}_{\tau_2}),\tau_2)\Big],$$

$$A(w(\mathbf{h}_{\tau_p}),\tau_p) = \frac{\sum_{\alpha=even,0\le\alpha\le\min\{\tau_p,w(\mathbf{h}_{\tau_p})\}}\binom{\tau_p}{\alpha}\binom{K-\tau_p}{w(\mathbf{h})-\alpha}}{\binom{K}{w(\mathbf{h})}},$$

$$\overline{A}(w(\mathbf{h}_{\tau_p}),\tau_p) = \frac{\sum_{\alpha=odd,\alpha\le\min\{\tau_p,w(\mathbf{h}_{\tau_p})\}}\binom{\tau_p}{\alpha}\binom{K-\tau_p}{w(\mathbf{h})-\alpha}}{\binom{K}{w(\mathbf{h})}},$$

and $\mathbf{h}_{\tau_p}$ is the subvector of $\mathbf{h}$, τ_p is the length of $\mathbf{h}_{\tau_p}$, and $p\in\{0,1,2\}$.

V. Numerical and Simulation Results

In this section, we present numerical and simulation results for the LT-based DNC scheme and the proposed Raptor-based DNC scheme over quasi-static Rayleigh fading channels. Due to the time-varying channels of the wireless network, within each transmission round, a Raptor code is generated on-the-fly to match the instantaneous network topology. Thus, the BER performance of an ensemble of codes is analyzed. We consider the case of $K=100$, $K_L=50$ and 98, which correspond to the pre-code rate $r=0.5$ and 0.98, respectively. We evaluate the upper and lower bounds on the bit error probability under ML decoding by using the degree distribution $\Omega(x)=rx+(1-r)x^4$ for the pre-code in the precoding phase and the degree distribution described in [24][6] for the LT code in the LT-coding phase:

$$\begin{aligned}\mu(x) \;=\; & 0.007969x+0.493570x^2+0.166220x^3\\ & +0.072646x^4+0.082558x^5+0.056058x^8\\ & +0.037229x^9+0.055590x^{19}+0.025023x^{65}\\ & +0.003135x^{66}. \qquad (22)\end{aligned}$$

Fig. 3 shows the BER bounds of the constructed LT codes and Raptor codes ensemble, respectively, at an average E_s/N_0 of 7dB, where E_s is the average energy per transmitted information symbol. It can be seen that the lower bound of the LT codes we derived in this paper is asymptotically tight to the upper bound given in [25] as the expanding coefficient grows. Also, from the figure we can see that the BER performance of the proposed Raptor-based DNC scheme is much better than that of the LT-based DNC scheme regardless of the expanding coefficient. The proposed DNC scheme on the basis of the Raptor codes has about two orders of magnitude improvement of BER performance compared with the scheme based on the LT codes if the expanding coefficient is large enough, e.g., $\eta=2$. The analytical upper bound of the proposed Raptor-based DNC scheme is asymptotically tight to the lower bound in a greater expanding coefficient extent.

[6]We only use this degree distribution as an example. In our future work, we will design the optimal degree distribution function for LT codes over the Rayleigh fading channel. This can possibly be based on our developed analytical bounds.

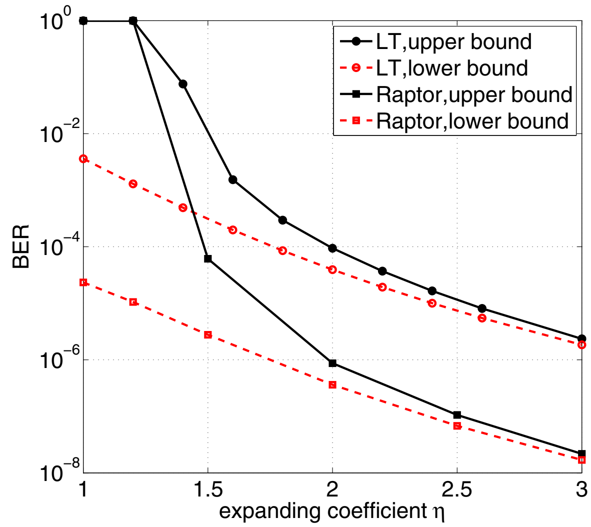

Fig. 3. BER bounds comparison for LT-based and Raptor-based DNC schemes over Rayleigh fading channels with $K_L=98$, $K=100$ and $E_s/N_0=7$dB.

