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Network Code Division Multiplexing for Wireless Relay Networks

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***Abstract*— In this paper, we investigate the performance of a wireless relay network with multiple transmission sessions, in which multiple groups of source nodes communicate with their respective destination nodes via a shared wireless relay network. A multiple transmission session model with network code division multiplexing (NCDM) scheme is proposed to remove the inter-session interference at each destination. The fundamental idea of the NCDM scheme takes advantage of the property of $\mathbf{G} \odot \mathbf{H}^T = 0$ of the low-density generator matrix (LDGM) codes. Based on the analysis of the NCDM scheme, we investigate the relationship among the equivalent received signal vector, the number of sessions and the column weight of the generator matrix. New code design criteria for the construction of the generator matrix is proposed. We further evaluate the multiple transmission session model with the proposed NCDM scheme in terms of throughput and complexity. Our evaluation demonstrates that the proposed scheme not only has a linear computational complexity, but also shows a similar error performance in the AWGN case and a considerable throughput improvement compared with its counterpart, which is referred to as a serial session scheme, where groups of source nodes communicate with their respective destinations in a time division manner.**

***Index Terms*— Distributed network coding, low density generator matrix codes, parallel session model, network code division multiplexing, code design criteria.**

I. Introduction

Distributed coding is a special channel coding strategy developed for cooperative communication networks [1], [2]. It has also been applied in the design of conventional channel codes, forming such as distributed turbo codes [3], distributed space-time codes [4] and distributed low-density parity-check (LDPC) codes [5]. In the above distributed coding schemes, the transmission reliability over point-to-point wireless communication channels has been efficiently improved.

The distributed coding schemes discussed above [1]–[5] are developed for small-scale unicast relay networks, in which messages are sent from a single source to a single destination through single/multi-hop relay nodes. In large-scale wireless sensor networks (WSNs), a large number of sensor nodes are often deployed to gather information from the surrounding environment. The information gathered from these sensor nodes, which are referred to as source nodes, are delivered then to a common destination through other sensor nodes, serving as relay nodes. As the source and the relay nodes are in different spatial locations, their signals can be combined at the destination node to achieve spatial diversity. Clearly, if a destination node receives replicas of the transmitted signal via multiple relay nodes, its signal will have higher spacial diversity and better performance [6].

Despite the potential benefits of relay nodes, they may consume considerable amounts of radio spectrum and energy, and cause further spectrum congestion and interference if not used properly. Therefore, for dense large-scale wireless networks, we need a novel approach to reduce bandwidth consumption of relay nodes while harnessing their benefits.

A novel approach that minimizes the bandwidth consumption is network coding (NC) [7]. Network coding is introduced as a generalization of routing. In routing, the relay nodes simply store and forward the received packets to the destination. In the NC scheme, the relay nodes are allowed to encode the packets received from multiple source nodes and send the encoded packets instead. Coding operations enable the relay nodes to compress the information, and whenever possible, to reduce the number of transmissions and bandwidth consumption. Prior work shows that network coding can achieve the multi-cast network capacity by making linear combinations of packets they received [7]–[9]. The idea of NC can be easily extended to wireless networks. A considerable amount of research that has been dedicated to exploiting NC on cooperative communication networks have been done, some of them focus on the research at the physical layer [2], [10]–[13], while others are for the network layer [14]–[16]. The idea of NC can be further applied to WSNs , where multiple source nodes communicate with a common destination through multi-hop relay nodes [17]. In [18], extrinsic information transfer (EXIT) charts are employed to design the irregular low-density

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generator matrix (LDGM) codes [19] based on a single session model. By a *single session*, we refer to a group of source nodes transmitting to a common destination node, while *multiple sessions* refer to multiple groups of source nodes transmitting to multiple respective destination nodes.

In this paper, for the applications where at each destination node, only the information from its own session is required, we propose to design novel network coding schemes for wireless networks. In wireless networks, multiple relays are shared among multiple sessions. We assume that at each destination node, the information from other sessions are not allowed to be decoded due to the security reasons. This assumption is realistic in the application scenario in smart meter networks, where multiple sensors, measuring water, gas and energy consumptions, communicate with multiple utility control centers [20]. The information from various utilities, such as for water, gas and energy consumptions, cannot be shared. The bandwidth consumption will be reduced by sharing dedicated relay stations among various sessions and the use of novel network coding mechanisms, referred to as network code division multiplexing (NCDM). The NCDM scheme can provide security protection on physical layer. Traditionally, the research of the security techniques are focused on the key-based enciphering technique on upper layers [21]. However, in wireless network, it requires a large number of computational resources to deal with the encryption key. We consider the security protection problem from the physical layer [22]. Since the physical layer security techniques can operate essentially independently from the upper layers, the NCDM scheme can be used to enhance the security of the data transmission. A common set of relays, as opposed to separate relay nodes for each session, can save spectrum, energy, communication infrastructure and bring additional spatial diversity. The signals from multiple sessions will be mixed in a controlled manner at the relay nodes to cooperatively produce a distributed network code. As inter-session interference is introduced in the cooperative process [23], novel network codes, with a special algebraic structure, enabling interference removal at the destinations and high coding gains, will be designed. Transmission of each session will be assisted by all relay nodes resulting in a maximum spatial diversity.

Particularly, the signal processing methods at the relays dominate the overall system performance. On the one hand, if all relays belonging to different transmission sessions are cooperative with each other, a maximum spatial diversity can be potentially achieved. On the other hand, inter-session interference is minimized if signals, belonging to different sessions, are transmitted from relays in an orthogonal manner. A usual way to remove interference is dividing the relay transmission time into L relay periods, corresponding to L sessions in a wireless relay network. One relay period contains N time slots, where N is the number of relays in the network. All relays process and transmit data for the ith session in a relay period i $(i = 1, \cdots, L)$. In this way, data from different sessions are separated by time division multiplexing and there will be no interference in the transmitted signals from multiple sessions. For convenience, in the following, we refer to the above described model as a *serial session* model. In this model, however, transmission in separate relay periods may result in a lower throughput or more spectrum consumption, compared to the transmission of all signals from multiple sessions in one relay period.

To mitigate interference while maximizing the spectrum efficiency, we propose to combine data from different sessions to form a network code, and forward such coded signals to the destinations, within one single relay period. In contrast to the *serial session* model, we refer to our proposed scheme as a *parallel session* model. The combined operation of data from different sessions in *parallel session* model will cause inter-session interference at the destinations. By using the proposed NCDM scheme at the destination nodes by orthogonalizing the equivalent sub-channels of different sessions, inter-session interference among multiple sessions will be minimized. Furthermore, we will maximize the coding gain of these network codes, by optimizing the network code topology. Please note that the NCDM scheme is different from the interference alignment approach given in [24]–[26]. The key idea of the interference alignment is to construct precoding vectors at the transmitters, and concentrate the interference into a partial signal space while the desired signal becomes free of interference at each receiver [26]. The NCDM scheme does not use a precoding operation at the transmitters, it only deals with the inter-session interference at the destinations. The NCDM scheme is also different from the aligned interference neutralization schemes, which is proposed in [27] for the relay-aided networks. In the aligned interference neutralization scheme, for the design of the beamforming vectors, the channel state information (CSI) must be known at both the transmitters and the receivers or at the receivers. Also, it requires full connection between the source and the relay nodes, and between the relay and the destination nodes. While for the NCDM scheme, the CSI is only required at the receivers. The situations that no full connection exists between the source and the relay nodes, and between the relay and the destination nodes are also considered in the NCDM scheme.

In this paper, we investigate the impact of the number of sessions on bit error rate (BER) performance, and examine the scalability of the proposed NCDM scheme when the number of nodes increases. In the analysis of the proposed NCDM process, we focus on the relationship among the information and the noise parts of the equivalent received signal vector, the number of sessions and the column weight of the formed generator matrix. We analyze the reasons which cause the degradation of the BER performance of the *parallel session* model during the NCDM process and investigate possible approaches to solving these problems. The theoretical analysis is followed by the derivation of the code design criteria to guide the selection of source nodes at each relay node. By implementing the proposed code design criteria, a desired code performance can be achieved. It is shown that the proposed NCDM scheme has a linear computational complexity of N and L, and a similar error performance as a *serial session* scenario, while achieving a considerable throughput improvement compared with the *serial session* model.

The remainder of the paper is organized as follows. Section II presents the system model and briefly introduces the

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NCDM scheme and the distributed LDGM codes from the system point of view. Section III introduces the proposed NCDM scheme for the *parallel session* model. Section IV analyzes the relationship among the equivalent received signal vector, the number of sessions and the column weight of the generator matrix. On the basis of the analytical results, code design criteria are derived for the *parallel session* model. Section V evaluates the performance of the NCDM scheme in terms of the BER, throughput and complexity. Finally, the conclusions are drawn in Section VI.

II. System Model

We consider a wireless network containing L transmission sessions, shown in Fig. 1. In each session, a group of source nodes communicate with their common destination node through a group of relay nodes. We use φ_i to denote the ith session and assume that φ_i has S_i source nodes, where $i \in \{1, 2, \cdots, L\}$. In total, there are $M = \sum_{i=1}^{L} S_i$ source nodes in the L sessions. Denote the destination node of session φ_i by D_i. We assume that the network consists of N relay nodes which are shared among all L sessions. The jth relay node is denoted by R_j, $j \in \{1, 2, \cdots, N\}$.

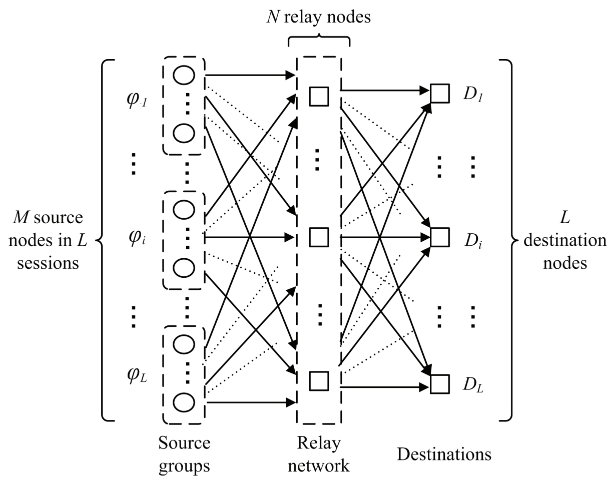

Fig. 1. A network graph used to describe the system model.

As it is described in Section I, the model where groups of source nodes in different sessions communicate with their respective destinations is referred to as a *serial session* model. In the *serial session* model, transmission in separate relay periods may result in a lower throughput or more spectrum consumption, compared to the transmission of all signals from multiple sessions in one relay period. To mitigate interference while maximizing the spectrum efficiency, we propose to combine data from different sessions to form a network code, and forward such coded signals to the destinations, within one single relay period. We refer to this model as a *parallel session* model.

For the *parallel session* model, the data transmission from the source nodes to their destination nodes is carried out in two phases: a broadcast phase and a relay phase. We take the operations of the source nodes in session φ_i in the broadcast phase, the jth relay node in the relay phase, and the destination node D_i, as an example to illustrate the data delivery process. The system diagram is shown in Fig. 2.

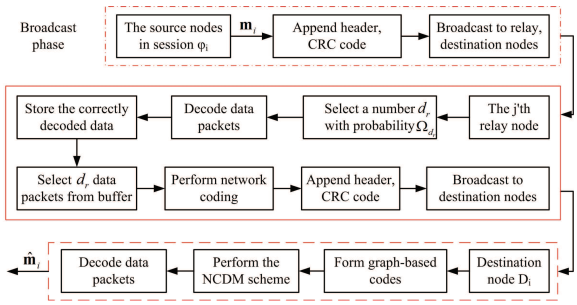

Fig. 2. The data delivery processing procedure.

- **Broadcast phase**. In this phase, all the source nodes broadcast their data packets to all the relay and the destinations. Each data packet is composed of three parts: an information data part, a Cyclic Redundancy Check (CRC) part and a header. The indices of the source nodes and the session to which the source node belongs are contained in the header. The information data parts of all data packets have the same length, as well as the CRC parts and the headers. In case the length of the information data part of one source node is shorter, we just add "0" to the end of the data to make it the same length as that in other source nodes. The performance will not be affected by this operation. A certain MAC layer protocol, e.g., Time-Division Multiple Access (TDMA), Carrier Sense Multiple Access (CSMA), or Orthogonal Frequency-Division Multiple Access (OFDMA) [28], [29], is assumed to be employed. This ensures that the source node transmissions do not cause interference to each other. The transmitted data packet from each source node in session φ_i is a vector of size $\ell \times 1$. Thus, the data packets from session φ_i form a data matrix $\mathbf{m}_i$ with size $\ell \times S_i$. We consider both additive white Gaussian noise (AWGN) and Rayleigh fading channels. We model channel coefficient $\boldsymbol{\alpha}_i$ as a zero-mean, circularly symmetric complex Gaussian random variable with a unit variance. In this work, we assume binary phase shift keying (BPSK) modulation for all the transmitted data.
- **Relay phase**. In this phase, each relay node first selects a number d_r with[1] probability Ω_{d_r} according to the degree distribution $\Omega(x) = \sum \Omega_{d_r} x^{d_r}$. Then each relay node listens to the transmission from the source nodes, decodes the received data packets from the source nodes, checks

[1] After applying the network code design criteria, which is proposed in Section IV-D, a set of column weights for the generator matrix, denoted by $\{d_r\}$, can be obtained. In our work, we only consider $\Omega_{d_r} = 1$. Thus, the set $\{d_r\}$ only contains one value, d_r. The indices of the selected d_r data corresponding to the d_r positions of non-zero elements in a column of the generator matrix. If irregular LDGM code is considered, each column of the generator matrix will has different column weight. Then, d_r data will be selected at each relay node with probability Ω_{d_r}, where $0 < \Omega_{d_r} < 1$ and $\sum \Omega_{d_r} = 1$. The network coding operation at relay nodes is similar to random linear network coding but with some constraints, i.e., the network coding operation must satisfy the parameters setting of the LDGM codes, for example, the selection of column weight of the generator matrix should follow the code design criteria.

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the correctness of the decoded packets by using CRC and puts the correctly decoded packets into its buffer. Since each relay node only performs hard decision decoding and CRC check, the decoding process is very fast and the delay due to the decoding at each relay node can be neglected. After that, each relay node selects uniformly at random a number of d_r data packets from its buffer to perform network coding by using linear combinations of the packets in the field of GF(2) [30]. Since the number d_r is much smaller than the total number of source nodes in the transmission sessions, i.e., $d_r \ll M$, the probability of the case that the number of data packets in a relay node's buffer is smaller than d_r is negligibly small and we can always assume that the network coding process is achievable. In case the number of data packets in the buffer of a relay node is less than d_r, then all data packets in the buffer will be selected for network coding. Then at each relay node, a network-coded data packet is formed. The connection information of the source and the relay nodes is contained in the header of each network-coded data packet. The number of selected source nodes is relatively small to reflect the fact of low density for the formed generator matrix. The length of the header in each data packet is much shorter than the information data part, therefore, the throughput loss due to the header can be neglected. All the network-coded data packets have the same length ℓ. Finally, all the relay nodes broadcast their network-coded data packets to the destination nodes using a TDMA, CSMA or OFDMA MAC layer protocol.

At the destination nodes, data packets received from the source and the relay nodes in the broadcast and relay phases, respectively, are organized into a data matrix of size $\ell \times (M+N)$. We consider a quasi-static Rayleigh fading channel. Without loss of generality, we consider the ith row of the received data matrix. At the destination node D_i, the received signal can be expressed as $\mathbf{r}_i = \boldsymbol{\alpha}_i \otimes \sqrt{E_b}(\mathbf{J} - 2\boldsymbol{\beta}) + \mathbf{n}_i$, where $\mathbf{r}_i$ is a row vector of size $1 \times (M+N)$. $\boldsymbol{\alpha}_i$ is the fading coefficient vector of size $1 \times (M+N)$. The fading coefficients remain constant over the length of one data packet but change independently between adjacent packets. Thus, we can assume perfect CSI at the destination nodes. $\otimes$ represents the element-wise multiplication of the vector. For example, there are two vectors $\mathbf{a} = [a_1, \cdots, a_i, \cdots, a_N]$ and $\mathbf{b} = [b_1, \cdots, b_i, \cdots, b_N]$, $\mathbf{a} \otimes \mathbf{b} = [a_1 b_1, \cdots, a_i b_i, \cdots, a_N b_N]$. E_b is the average transmitted energy per bit. $\mathbf{J}$ is a $1 \times (M+N)$ vector with all elements equal to 1. The binary sequence $\boldsymbol{\beta}$ is a random row of the data matrix $[\mathbf{m}_1 \cdots \mathbf{m}_i \cdots \mathbf{m}_L] \odot \mathbf{G}_i$, where $\odot$ represents the multiplication operation over the field of GF(2). $[\mathbf{m}_1 \cdots \mathbf{m}_i \cdots \mathbf{m}_L]$ represents the information matrix of the L sessions of size $\ell \times M$. $\mathbf{G}_i$ is[2] the corresponding LDGM generator matrix seen from D_i of size $M \times (M+N)$. Thus, $\boldsymbol{\beta}$ is of size $1 \times (M+N)$. The parameter $\mathbf{n}_i = [n_{i,1}, n_{i,2}, \cdots, n_{i,M+N}]$ is the additive white Gaussian noise with a zero mean and a double sided power spectral density of $N_0/2$. Next, each destination node implements the NCDM scheme to remove the interference from the source nodes of other sessions. After using the NCDM scheme, each destination node only receives the data packets from the source nodes of its own session. The particular description of the NCDM scheme will be given in Section III.

Note that since all the connection information are contained in the header of each data packet, the destination nodes know how the network coded data packets are formed at the relay nodes and can correspondingly replicate the code graph and perform message-passing decoding.

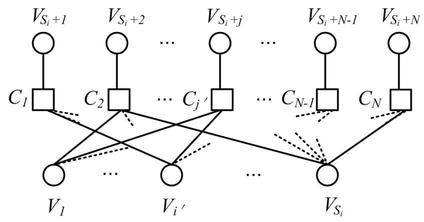

Fig. 3. The bipartite graph.

At each destination node, after implementing the NCDM scheme, a regular systematic LDGM code is obtained by combining the broadcast information data packets from the broadcast phase and the corresponding network-coded data packets from the relay phase, as shown in Fig. 3. The LDGM code formed at the ith destination node D_i consists of S_i systematic symbols $V_{i'}, i' \in \{1, 2, \cdots, S_i\}$, corresponding to the S_i source nodes from φ_i, and N coded symbols, corresponding to the N relay nodes. $\{C_N\}$ are the check nodes associated with the syndromes. The decoding can be implemented by the Belief Propagation (BP) algorithm [31] based on the soft information of the received data packets.

III. The NCDM Scheme

In this section, we introduce the NCDM scheme for the *parallel session* model, which can be used to remove the inter-session interference at each destination. The fundamental idea behind the process takes advantage of the property of $\mathbf{G} \odot \mathbf{H}^T = \mathbf{0}$. Here $\mathbf{G}$ and $\mathbf{H}$ are the generator matrix and the parity check matrix of the LDGM codes, respectively. $\mathbf{H}^T$ represents the transpose of the matrix $\mathbf{H}$. Throughout the paper, we denote the LDGM generator matrix seen from the ith destination node D_i by $\mathbf{G}_i$ and the corresponding parity check matrix by $\mathbf{H}_i$. In order to remove the inter-session interference, $\mathbf{G}_i$ can be split into L separate parts, so that

$$\mathbf{G}_i = \left[\left(\mathbf{G}_i^1\right)^T \cdots \left(\mathbf{G}_i^t\right)^T \cdots \left(\mathbf{G}_i^L\right)^T \right]^T, \tag{1}$$

where $\mathbf{G}_i$ is of size $M \times (M+N)$. $\mathbf{G}_i^t$ is the tth submatrix of $\mathbf{G}_i$ of size $S_t \times (M+N)$, $t \in \{1, 2, \cdots, L\}$. S_t is the number of source nodes in the tth session.

Since $\mathbf{G}_i$ is a systematic matrix, it can be written as

$$\mathbf{G}_i = [\mathbf{I}_M \, \mathbf{P}_i], \tag{2}$$

where $\mathbf{I}_M$ represents an $M \times M$ identity matrix. $\mathbf{P}_i$ is an $M \times N$ parity part matrix.