Fig. 4 shows the BER bounds of the constructed Raptor codes ensemble at an average E_s/N_0 of 7dB and 10 dB, respectively. The information length of the LDPC codes is $K_L=98$, and that of the LT codes is $K=100$, which correspond to 98 source nodes and 100 relay nodes in $\mathbb{R}_1$, respectively. It can be seen from the figure that as E_s/N_0 increases, e.g., E_s/N_0 changes from 7dB to 10dB, the upper and lower BER bounds of the proposed DNC scheme based on the Raptor codes become tighter, especially when the expanding coefficient is large enough, e.g., $\eta\ge 1.5$. This phenomenon can be explained by examining the difference between the upper and lower bounds given by *Theorem 1* and *Theorem 2* in Section IV, respectively. The difference between the two bounds can be represented by

$$\xi_U^{Raptor}-\xi_L^{Raptor} = \sum_{w(\mathbf{e})=t=1}^{K}\frac{t}{K}\binom{K}{t}\Bigg[\Bigg(\sum_{w(\mathbf{h})}\Omega_{w(\mathbf{h})}\varsigma_t^{R,w(\mathbf{h})}\Bigg)^{K-K_L} \cdot\left(\xi_U^t(1-\xi_U)^{K-t}-\xi_L^t(1-\xi_L)^{K-t}\right)$$

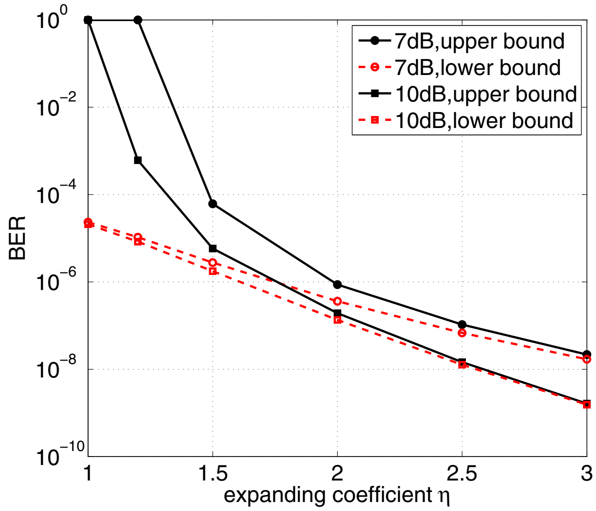

Fig. 4. BER bounds of constructed Raptor codes ensemble over the Rayleigh fading channel with $K_L = 98$, $K = 100$, $E_s/N_0 = 7$dB and 10dB, respectively.

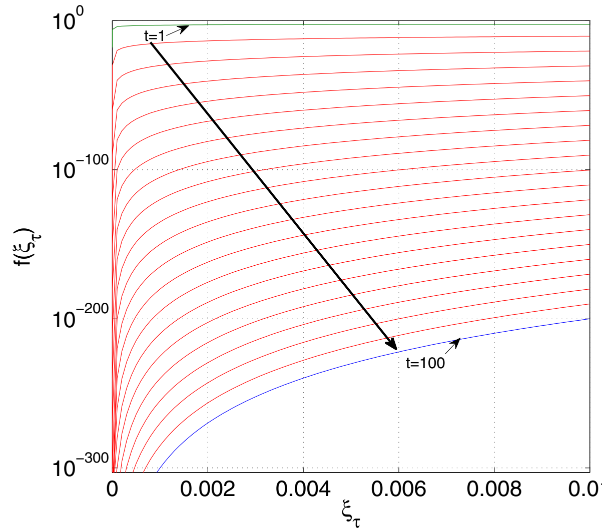

Fig. 5. $f(\xi_\tau)$ with $K = 100$.

$$-\frac{1}{2}\xi_U^t(1-\xi_U)^{K-t} \cdot \sum_{\tau_0=0}^{t}\sum_{\tau_1=t-\tau_0}\sum_{\tau_2=0}^{K-\tau_0}\frac{\binom{t}{\tau_0}}{2^t}\frac{\binom{K-\tau_0}{\tau_1}}{2^{K-\tau_0}}\frac{\binom{K-\tau_0}{\tau_2}}{2^{K-\tau_0}} \cdot \left(\sum_{w(\mathbf{h})}\Omega_{w(\mathbf{h})}\zeta(\tau_0,\tau_1,\tau_2,t)\right)^{K-K_L}\Bigg]. \tag{23}$$

We only consider the effect from E_s/N_0 to $\xi_U^{Raptor} - \xi_L^{Raptor}$. Let $B(t)$ and $A(t)$ denote the terms of $\sum_{\tau_0=0}^{t}\sum_{\tau_1=t-\tau_0}\sum_{\tau_2=0}^{K-\tau_0}\frac{\binom{t}{\tau_0}}{2^t}\frac{\binom{K-\tau_0}{\tau_1}}{2^{K-\tau_0}}\frac{\binom{K-\tau_0}{\tau_2}}{2^{K-\tau_0}}\Big(\sum_{w(\mathbf{h})}\Omega_{w(\mathbf{h})}$ $\zeta(\tau_0,\tau_1,\tau_2,t)\Big)^{K-K_L}$ and $\left(\sum_{w(\mathbf{h})}\Omega_{w(\mathbf{h})}\varsigma_t^{R,w(\mathbf{h})}\right)^{K-K_L}$ in (23), respectively. These two terms do not depend on E_s/N_0. Therefore, (23) can be rewritten as

$$\xi_U^{Raptor} - \xi_L^{Raptor} = \sum_{w(\mathbf{e})=t=1}^{K}\frac{t}{K}\binom{K}{t}\left[\left(A(t)+\frac{1}{2}B(t)\right)\xi_U^t(1-\xi_U)^{K-t} - A(t)\xi_L^t(1-\xi_L)^{K-t}\right]. \tag{24}$$