[2] In this paper, we consider a general situation that due to the channel difference between the relay and the destination nodes, some destination nodes may not receive all transmissions from the relay nodes, in this case, the generator matrices observed by the destination nodes maybe different. Thus, we use $\mathbf{G}_i$ to represent the generator matrix seen from the ith destination node D_i. The situation that all destination nodes observe the same generator matrices is only a special case in our work.

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The parity part matrix $\mathbf{P}_i$ is given by

$$\mathbf{P}_i = \left[\left(\mathbf{P}_i^1\right)^T \cdots \left(\mathbf{P}_i^t\right)^T \cdots \left(\mathbf{P}_i^L\right)^T \right]^T, \tag{3}$$

where $\mathbf{P}_i^t$ is an $S_t \times N$ parity part matrix of $\mathbf{G}_i^t$. The parity-check matrix $\mathbf{H}_i^t$ corresponding to the generator matrix $\mathbf{G}_i^t$ is

$$\mathbf{H}_i^t = \begin{bmatrix} \mathbf{I}_{A_t} & \mathbf{0}_{A_t \times S_t} & \mathbf{0}_{A_t \times B_t} & \mathbf{0}_{A_t \times N} \\ \mathbf{0}_{B_t \times A_t} & \mathbf{0}_{B_t \times S_t} & \mathbf{I}_{B_t} & \mathbf{0}_{B_t \times N} \\ \mathbf{0}_{N \times A_t} & \left(\mathbf{P}_i^t\right)^T & \mathbf{0}_{N \times B_t} & \mathbf{I}_N \end{bmatrix}, \tag{4}$$

where $A_t = \sum_{i'=1}^{t-1} S_{i'}$, $(2 \le t \le L)$ and $A_t = 0$, when $t = 1$. $B_t = \sum_{i'=t+1}^{L} S_{i'}$, $(1 \le t \le L-1)$ and $B_t = 0$, when $t = L$. $\left(\mathbf{P}_i^t\right)^T$ is the transpose of the matrix $\mathbf{P}_i^t$ [32].

Similarly, the 1st and the Lth parity check matrices corresponding to $\mathbf{G}_i^1$ and $\mathbf{G}_i^L$ are respectively given by

$$\mathbf{H}_i^1 = \begin{bmatrix} \mathbf{0}_{(M-S_1) \times S_1} & \mathbf{I}_{(M-S_1)} & \mathbf{0}_{(M-S_1) \times N} \\ \left(\mathbf{P}_i^1\right)^T & \mathbf{0}_{N \times (M-S_1)} & \mathbf{I}_N \end{bmatrix}, \tag{5}$$

$$\mathbf{H}_i^L = \begin{bmatrix} \mathbf{I}_{(M-S_L)} & \mathbf{0}_{(M-S_L) \times S_L} & \mathbf{0}_{(M-S_L) \times N} \\ \mathbf{0}_{N \times (M-S_L)} & \left(\mathbf{P}_i^L\right)^T & \mathbf{I}_N \end{bmatrix}. \tag{6}$$

A. The NCDM Scheme for a Parallel Session Model with Two Sessions

When $L = 2$, we can express the generator matrix $\mathbf{G}_i$ seen from the D_i, $i \in \{1, 2\}$, as

$$\mathbf{G}_i = \begin{bmatrix} \mathbf{G}_i^1 \\ \mathbf{G}_i^2 \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{S_1} & \mathbf{0}_{S_1 \times S_2} & \mathbf{P}_i^1 \\ \mathbf{0}_{S_2 \times S_1} & \mathbf{I}_{S_2} & \mathbf{P}_i^2 \end{bmatrix}, \tag{7}$$

where parity part matrices $\mathbf{P}_i^1$ and $\mathbf{P}_i^2$ are of sizes $S_1 \times N$ and $S_2 \times N$, respectively.

The parity check matrices of $\mathbf{G}_i^1$ and $\mathbf{G}_i^2$ are respectively given by

$$\mathbf{H}_i^1 = \begin{bmatrix} \mathbf{0}_{(M-S_1) \times S_1} & \mathbf{I}_{M-S_1} & \mathbf{0}_{(M-S_1) \times N} \\ \left(\mathbf{P}_i^1\right)^T & \mathbf{0}_{N \times (M-S_1)} & \mathbf{I}_N \end{bmatrix}, \tag{8}$$

$$\mathbf{H}_i^2 = \begin{bmatrix} \mathbf{I}_{M-S_2} & \mathbf{0}_{(M-S_2) \times S_2} & \mathbf{0}_{(M-S_2) \times N} \\ \mathbf{0}_{N \times (M-S_2)} & \left(\mathbf{P}_i^2\right)^T & \mathbf{I}_N \end{bmatrix}. \tag{9}$$

The generator matrix after using the NCDM scheme at the destination node D_i is named as the equivalent generator matrix, denoted by $\widetilde{\mathbf{G}}_i$. The equivalent generator matrix seen from D_i for the session φ_i when $i = 1$, is then

$$\begin{aligned} \widetilde{\mathbf{G}}_1 &= \mathbf{G}_1 \odot (\mathbf{H}_1^2)^T = \begin{bmatrix} \mathbf{G}_1^1 \\ \mathbf{G}_1^2 \end{bmatrix} \odot (\mathbf{H}_1^2)^T \\ &= \begin{bmatrix} \mathbf{I}_{S_1} & \mathbf{P}_i^1 \\ \mathbf{0}_{S_2 \times S_1} & \mathbf{0}_{S_2 \times N} \end{bmatrix}. \end{aligned} \tag{10}$$

Let $[\underline{m_1} \ \underline{m_2}]$ represent an arbitrary row of the data matrix $[\mathbf{m}_1 \ \mathbf{m}_2]$. At the destination node D_1, the equivalent received vector after using the NCDM scheme is

$$\begin{aligned} \mathbf{r}_1 \left(\mathbf{H}_1^2\right)^T = & \left[\boldsymbol{\alpha}_1 \otimes \sqrt{E_b} \left(\mathbf{J} - 2[\underline{m_1} \ \underline{m_2}] \odot \begin{bmatrix} \mathbf{G}_1^1 \\ \mathbf{G}_1^2 \end{bmatrix} \right) \right] (\mathbf{H}_1^2)^T \\ & + \mathbf{n}_1 (\mathbf{H}_1^2)^T, \end{aligned} \tag{11}$$

where $(\mathbf{H}_1^2)^T$ is the transpose of the parity check matrix $\mathbf{H}_1^2$. $\boldsymbol{\alpha}_1$ is the fading coefficient vector of size $1 \times (M + N)$. $\mathbf{J}$ is a row vector of size $1 \times (M + N)$ with all one elements. Ψ_j represents the set formed by the indices of positions of all the non-zero elements in the jth column of $\left(\mathbf{H}_1^2\right)^T$. Note that (11) cannot be calculated properly, because $\boldsymbol{\alpha}_1$ is the fading coefficient vector and $\mathbf{n}_1$ is a noise vector, while $(\mathbf{H}_1^2)^T$ is a binary matrix, and the multiplication of them is implemented over the complex field. To tackle this problem, in [33] we employed a soft processing algorithm at the destination nodes to process the soft value of the received symbols. The soft processing algorithm is as follows:

For a binary random variable $X \in \{\pm 1\}$, its LLR is defined as $\zeta(X) \doteq log\left(\frac{P_r\{X=+1\}}{P_r\{X=-1\}}\right)$. Subsequently, the LLR for a transmitted symbol x_k at the destination node is given by $\zeta_k = \frac{4\alpha_{i,k} r_k E_b}{N_0}$, where $\alpha_{i,k}$ is the fading coefficient, which is assumed to be known at the receiver, r_k is the received symbol corresponding to the transmitted symbol x_k.

Once the LLR values are obtained for all the transmitted symbols, we then need to find the corresponding input LLR values for each check node. This can be done by multiplying the LLR values with each column of the matrix $\left(\mathbf{H}_1^2\right)^T$. The obtained vectors of the LLR values after the multiplication are then the LLR values of the input symbols for the check nodes. Let z be the output symbol of the jth check node. Then $z = \oplus \sum_{k \in \Psi_j} x_k$, where $\oplus$ represents the addition operation over the field of GF(2). The output LLR values of each check node can be computed by $L_j = 2\tanh^{-1}\left(\Pi_{k \in \Psi_j} \tanh(\frac{\zeta_k}{2})\right)$.

$J_{i'}$ is the i'th element of $\mathbf{J}$. $\beta_{i'}$ is the i'th element of $\boldsymbol{\beta}$, and $\boldsymbol{\beta} = [\underline{m_1} \ \underline{m_2}] \odot \begin{bmatrix} \mathbf{G}_1^1 \\ \mathbf{G}_1^2 \end{bmatrix}$. When $\left(\oplus \sum_{i' \in \Psi_j} J_{i'}\right) - 2\left(\oplus \sum_{i' \in \Psi_j} \beta_{i'}\right) \ne 0$ for arbitrary j, $j \in \{1, 2, \cdots, M + N - S_2\}$, we can construct a coefficient $\boldsymbol{\alpha}_1'$, and $\boldsymbol{\alpha}_1'$ is of size $1 \times (M + N - S_2)$. Let $\alpha_{1,j}'$ be the jth element of $\boldsymbol{\alpha}_1'$, and $\alpha_{1,j}' = \frac{\sum_{i' \in \Psi_j} \alpha_{1,i'} (J_{i'} - 2\beta_{i'})}{\left(\oplus \sum_{i' \in \Psi_j} J_{i'}\right) - 2\left(\oplus \sum_{i' \in \Psi_j} \beta_{i'}\right)}$. Then (11) can be written as

$$\begin{aligned} \mathbf{r}_1 \left(\mathbf{H}_1^2\right)^T = & \boldsymbol{\alpha}_1' \otimes \sqrt{E_b} \left(\mathbf{J} \odot \left(\mathbf{H}_1^2\right)^T - 2[\underline{m_1}] \odot \widetilde{\mathbf{G}}_1^1 \right) \\ & + \mathbf{n}_1 \left(\mathbf{H}_1^2\right)^T. \end{aligned} \tag{12}$$

When $\left(\oplus \sum_{i' \in \Psi_j} J_{i'}\right) - 2\left(\oplus \sum_{i' \in \Psi_j} \beta_{i'}\right) = 0$, it will cause the problem of performance degradation. We will analyze this case in Section IV.

From (12) we can see that by using the NCDM scheme, destination node D_1 (and following a similar procedure, also D_2) only receives data from its own session. However, for the *parallel session* model with more than two sessions, directly multiplying generator matrix by the transpose of its submatrix' parity check matrix will lead to the interference calculation process stop. Therefore, the NCDM scheme proposed for the *parallel session* model with two sessions cannot be used directly in the *parallel session* model with more than two sessions. Next, we will introduce an effective method to adapt the NCDM scheme to the *parallel session* model with more than two sessions to minimize the inter-session interference.

B. The NCDM Scheme for a Parallel Session Model with More than Two Sessions

To obtain session φ_i's information at the destination D_i, $i \in \{1, 2, \cdots, L\}$, the interference introduced from other

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sessions to φ_i needs to be eliminated. This can be done by multiplying the generator matrix $\mathbf{G}_i$ by $(\mathbf{H}_i^t)^T$. Using the property that $\mathbf{G}_i^t \odot (\mathbf{H}_i^t)^T = \mathbf{0}$, $\forall t \in \{1, 2, \cdots, L\}\backslash i$, the interference from other sessions can be removed. However, different from the case with only two sessions, the parity check matrix $(\mathbf{H}_i^t)^T$ is also presented to other submatrices of $\mathbf{G}_i^j, j \in \{1, 2, \cdots, L\}\backslash t$, as an undesirable by-product. For example, when $t = 1$, after carrying out the first session $\mathbf{G}_i^1$'s interference cancelation process, the successive process of $\mathbf{G}_i^2 \odot (\mathbf{H}_i^1)^T \odot (\mathbf{H}_i^2)^T$ is not equal to zero due to the contribution of $(\mathbf{H}_i^1)^T$. As a result, the interference from the second session's information cannot be eliminated and the interference cancelation process stops. To solve this problem, in this paper, we deliberately introduce a term $\left(\left(\widetilde{\mathbf{H}}_i^1\right)^T\right)^{-1}$, so that the product of $(\mathbf{H}_i^1)^T$ and $\left(\left(\widetilde{\mathbf{H}}_i^1\right)^T\right)^{-1}$ approximately equals to an identity matrix $\mathbf{I}$. The interference cancelation process continues until φ_i's information at D_i is obtained. $\left(\widetilde{\mathbf{H}}_i^t\right)^T$ is a matrix obtained by setting the linear dependent rows in $(\mathbf{H}_i^t)^T$ to zeros and keeping other rows unchanged. $\left(\widetilde{\mathbf{H}}_i^t\right)^T$ and its inverse matrix are respectively expressed as follows:

$$\left(\widetilde{\mathbf{H}}_i^t\right)^T = \begin{bmatrix} \mathbf{I}_{A_t} & \mathbf{0}_{A_t\times B_t} & \mathbf{0}_{A_t\times N}\\ \mathbf{0}_{S_t\times A_t} & \mathbf{0}_{S_t\times B_t} & \mathbf{0}_{S_t\times N}\\ \mathbf{0}_{B_t\times A_t} & \mathbf{I}_{B_t} & \mathbf{0}_{B_t\times N}\\ \mathbf{0}_{N\times A_t} & \mathbf{0}_{N\times B_t} & \mathbf{I}_N \end{bmatrix}, \qquad (13)$$

$$\left(\left(\widetilde{\mathbf{H}}_i^t\right)^T\right)^{-1} = \begin{bmatrix} \mathbf{I}_{A_t} & \mathbf{0}_{A_t\times S_t} & \mathbf{0}_{A_t\times B_t} & \mathbf{0}_{A_t\times N}\\ \mathbf{0}_{B_t\times A_t} & \mathbf{0}_{B_t\times S_t} & \mathbf{I}_{B_t} & \mathbf{0}_{B_t\times N}\\ \mathbf{0}_{N\times A_t} & \mathbf{0}_{N\times S_t} & \mathbf{0}_{N\times B_t} & \mathbf{I}_N \end{bmatrix}. \qquad (14)$$

In the following, we denote by $\mathbf{U}_{\overline{[Y]}}$ and $\mathbf{U}_{\underline{[Y]}}$ the first Y and the last Y columns of a matrix $\mathbf{U}$, respectively. Let $\mathbf{U}_{[Y_1\to Y_2]}$ represent a part of the matrix $\mathbf{U}$ from the Y_1th column to the Y_2th column. When $i \neq L$, a detailed description of the process to remove φ_1's interference is shown in the first part of the right hand side of (15). Since the first S_1 columns of $\mathbf{G}_i$ are zeros, excluding the first identity submatrix $\mathbf{I}_{S_1}$, within the process, $\mathbf{G}_{i\overline{[S_1]}}^t \odot \mathbf{P}_i^1$ can be successfully eliminated when $t \in \{2, \cdots, L\}\backslash i$. $\mathbf{P}_i^1$ is the parity part matrix of $\mathbf{G}_i^1$, and $\mathbf{P}_i^1$ is of size $S_1 \times N$.

$$\begin{aligned}
\widetilde{\mathbf{G}}_i &= \begin{bmatrix} \mathbf{G}_i^1 \odot (\mathbf{H}_i^1)^T \\ \vdots \\ \mathbf{G}_i^t \odot (\mathbf{H}_i^1)^T \\ \vdots \\ \mathbf{G}_i^L \odot (\mathbf{H}_i^1)^T \end{bmatrix} \odot \left(\left(\widetilde{\mathbf{H}}_i^1\right)^T\right)^{-1} \\
&\quad \odot \left(\prod_{t, t\in\{2,\cdots,L-1\}\backslash i} (\mathbf{H}_i^t)^T \odot \left(\left(\widetilde{\mathbf{H}}_i^t\right)^T\right)^{-1} \right) \\
&\quad \odot \left(\mathbf{H}_i^L\right)^T \\
&= \begin{bmatrix} \mathbf{0}_{S_1\times S_1} & \mathbf{0}_{S_1\times(M-S_1)} & \mathbf{0}_{S_1\times N} \\ \mathbf{0}_{S_2\times S_1} & \mathbf{G}_{i[(S_1+1)\to M]}^2 & \mathbf{G}_{i\overline{[S_1]}}^2 \odot \mathbf{P}_i^1 + \mathbf{G}_{i\underline{[N]}}^2 \\ \vdots & \vdots & \vdots \\ \mathbf{0}_{S_t\times S_1} & \mathbf{G}_{i[(S_1+1)\to M]}^t & \mathbf{G}_{i\overline{[S_1]}}^t \odot \mathbf{P}_i^1 + \mathbf{G}_{i\underline{[N]}}^t \\ \vdots & \vdots & \vdots \\ \mathbf{0}_{S_L\times S_1} & \mathbf{G}_{i[(S_1+1)\to M]}^L & \mathbf{G}_{i\overline{[S_1]}}^L \odot \mathbf{P}_i^1 + \mathbf{G}_{i\underline{[N]}}^L \end{bmatrix} \\
&\quad \odot \left(\prod_{t, t\in\{2,\cdots,L-1\}\backslash i} (\mathbf{H}_i^t)^T \odot \left(\left(\widetilde{\mathbf{H}}_i^t\right)^T\right)^{-1} \right) \\
&\quad \odot \left(\mathbf{H}_i^L\right)^T \\
&= \begin{bmatrix} \mathbf{0}_{S_1\times S_1} & \mathbf{0}_{S_1\times(M-S_1)} & \mathbf{0}_{S_1\times N} \\ \mathbf{0}_{S_2\times S_1} & \mathbf{G}_{i[(S_1+1)\to M]}^2 & \mathbf{G}_{i\underline{[N]}}^2 \\ \vdots & \vdots & \vdots \\ \mathbf{0}_{S_t\times S_1} & \mathbf{G}_{i[(S_1+1)\to M]}^t & \mathbf{G}_{i\underline{[N]}}^t \\ \vdots & \vdots & \vdots \\ \mathbf{0}_{S_L\times S_1} & \mathbf{G}_{i[(S_1+1)\to M]}^L & \mathbf{G}_{i\underline{[N]}}^L \end{bmatrix} \\
&\quad \odot \left(\prod_{t, t\in\{2,\cdots,L-1\}\backslash i} (\mathbf{H}_i^t)^T \odot \left(\left(\widetilde{\mathbf{H}}_i^t\right)^T\right)^{-1} \right) \\
&\quad \odot \left(\mathbf{H}_i^L\right)^T. \qquad (15)
\end{aligned}$$

(15) shows how to keep the ith session φ_i's information and remove other sessions' information simultaneously. The equivalent generator matrix corresponds to the ith session φ_i after using the NCDM scheme at D_i, can be expressed as

$$\begin{aligned}
\widetilde{\mathbf{G}}_i^i &= \begin{bmatrix} \mathbf{0}_{S_i\times A_i} & \mathbf{I}_{S_i} & \mathbf{0}_{S_i\times(B_i-S_L)} & \mathbf{G}_{i\underline{[N]}}^i \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{0}_{S_i\times A_i} & \mathbf{I}_{S_i} & \mathbf{0}_{S_i\times(B_i-S_L)} & \mathbf{P}_i^i \end{bmatrix}. \qquad (16)
\end{aligned}$$