Let $f(\xi_\tau)$ denote $\xi_\tau^t(1-\xi_\tau)^{K-t}$ in (24), where $\tau \in \{U, L\}$. From Fig. 3, it can be seen that when the expanding coefficient $\eta \geq 1.5$, the values of both the upper and lower BER bounds of LT codes are smaller than 10^{-2}. Fig. 5 illustrates the change of $f(\xi_\tau)$ with ξ_τ when $\xi_\tau \leq 0.01$. In this figure, green and blue lines denote the cases when $t = 1$ and 100, respectively. Red lines between the green and blue ones denote the cases when $1 < t < 100$.

From Fig. 5 we can see that when $\xi_\tau \leq 0.01$, $f(\xi_\tau)$ is a monotonically increasing function. Since ξ_U is always bigger than ξ_L when $\eta \leq 1.5$, $f(\xi_U)$ can be represented by the function $f(\xi_L)$, that is $f(\xi_U) = f(\xi_L) + \Delta$, where Δ represents the difference between $f(\xi_U)$ and $f(\xi_L)$. Thus, (24) becomes

$$\xi_U^{Raptor} - \xi_L^{Raptor} = \sum_{w(\mathbf{e})=t=1}^{K}\frac{t}{K}\binom{K}{t}\left(\frac{1}{2}B(t)f(\xi_U) + A(t)\Delta\right). \tag{25}$$

As the E_s/N_0 increases, ξ_U described in *Theorem 1* becomes smaller. Since $f(\xi_U) = \xi_U^t(1-\xi_U)^{K-t}$ is a monotonically increasing function in the region of $0 \leq \xi_U \leq 0.01$, $f(\xi_U)$ becomes smaller with the increases of E_s/N_0.

From the *Theorem 1* and the *Lemma*, we can express ξ_U as $\xi_U = \xi_L + \Delta_o$, where

$$\Delta_o = \sum_{\kappa=2}^{K}\frac{\kappa}{K}\binom{K}{\kappa}\left[\left(\sum_{w(\mathbf{c})}\mu_{w(\mathbf{c})}\varsigma_\kappa^{w(\mathbf{c})}\right)^Q + \sum_{k=1}^{Q}\frac{1}{\pi}\binom{Q}{k}\beta_\kappa^k(1-\beta_\kappa)^{Q-k} \cdot \int_0^{\pi/2}\left(\frac{1}{1+\frac{E_s^k}{N_0\sin^2(\theta)}}\right)^k d\theta\right]. \tag{26}$$

Thus, $\Delta = f(\xi_U) - f(\xi_L)$ can also be written as $\Delta = f(\xi_L + \Delta_o) - f(\xi_L)$. From the *Lemma* and (26) we can see that with the increases of E_s/N_0, both ξ_L and Δ_o become smaller. Let Δ_{ξ_L} and Δ_{Δ_o} represent the variations of ξ_L and Δ_o, respectively, with the changes of E_s/N_0. That is, if the transmitted symbol energy at a particular node is changed from E_{s1} to E_{s2}, then $\Delta_{\Delta_g} = \Delta_g(E_{s2}/N_0) - \Delta_g(E_{s1}/N_0)$, where $g \in \{\xi_L, o\}$. With the above notations, the ratio between Δ_{ξ_L} and Δ_{Δ_o} can be represented by

$$\frac{\Delta_{\xi_L}}{\Delta_{\Delta_o}} = \frac{\sum_{k=1}^{Q}\frac{1}{\pi}\binom{Q}{k}\beta_1^k(1-\beta_1)^{Q-k}}{\sum_{\kappa=2}^{K}\frac{\kappa}{K}\binom{K}{\kappa}\sum_{k=1}^{Q}\frac{1}{\pi}\binom{Q}{k}\beta_\kappa^k(1-\beta_\kappa)^{Q-k}}. \tag{27}$$

It can be seen that E_s/N_0 is not included in (27). This means that no matter how the E_s/N_0 changes, the relationship