Let $\mathbf{\Phi}_i$ represent the term of $\left(\prod_{t,t\in\{1,\cdots,L-1\}\backslash i} (\mathbf{H}_i^t)^T \odot \left(\left(\widetilde{\mathbf{H}}_i^t\right)^T\right)^{-1}\right) \odot (\mathbf{H}_i^L)^T$. In a matrix form, $\mathbf{\Phi}_i$ can be expressed as

$$\mathbf{\Phi}_i = \Bigg[\underbrace{\begin{matrix} \mathbf{0}_{S_1\times A_i} & \mathbf{0}_{S_1\times S_i} & \mathbf{0}_{S_1\times(B_i-S_L)} \\ \vdots & \vdots & \vdots \\ \mathbf{0}_{S_{i-1}\times A_i} & \mathbf{0}_{S_{i-1}\times S_i} & \mathbf{0}_{S_{i-1}\times(B_i-S_L)} \\ \mathbf{0}_{S_i\times A_i} & \mathbf{I}_{S_i} & \mathbf{0}_{S_i\times(B_i-S_L)} \\ \mathbf{0}_{S_{i+1}\times A_i} & \mathbf{0}_{S_{i+1}\times S_i} & \mathbf{0}_{S_{i+1}\times(B_i-S_L)} \\ \vdots & \vdots & \vdots \\ \mathbf{0}_{S_L\times A_i} & \mathbf{0}_{S_L\times S_i} & \mathbf{0}_{S_L\times(B_i-S_L)} \\ \mathbf{0}_{N\times A_i} & \mathbf{0}_{N\times S_i} & \mathbf{0}_{N\times(B_i-S_L)} \end{matrix}}_{(M-S_L)\ \text{columns}} \quad \underbrace{\begin{matrix} \mathbf{P}_i^1 \\ \vdots \\ \mathbf{P}_i^{i-1} \\ \mathbf{0}_{S_i\times N} \\ \mathbf{P}_i^{i+1} \\ \vdots \\ \mathbf{P}_i^L \\ \mathbf{I}_N \end{matrix}}_{N\ \text{columns}}\Bigg], \qquad (17)$$

where for the first $M - S_L$ columns, all the elements are equal to zero except an identity submatrix $\mathbf{I}_{S_i}$ of size $S_i \times S_i$, see (17). For the last N columns of $\mathbf{\Phi}_i$, the first L submatrices correspond to the parity part matrices of $\mathbf{G}_i^t, t \in \{1, \cdots, L\}\backslash i$, which are given by (1). The last N rows form an identity matrix, as shown in (17).

Let $[\underline{m_1} \ \cdots \ \underline{m_i} \ \cdots \ \underline{m_L}]$ represent an arbitrary row of the data matrix $[\mathbf{m}_1 \ \cdots \ \mathbf{m}_i \ \cdots \ \mathbf{m}_L]$. At destination node D_i, after the NCDM scheme, we can obtain the equivalent received signal as

$$\begin{aligned}
&\mathbf{r}_i\mathbf{\Phi}_i \\
= \ &\left[\boldsymbol{\alpha}_i \otimes \sqrt{E_b}\left(\mathbf{J} - 2[\underline{m_1} \ \cdots \ \underline{m_i} \ \cdots \ \underline{m_L}] \odot \mathbf{G}_i\right)\right]\mathbf{\Phi}_i \\
&+\mathbf{n}_i\mathbf{\Phi}_i, \qquad (18)
\end{aligned}$$

where $\boldsymbol{\alpha}_i$ is the fading coefficient of size $1 \times (M + N)$.

$J_{i'}$ is the i'th element of $\mathbf{J}$. $\beta_{i'}$ is the i'th element of $\boldsymbol{\beta}$, and $\boldsymbol{\beta} = [\underline{m_1} \ \cdots \ \underline{m_i} \ \cdots \ \underline{m_L}] \odot \mathbf{G}_i$. Ψ_j represents the set

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formed by the indices of positions of all the non-zero elements in the jth column of $\mathbf{\Phi}_i$.

When $\left(\oplus \sum_{i'\in\Psi_j} J_{i'}\right) - 2\left(\oplus \sum_{i'\in\Psi_j} \beta_{i'}\right) \neq 0$ for arbitrary j, $j \in \{1,2,\cdots,M+N-S_L\}$, we can construct a coefficient $\boldsymbol{\alpha}'_i$, and $\boldsymbol{\alpha}'_i$ is of size $1\times(M+N-S_L)$. $\alpha'_{i,j}$ is the jth element of $\boldsymbol{\alpha}'_i$, and $\alpha'_{i,j} = \frac{\sum_{i'\in\Psi_j}\alpha_{i,i'}(J_{i'}-2\beta_{i'})}{\left(\oplus\sum_{i'\in\Psi_j} J_{i'}\right)-2\left(\oplus\sum_{i'\in\Psi_j}\beta_{i'}\right)}$. Then (18) can be written as

$$\mathbf{r}_i\mathbf{\Phi}_i = \boldsymbol{\alpha}'_i \otimes \sqrt{E_b}\left(\mathbf{J}\odot\mathbf{\Phi}_i - 2\underline{[m_i]}\odot\widetilde{\mathbf{G}}_i^i\right) + \mathbf{n}_i\mathbf{\Phi}_i, \tag{19}$$

When $\left(\oplus \sum_{i'\in\Psi_j} J_{i'}\right) - 2\left(\oplus \sum_{i'\in\Psi_j} \beta_{i'}\right) = 0$, the performance becomes degraded. We will analyze this case in Section IV.

It can be seen from (19) that only session φ_i's data packet $\underline{[m_i]}$ is included in the equivalent received signal vector $\mathbf{r}_i\mathbf{\Phi}_i$. When $i = L$, the same conclusion can be obtained.

IV. Performance Analysis and the Generator Matrix Design Criteria

In this section, we analyze the performance of the *parallel session* model with the proposed NCDM scheme at the destination nodes. At first, based on the equivalent received signal vector after implementing the NCDM scheme defined in (19), the impact of the proposed NCDM scheme on the information and noise parts of the received signal $\mathbf{r}_i$ is analyzed. Then the LDGM codes design criteria are introduced based on the analytical results.

It can be seen from (19) that the equivalent received signal vector $\mathbf{r}_i\mathbf{\Phi}_i$ consists of two parts, the information part $\mathbf{J}\odot\mathbf{\Phi}_i - 2\underline{[m_i]}\odot\widetilde{\mathbf{G}}_i^i$ and the noise part $\mathbf{n}_i\mathbf{\Phi}_i$. Also, it can be seen from (19) that the equivalent received signal vector only contains the data packets from the ith session φ_i, $\underline{[m_i]}\odot\widetilde{\mathbf{G}}_i^i$. However, an extra matrix $\mathbf{\Phi}_i$ is introduced into the received signal vector. A row vector $\mathbf{J}\odot\mathbf{\Phi}_i$ is given by

$$\mathbf{J}\odot\mathbf{\Phi}_i = \widetilde{\mathbf{J}} - \mathbf{\Delta}, \tag{20}$$

where $\widetilde{\mathbf{J}}$ is a row vector of size $1\times(M+N-S_L)$ with all elements equal to one. $\mathbf{\Delta} = [\Delta_1,\cdots,\Delta_k,\cdots,\Delta_{M+N-S_L}]$ is a row vector of size $1\times(M+N-S_L)$, in which the element Δ_k, $k\in\{1,2,\cdots,M+N-S_L\}$ is reciprocal of the corresponding element in $\mathbf{J}\odot\mathbf{\Phi}_i$ in the field of GF(2).

The noise part of (19), $\mathbf{n}_i\mathbf{\Phi}_i$, is denoted by $\widetilde{\mathbf{n}}_i$, $\widetilde{\mathbf{n}}_i = [\widetilde{n}_{i,1},\cdots,\widetilde{n}_{i,k},\cdots,\widetilde{n}_{i,M+N-S_L}]$. The relationship between $\mathbf{n}_i$ and $\widetilde{\mathbf{n}}_i$ can be represented as

$$\widetilde{\mathbf{n}}_i = \mathbf{n}_i\mathbf{\Phi}_i = [n_{i,1},n_{i,2},\cdots,n_{i,M+N}]\mathbf{\Phi}_i. \tag{21}$$

Based on (20) and (21), (19) can be rewritten as

$$\begin{aligned}\mathbf{r}_i\mathbf{\Phi}_i &= \boldsymbol{\alpha}'_i\otimes\sqrt{E_b}(\mathbf{J}\odot\mathbf{\Phi}_i - 2\underline{[m_i]}\odot\widetilde{\mathbf{G}}_i^i) + \mathbf{n}_i\mathbf{\Phi}_i \\ &= \boldsymbol{\alpha}'_i\otimes\sqrt{E_b}(\widetilde{\mathbf{J}} - \mathbf{\Delta} - 2\underline{[m_i]}\odot\widetilde{\mathbf{G}}_i^i) + \widetilde{\mathbf{n}}_i \qquad (22) \\ &= \boldsymbol{\alpha}'_i\otimes\sqrt{E_b}(\widetilde{\mathbf{J}} - 2\underline{[m_i]}\odot\widetilde{\mathbf{G}}_i^i) + \widetilde{\mathbf{n}}_i - \boldsymbol{\alpha}'_i\sqrt{E_b}\mathbf{\Delta}. \qquad (23)\end{aligned}$$

From (22), the kth value of $\mathbf{r}_i\mathbf{\Phi}_i$, denoted by $(r_i\Phi_i)_k$, is given by

$$\begin{cases}\alpha'_{i,k}\sqrt{E_b}(1-2(\underline{[m_i]}\odot\widetilde{G}_i^i)_k)+\widetilde{n}_{i,k}, & if\ \Delta_k = 0\\ \alpha'_{i,k}\sqrt{E_b}(0-2(\underline{[m_i]}\odot\widetilde{G}_i^i)_k)+\widetilde{n}_{i,k}, & if\ \Delta_k = 1\end{cases}, \tag{24}$$

where $\alpha'_{i,k}$ is the kth element of $\boldsymbol{\alpha}'_i$, $\alpha'_{i,k} = 1$ for the AWGN channel. $(\underline{[m_i]}\odot\widetilde{G}_i^i)_k$ represents the kth element of $\underline{[m_i]}\odot\widetilde{\mathbf{G}}_i^i$.

In the following, we first present the relationship between the column weight of the matrix $\mathbf{\Phi}_i$ with the number of sessions being L and column weight ρ of the generator matrix $\mathbf{G}_i^t$, $t\in\{1,\cdots,L\}\backslash i$. This relationship tells us how to set the column weight ρ of $\mathbf{G}_i^t$ given the number of sessions L in the system, so that the introduced term of $\mathbf{\Phi}_i$ in (19) has the minimum effect on the system performance. Then the impact of the matrix $\mathbf{\Phi}_i$ on the information part and the noise part of the received signal vector will be analyzed separately. Next, based on these analytical results, the equivalent received signal-to-noise ratio (SNR) is derived. Finally, the network code design criteria are summarized.