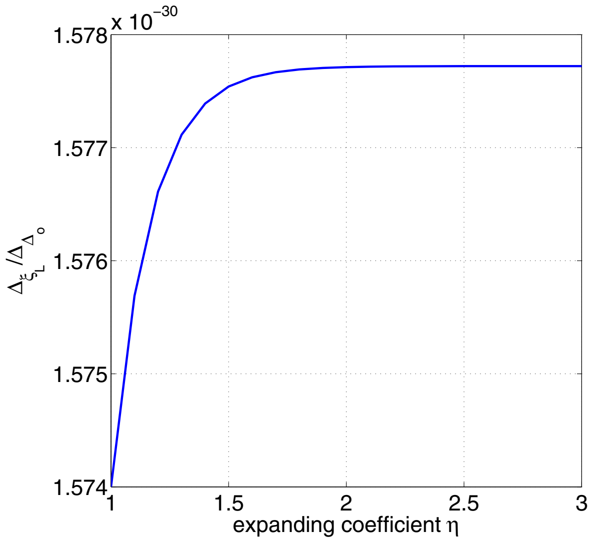

Fig. 6. The ratio $\frac{\Delta_{\xi_L}}{\Delta_{\Delta_o}}$.

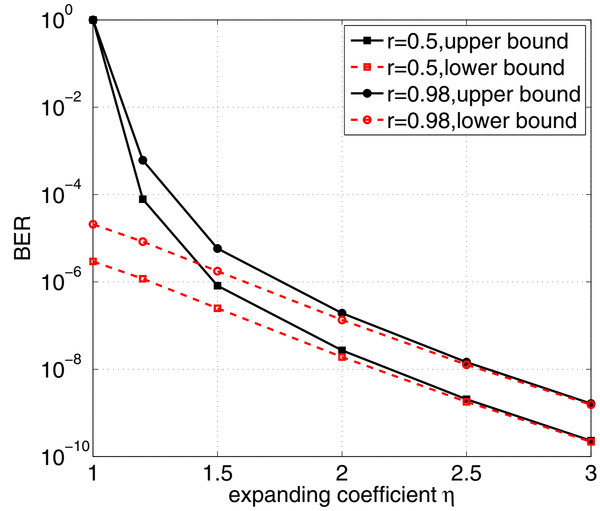

Fig. 7. BER performance bounds of constructed Raptor codes ensemble over the Rayleigh fading channel with $K_L = 50$ and 98, respectively, $K = 100$ and $E_s/N_0 = 10$dB.

between the variations Δ_{ξ_L} and Δ_{Δ_o} is always the same. Fig. 6 shows the ratio value changes as a function of the expanding coefficient η. From Fig. 6 we can see that $\frac{\Delta_{\xi_L}}{\Delta_{\Delta_o}}$ is much smaller than 1 over all the expanding coefficients. It means that the variation of ξ_L is much smaller than that of Δ_o. Corresponding to this, $f(\xi_L + \Delta_o)$ changes much greater than $f(\xi_L)$ with the increase of E_s/N_0. Thus, the change of $\Delta = f(\xi_L + \Delta_o) - f(\xi_L)$ is mainly determined by $f(\xi_L + \Delta_o)$, that is to say, as E_s/N_0 increases, Δ becomes smaller.

From what has been discussed above, we may finally draw the conclusion that as E_s/N_0 increases, both $f(\xi_U)$ and Δ become smaller. Therefore, as a result of the increases of E_s/N_0, the difference between the upper bound ξ_U^{Raptor} of the proposed DNC scheme based on the Raptor codes and the lower bound ξ_L^{Raptor} becomes smaller.

Fig. 7 shows the BER bounds of the constructed Raptor codes ensemble with $K_L = 50$ and 98, which correspond to the code rate of 0.5 and 0.98, respectively, at an average E_s/N_0 of 10 dB. It can be seen that as the code rate of the LDPC codes, which are used in the construction of the Raptor codes, decreases, the BER bounds get better. The reason is that if take code rate of the pre-code in the precoding phase into consider, the overall expanding coefficient equals to $\frac{\eta}{r}$. As r changes to smaller, the overall expanding coefficient becomes larger. Thus, the BER performance of the Raptor codes get better as the code rate of the LDPC codes decreases.

VI. Conclusion

In this work, we proposed a Raptor-based DNC scheme for WSNs. We proposed to use such a coding scheme for matching a code-on-graph to a network-on-graph to achieve a combined network-channel coding gain. We first developed lower BER bound of the LT-based DNC scheme over fast Rayleigh fading channels under ML decoding. Then, we derived analytical upper and lower bounds on ML decoding for the proposed Raptor-based DNC scheme. From the numerical and simulation results, we can see that the derived upper and lower BER bounds come closer, particularly as the length of the codewords is increased with a larger expanding coefficient. In our future work, we will use the developed bounds to optimize degree distributions of rateless codes over Rayleigh fading channels.

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