A. Relationship between Column Weight of the Matrix $\mathbf{\Phi}_i$, the Number of Sessions L and Column Weight ρ of $\mathbf{G}_i^t$

The number of sessions L of the system, together with the column weight ρ of the matrix $\mathbf{G}_i^t$ of the LDGM codes, determine the column weight of the matrix $\mathbf{\Phi}_i$, which affects the received signal vector in (19).

As shown in (20) and (21), $\mathbf{\Delta}$ and $\widetilde{\mathbf{n}}_i$ are directly related to the column weight of $\mathbf{\Phi}_i$. According to (20), $\mathbf{\Delta}$ can be expressed as

$$\mathbf{\Delta} = \widetilde{\mathbf{J}} - \mathbf{J}\odot\mathbf{\Phi}_i. \tag{25}$$

The nonzero submatrix of the first $M-S_L$ columns in (17) is $\left[\mathbf{0}_{S_i\times A_i}\ \mathbf{I}_{S_i}\ \mathbf{0}_{S_i\times(B_i-S_L)}\right]$. The first $M-S_L$ elements of $\mathbf{\Delta}$ are $[\mathbf{0}_{1\times A_i}\ \mathbf{J}_{1\times S_i}\ \mathbf{0}_{1\times(B_i-S_L)}]$, where $\mathbf{J}_{1\times S_i}$ represents a vector of size $1\times S_i$ with all one elements. Now, let us move to the last N elements of $\mathbf{\Delta}$. As shown in (17), the column weight of $\mathbf{\Phi}_i$ equals to the sum of the column weights of the $L-1$ parity part matrices $\mathbf{P}_i^t$ plus one[3], where $t\in\{1,\cdots,L\}\backslash i$. As $\mathbf{P}_i^t$ is a part of $\mathbf{G}_i^t$, we only need to focus on the column weight of $\mathbf{G}_i^t$. That is, the two terms $\mathbf{\Delta}$ and $\widetilde{\mathbf{n}}_i$ given in (21) and (25), respectively, are related to the generator matrix of the LDGM codes. Here, we only consider regular LDGM codes, where the column weight ρ of $\mathbf{G}_i^t$ remains unchanged.

Let us first derive the relationship between the system parameter, i.e., the number of sessions L, the LDGM codes' parameter, i.e., the column weight ρ and $\mathbf{\Delta}$. In the following, we discuss separately the situations that the parameters L and ρ are odd and/or even numbers.

- When the number of sessions L is an even number
  - If ρ is an odd number, then $(L-1)\rho$ is also an odd number. As the column weight of $\mathbf{\Phi}_i$ equals to the sum of $L-1$ columns' weight of parity-check part in $\mathbf{G}_i^t$ plus 1, the column weight of $\mathbf{\Phi}_i$ is even and the last N elements of the row vector $\mathbf{J}\odot\mathbf{\Phi}_i$ are all zeros. According to (20) and (25), $\mathbf{J}\odot\mathbf{\Phi}_i = \widetilde{\mathbf{J}} - \mathbf{\Delta}$, $\widetilde{\mathbf{J}}$ is a row vector with all one elements. Thus $\mathbf{\Delta}$ is a

[3]The generator matrix of systematic LDGM codes can be expressed as $\mathbf{G} = [\mathbf{I}\ \mathbf{P}]$, so the parity-check matrix $\mathbf{H} = [\mathbf{P}^T\ \mathbf{I}]$ and the transpose matrix of $\mathbf{H}$ is given by $\mathbf{H}^T = \begin{bmatrix}\mathbf{P}\\ \mathbf{I}\end{bmatrix}$. Thus, the column weight of $\mathbf{H}^T$ equals to that of $\mathbf{P}$ plus 1.

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row vector with all elements equal to one. It follows that the kth element of $\mathbf{\Delta}$, Δ_k, can be expressed as $\Delta_k = 1$, if ρ is an odd number.

- If ρ is an even number, then $(L-1)\rho$ is also an even number, the column weight of $\mathbf{\Phi}_i$ is an odd number. Thus the row vector $\mathbf{J} \odot \mathbf{\Phi}_i$ has all elements equal to one. By using the above method, the value of $\mathbf{\Delta}$ can be derived. It is a row vector with all elements equal to zero. Consequently, Δ_k can be expressed as $\Delta_k = 0$, if ρ is even.

From the above descriptions, Δ_k can be generally expressed as

$$\Delta_k = \begin{cases} 1, & if\ L\ is\ an\ even\ number \\ & and\ \rho\ is\ an\ odd\ number \\ 0, & if\ L\ is\ an\ even\ number \\ & and\ \rho\ is\ an\ even\ number \end{cases}. \quad (26)$$

- When the number of sessions L is an odd number
  - No matter whether ρ is an odd or even number, $(L-1)\rho$ is always an even number. It follows that the column weight of $\mathbf{\Phi}_i$ equals to $(L-1)\rho+1$, which is an odd number. Thus the row vector $\mathbf{J} \odot \mathbf{\Phi}_i$ has all elements equal to one. Based on (20) and (25), $\mathbf{J} \odot \mathbf{\Phi}_i = \widetilde{\mathbf{J}} - \mathbf{\Delta}$, $\widetilde{\mathbf{J}}$ is a row vector with all elements equal to one. Consequently, $\mathbf{\Delta}$ is a row vector with all elements equal to zero. Therefore, Δ_k can be expressed as

$$\Delta_k = 0, \quad if\ L\ is\ odd. \quad (27)$$

B. The Impact of the Matrix $\mathbf{\Phi}_i$ on the Noise Part of the Received Signal Vector

The number of sessions L of the system and the column weight ρ of the generator matrix of the formed LDGM code for each session determine the received signal vector through the column weight of the matrix $\mathbf{\Phi}_i$. In this subsection, we first describe the impact of the matrix $\mathbf{\Phi}_i$ on the noise part of the received signal vector. Then, we derive the equivalent received SNR based on the relationships described above. The equivalent received SNR can be used to compare the performance of the formed LDGM codes with various L and ρ values in the latter part of this section.

From (24), we can see that no matter what the value of Δ_k is, the noise part is always $\widetilde{n}_{i,k}$. In other words, $\mathbf{\Phi}_i$ always affects the noise part of the received signal vector as $\widetilde{\mathbf{n}}_i = \mathbf{n}_i \mathbf{\Phi}_i$.

The kth element of $\widetilde{\mathbf{n}}_i$ in (21) can be rewritten as

$$\widetilde{n}_{i,k} = \sum_{q \in \Omega_k} n_{i,q}, \quad (28)$$

where Ω_k denotes a set, in which the elements are the row indices of nonzero elements in the kth column of matrix $\mathbf{\Phi}_i$. Ω_k has $(L-1)\rho+1$ elements. $\widetilde{n}_{i,k}$ equals to the sum of $(L-1)\rho+1$ elements in $\mathbf{n}_i$, where $\mathbf{n}_i$ is the channel noise and $\widetilde{n}_{i,k}$ is the kth element of $\widetilde{\mathbf{n}}_i$. It can be seen that when the value of ρ or L increases, the size of Ω_k gets larger and subsequently $\widetilde{n}_{i,k}$ increases. The impact of ρ or L on the noise part $\widetilde{\mathbf{n}}_i$ in (21) increases.

Based on (23), the kth equivalent received signal $(r_i\Phi_i)_k$ can also be represented as

$$\begin{cases} \alpha'_{i,k}\sqrt{E_b}(1-2(\underline{[m_i]} \odot \widetilde{G}_i^i)_k) + \widetilde{n}_{i,k}, \ if\ \Delta_k = 0 \\ \alpha'_{i,k}\sqrt{E_b}(1-2(\underline{[m_i]} \odot \widetilde{G}_i^i)_k) + \widetilde{n}_{i,k} - \alpha'_{i,k}\sqrt{E_b}, \\ \quad if\ \Delta_k = 1 \end{cases}. \quad (29)$$

To simplify the analytical expression, we assume that different sessions have the same number of source nodes, denoted by M_e $(S_i = M_e,\ i \in \{1, 2, \cdots, L\})$. From (24), it can be seen that $\mathbf{\Phi}_i$ always affects the received signal vector, regardless of the value of Δ_k. Let $P(\Delta_k = x)$, $x \in \{0, 1\}$, be the probability of the event that Δ_k equals to x. From (29) and based on the relationship among the row vector $\mathbf{\Delta}$, noise $\widetilde{\mathbf{n}}_i$ and column weight of $\mathbf{G}_i^t$, an equivalent SNR, which is denoted by $\overline{SNR}$, can be expressed as

$$\begin{aligned} \overline{SNR} \; &= \; \frac{1}{M_e+N}\Bigg\{\sum_{k=1}^{M_e}\frac{|\alpha'_{i,k}|^2E_b}{N_0} \\ &+ \sum_{k=M_e+1}^{M_e+N}\Bigg[P(\Delta_k=0)\frac{|\alpha'_{i,k}|^2E_b}{(L-1)\,\rho N_0+N_0} \\ &+ P(\Delta_k=1)\frac{|\alpha'_{i,k}|^2E_b}{|\alpha'_{i,k}|^2E_b+(L-1)\,\rho N_0+N_0}\Bigg]\Bigg\} \end{aligned} \quad (30)$$

From (26) and (30), we can see that the value of $\overline{SNR}$ is affected by the column weight ρ of $\mathbf{G}_i^t$ and the number of sessions L.

C. Unstable Zero Elements

In this part, we analyze the performance of systems with different number of sessions and with different column weights of the formed LDGM codes. The problem, unstable zero elements, which is caused by right multiplication of $\mathbf{\Phi}_i$, is elaborated. The impacts of these unstable zero elements on the performance of the formed LDGM codes with different L and ρ values are analyzed, by using the equivalent received SNR derived in Section IV-B.

In Section IV-B, the relationship between the noise part in (21) and ρ has been investigated. Here we investigate the relationship between the information part $\mathbf{J} \odot \mathbf{\Phi}_i - 2\underline{[m_i]} \odot \widetilde{\mathbf{G}}_i^i$ and ρ.

The information part, $\mathbf{J} \odot \mathbf{\Phi}_i - 2\underline{[m_i]} \odot \widetilde{\mathbf{G}}_i^i$, is denoted by $\mathbf{X}$, $\mathbf{X} = [x_1, \cdots, x_k, \cdots, x_{M+N-S_L}]$. From (20), we can see that the elements of $\mathbf{J} \odot \mathbf{\Phi}_i$ are determined by $\mathbf{\Delta}$. Similar to the previous subsection, we discuss the situations when L and ρ are even and/or odd numbers separately.

- **When L is an even number**, as shown in (26), the value of each element in $\mathbf{\Delta}$ depends on ρ. When ρ is an odd number, Δ_k equals to 1; when ρ is an even number, Δ_k equals to 0. Therefore, each element in $\mathbf{J} \odot \mathbf{\Phi}_i$ has two possible values, 0 or 1. Moreover, $(\underline{[m_i]} \odot \widetilde{G}_i)_k$ may equal to 0 or 1. Thus, an element of $\mathbf{X}$, such as x_k has four possible values, shown below

$$x_k = \begin{cases} +1, & if\ \Delta_k = 0, (\underline{[m_i]} \odot \widetilde{G}_i^i)_k = 0 \\ -1, & if\ \Delta_k = 0, (\underline{[m_i]} \odot \widetilde{G}_i^i)_k = 1 \\ 0, & if\ \Delta_k = 1, (\underline{[m_i]} \odot \widetilde{G}_i^i)_k = 0 \\ -2, & if\ \Delta_k = 1, (\underline{[m_i]} \odot \widetilde{G}_i^i)_k = 1 \end{cases}. \quad (31)$$

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The kth equivalent received signal $\tilde{r}_k$ can be expressed as $\tilde{r}_k = x_k + \tilde{n}_k$. With BPSK demodulation rule, $\tilde{r}_k$ can be detected as either $+1$ or -1. Apparently, when $x_k \in \{+1, -1, -2\}$, $\tilde{r}_k$ can be easily detected as either $+1$ or -1. However, when $x_k = 0$, $\tilde{r}_k$ has a probability of $\frac{1}{2}$ to be correctly detected. Thus, the incorrectly detected values associated with zero elements introduce errors. We refer to these zero elements as the ***unstable zero elements***.

The unstable zero elements are introduced by right multiplication of $\mathbf{\Phi}_i$ by the received signal vector. The probability of generating unstable zero elements can be expressed as

$$P(x_k = 0) = P\left(\Delta_k = 1\right) P\left(([\underline{m_i}] \odot \widetilde{G}_i^i)_k = 0\right), \quad (32)$$

where $P\left(\Delta_k = 1\right)$ denotes the probability of the event $\Delta_k = 1$. $P\left(([\underline{m_i}] \odot \widetilde{G}_i^i)_k = 0\right)$ denotes the probability of the event $([\underline{m_i}] \odot \widetilde{G}_i^i)_k = 0$. If $P\left(\Delta_k = 1\right) = 0$, then $P(x_k = 0) = 0$.

We assume that there are N relay nodes in the network. If source nodes are selected randomly in the relay phase, then (32) can be rewritten as $P(x_k = 0) = \frac{1}{2} \frac{N}{M_e + N} \times \frac{1}{2} = \frac{N}{4(M_e + N)}$.

These unstable zero elements, which are generated by right multiplication of $\mathbf{\Phi}_i$ by the received signal vector, cause extra errors with probability $\frac{1}{2} P(x_k = 0)$. These errors would result in lower equivalent SNR and degrade the decoding performance.

- **When L is an odd number**, Δ_k always equals to 0. From (31) we can see that x_k is $+1$ or -1. Therefore, when L is an odd number, x_k can be decoded correctly with a high probability.

  When the column weight of each generator matrix is an even number, the session has a better performance compared with the one whose generator matrix has an odd column weight.

  – If the column weight of the matrix $\mathbf{G}_i^t$ is an even number, the probability $P(\Delta_k = 1)$ in (32) equals to 0. Thus, in case of $P(x_k = 0) = 0$, no unstable zero elements are generated in the information part of the signal vector.
  – If the column weight is an odd number, some unstable zero elements in the information part of the signal vector will be generated. These zero elements lead to the worse performance.

  If the column weight of $\mathbf{G}_i^t$ is a smaller value, the equivalent noise in (28) reduces, so the code performance is better than that with a bigger column weight.

  As we discussed in (27), when L is odd, $\Delta_k = 0$. It means that no unstable zero elements are produced regardless of the value of ρ. In addition, the code with a smaller column weight has a better performance.

Based on the previous analysis, we can design network codes to achieve desired performance by properly selecting the code parameters according to the situation of the entire system. The code design criteria is summarized in the following.

D. Network Codes Design Criteria

As mentioned in (1), $\mathbf{G}_i^t$ is the session φ_t's generator matrix, seen from D_i. $\mathbf{H}_i^t$ is the corresponding parity-check matrix of $\mathbf{G}_i^t$. On the one hand, we should notice that when L is an even number, it is necessary to make sure that after the NCDM scheme, the generator matrices have desired column weights. As indicated in (24), (26) and (27), the column weight ρ of $\mathbf{G}_i^t$ must be chosen to be an even number if L is even. When L is an odd number, the column weight of $\mathbf{G}_i^t$ can be chosen as either an even or odd number, such that, $P(\Delta_k = 1) = 0$ and $P(x_k = 0) = 0$ do not produce unstable zeros elements. On the other hand, we need to select the column weight of $\mathbf{G}_i^t$ as small as possible to reduce the effect from the noise part of (19). Based on the above explanation, to minimize a code error performance, the code design criteria can be summarized as

- Make sure that the column weight of each generator matrix $\mathbf{G}_i^t$ is an even number if L is an even number. If L is an odd number, the column weight of each generator matrix $\mathbf{G}_i^t$ can be either an even or odd number.
- Minimize the column weight of the generator matrix $\mathbf{G}_i^t$, i.e., $\rho = 2$ when L is an even number; $\rho = 2$ or $\rho = 3$ when L is an odd number.

V. Performance Evaluation

In this section, we present simulation results for the proposed NCDM scheme. Due to the time-varying channels in the wireless network, within each transmission round, the LDGM code is generated on-the-fly to match the instantaneous network topology. The performance[4] of the designed code is based on the ensemble average performance. We consider a wireless network with 3 or 4 sessions, respectively. In the simulation, each session has 300 source nodes[5], sending data packets to its corresponding destination node via a common relay network. For convenience, we assume that the number of relay nodes equals the total number of source nodes in the investigated networks.

A. BER Performance

We assume that all source nodes have data packets to transmit. All the source-relay channels in the network are

[4]In this paper, the effect from asynchronisation to the performance of both the *serial session* model and the *parallel session* model is not considered. Asynchronisation in distributed wireless network will cause collisions and idle periods among transmissions from different sensor nodes. Collisions cause the BER performance to be deteriorated for both the *serial session* model and the *parallel session* model. The throughput of both the two session models will decrease with the increasing of idle periods.

[5]The simulation parameter, 300 source nodes in each transmission session, is arbitrarily chosen. In a practical network, the number of source nodes is determined by the topology of the network. We also can set different values to the parameter according to the requirements in different applications. Note that there exists a trade-off between the end-to-end delay and the performance. Assume that the threshold of the end-to-end delay is Γ. ℓ_H, ℓ_C and ℓ_I represent the length of the header, the CRC part, and the information data part of one data packet, respectively. When $\frac{\ell_H + \ell_C}{\ell_I} < \frac{\Gamma}{M - \Gamma}$, the additional time required for transmitting the information data in the *parallel session* model is tolerable. In general, the length of the information data part is much larger than the length of the header and the CRC part in one data packet, i.e., $\ell_I \gg \ell_H + \ell_C$. Thus, $\frac{\ell_H + \ell_C}{\ell_I}$ is a very small value. Therefore, we can come to the conclusion that in the general case, the end-to-end delay in the *parallel session* model is small and tolerable.

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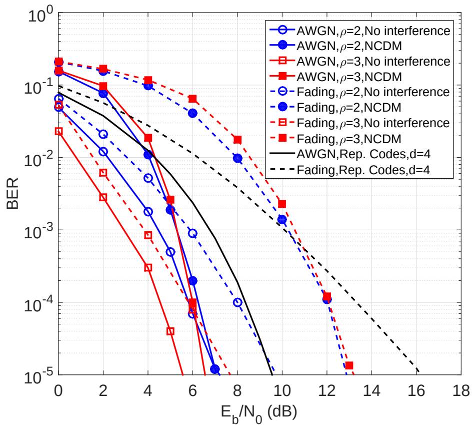
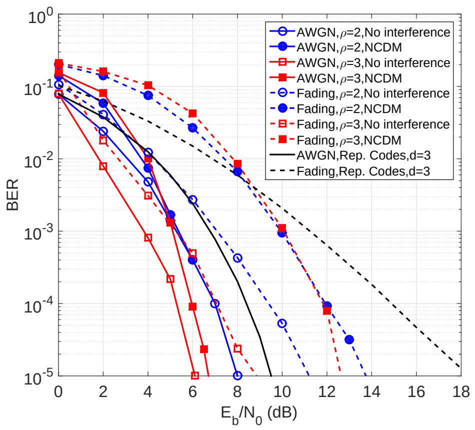
spatially independent and have equal transmitting power. By a *serial session* model, we refer to multiple groups of source nodes communicating with their respective destinations through a common relay network one by one. On the other hand, the *parallel session* model refers to multiple groups of source nodes communicating with their respective destinations through a common relay network simultaneously. In the presented figures, the label “no interference” corresponds to the *serial session* model. “NCDM” refers to the *parallel session* model and at each destination the NCDM scheme is used.

Fig. 4 (a) presents the performance comparison of various column weights of $\mathbf{G}_i^t$, (i.e., $\rho = 2$ and $\rho = 3$), for an even number of sessions (i.e., $L = 4$). As we discussed in Section IV-C, when the number of sessions L is an even number, if the column weight ρ of $\mathbf{G}_i^t$ is an even number, only the noise part in (19) is affected. As a result, the BER performance gap between the *serial session* model and *parallel session* model is smaller than the one for the case when ρ is an odd number. Note that when the column weight ρ is an odd number, both the information part and the noise part in (19) are affected, and the BER performance gets worse. The observation is consistent with the theoretical analysis results.



(a) The number of sessions is $L = 4$.



(b) The number of sessions is $L = 3$.

Fig. 4. The BER performance comparison between two different column weights of generator matrix, i.e., $\rho = 2$ and $\rho = 3$, over AWGN and Rayleigh fading channels, respectively. The number of sessions is $L = 3$ and 4.

Fig. 4 (b) presents the performance comparison of various column weights of $\mathbf{G}_i^t$, (i.e., $\rho = 2$ and $\rho = 3$), with an odd number of sessions (i.e., $L = 3$). As we discussed in Section IV-C, when the number of sessions is an odd number, only the noise part in (19) is affected. The smaller the column weight ρ of $\mathbf{G}_i^t$ is, the closer the BER performance of the *parallel session* model with NCDM to that of the *serial session* model. From Fig. 4 (b) we can see that when $\rho = 2$, the BER performance of the *parallel session* model is very close to the one for the *serial session* model.

The idea of the repetition code is to repeat the message several times. We consider that there are $L = 3$ and 4 sessions in the network. After using the NCDM scheme, the code rate becomes $\frac{1}{L+1}$. Thus we compare the performance of our proposed scheme with the repetition code with parameter $d = L$, i.e., the messages are repeated L times. From Fig. 4, it can be seen that the proposed system achieves a significant BER performance gain compared with the system with the repetition codes in a high SNR region.

For the Rayleigh fading channel, the BER performance gap between the *serial session* and *parallel session* model is much larger than that of the AWGN channel, as shown in Fig. 4. This is due to the information loss caused by LLR processing within the NCDM scheme. For the fading channel, the difference among the LLR values in the vector is much larger than that of the AWGN channel. The smaller LLR values result in worse BER performance gap between the *serial session* and *parallel session* model over the fading channel.

B. *System Throughput*

The throughput performance of the *parallel session* model with the NCDM scheme over AWGN and Rayleigh fading channels are shown in Fig. 5. Let $\triangle t_b$ represent the transmission time for one bit. Here, the throughput is defined as the number of correctly delivered information bits by all the source nodes in $\triangle t_b$. $\triangle t$ represents one time slot used to transmit one data packet, and $\triangle t = \ell \triangle t_b$. ℓ is the length of one data packet. One transmission period $\triangle T$ is defined as the time required to transmit all the sources' data packets and relays' data packets in one round. Assume that the source nodes in the ith session φ_i have S_i data packets to transmit in one transmission period, and the number of correctly delivered data packets at D_i is B_i for the *parallel session* model and C_i for the *serial session* model, respectively. Then, the number of transmitted data packets from all the sessions in one transmission period is $M = \sum_{i=1}^{L} S_i$, and the number of correctly delivered data packets at all the destination nodes is $B = \sum_{i=1}^{L} B_i$ for the *parallel session* model and $C = \sum_{i=1}^{L} C_i$ for the *serial session* model, respectively.

Assume that TDMA is employed. For the *serial session* model, the source nodes in transmission session φ_i send their date packets to the relay nodes separately in a number of S_i time slots. After all the source nodes in φ_i sending their data packets, the relay nodes begin to decode the received data packets from φ_i, check the correctness of the decoded packets by using CRC and put the correctly decoded packets into their buffers. Then the relay nodes select a fix number

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of data from their buffers and perform network coding to form network-coded data packets. Finally, the relay nodes send their network-coded data packets to the destination nodes in a number of N time slots. The same operations between the transmission sessions and the relay nodes run alternately until the relay nodes assist all the L transmission sessions completing their transmissions. The total time slots for the data transmission from the source nodes to the destination nodes in the *serial session* model is $\sum_{i=1}^{L} S_i + LN$, i.e., $M + LN$.

For the *parallel session* model, the source nodes in all the L transmission sessions send their date packets to the relay nodes in a number of $\sum_{i=1}^{L} S_i$ time slots. After all the source nodes sending their data packets, the relay nodes deal with the received data packets, perform network coding, and send the formed network-coded data packets to the destinations in a number of N time slots. Thus, the total time slots for the date transmission from the source nodes to the destination nodes in the *parallel session* model is $\sum_{i=1}^{L} S_i + N$, i.e., $M + N$.

Let ℓ_H and ℓ_C represent the length of the header and the CRC part of one data packet, respectively. The throughput of the *parallel session* model with L source group-destination pairs and the *serial session* scheme are calculated as follows:

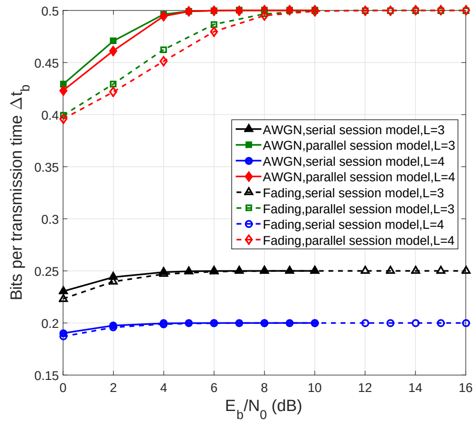
$$\begin{aligned}\frac{\ell B-(\ell_H+\ell_C)B}{(\triangle T)_{parallel\ sessioin}} &= \frac{\ell\sum_{i=1}^{L}B_i-(\ell_H+\ell_C)\sum_{i=1}^{L}B_i}{\sum_{i=1}^{L}S_i\triangle t+N\triangle t}\\ &= \frac{(\ell-(\ell_H+\ell_C))\sum_{i=1}^{L}B_i}{(\sum_{i=1}^{L}S_i+N)\ell\triangle t_b}\\ &= \frac{\left(1-\frac{\ell_H+\ell_C}{\ell}\right)\sum_{i=1}^{L}B_i}{M+N}/\triangle t_b \qquad (33)\end{aligned}$$

$$\begin{aligned}\frac{\ell C-(\ell_H+\ell_C)C}{(\triangle T)_{serial\ session}} &= \frac{\ell\sum_{i=1}^{L}C_i-(\ell_H+\ell_C)\sum_{i=1}^{L}C_i}{\sum_{i=1}^{L}S_i\triangle t+LN\triangle t}\\ &= \frac{(\ell-(\ell_H+\ell_C))\sum_{i=1}^{L}C_i}{(\sum_{i=1}^{L}S_i+LN)\ell\triangle t_b}\\ &= \frac{\left(1-\frac{\ell_H+\ell_C}{\ell}\right)\sum_{i=1}^{L}C_i}{M+LN}/\triangle t_b \qquad (34)\end{aligned}$$

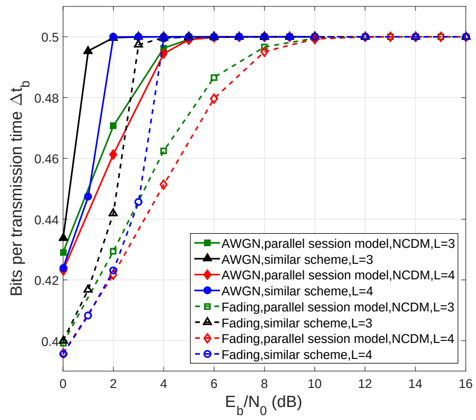
In general, $\frac{\ell_H+\ell_C}{\ell}$ is a very small value. The effect from the overhead to throughput is very small.

It can be seen from Fig. 5 (a), the throughput for the *parallel session* model with the NCDM scheme significantly outperforms the *serial session* scheme. This is due to the fact that at each time unit, the *serial session* model only allows one source group to be relayed, while the *parallel session* model with the NCDM scheme allows all the sessions' data packets to be relayed simultaneously. From (33) and (34) we can see, when $B = C$, the throughput of the *parallel session* model is $\frac{L+1}{2}$ times that of the *serial session* model. In addition, we should notice that the BER performance of the *parallel session* model with the NCDM scheme approaches that of the *serial session* model over AWGN channels, as shown in Fig. 4.

Next, we compare the throughput of the NCDM scheme with a similar scheme, which use the *parallel session* model but make BP decoding without canceling any interference. In the similar scheme, at each destination node, though only the information from its own session is required, the information from all the L transmission sessions are recovered. The comparison of the throughput of system with the NCDM scheme and the similar scheme is shown in Fig. 5 (b).



(a) Throughput comparison between the serial session and the parallel session models.



(b) The parallel session model with the NCDM scheme and the similar scheme.

Fig. 5. Throughput comparison between (a) the serial session and the parallel session models, (b) the parallel session model with the NCDM scheme and the similar scheme, for $L = 3$ and 4, $S_i = 300$, $N = 900$ and 1200, over AWGN and Rayleigh fading channels.

It can be seen from Fig. 5 (b), the throughput for the *parallel session* model with the NCDM scheme is worse than the similar scheme at low E_b/N_0 region. This is due to the fact that there are no interference problems among different sessions when using the similar scheme, while the *parallel session* model with the NCDM scheme has to deal with the inter-session interference. The throughput of the *parallel session* model with the NCDM scheme approaching to that of the similar scheme as the value of E_b/N_0 increases. At the region where $E_b/N_0 \geq$ 6dB for the AWGN channels and $E_b/N_0 \geq$ 10dB for the Rayleigh fading channels, the throughput of the *parallel session* model match that of the similar scheme. However, in the similar scheme, the information from all the sessions is recovered at all the destinations. Thus, though the similar scheme can achieve better throughput performance, it cannot satisfy the security requirement of the applications considered in this paper.

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C. Complexity

For simplicity, in our investigation we assume that the number of source nodes M from all the sessions equals the number of relay nodes N, i.e. $M = N$. We further assume that the number of source nodes M_e in each session is the same, that is $M_e = M/L$. We define the computational complexity as the total number of additions and multiplications. When $L >> 1$, $N >> 1$ and the column weight of the generator matrix $\mathbf{G}_i^t$, $\rho \geq 2$, the computational complexity for the NCDM scheme can be expressed by $(L-1)\rho N < L\rho N$.

Table I presents a summary of the complexity for the system models applying respectively repetition codes, the *serial session* model and the *parallel session* model with NCDM scheme. The system complexity includes the relaying complexity, the computational complexity of implementing interference cancelation by the proposed NCDM scheme and decoding complexity at the destination nodes.

As the decoding complexity of the three systems are the same, here, we only focus on discussing the computational complexity of implementing the proposed interference cancelation scheme. For the *parallel session* model with the proposed NCDM scheme, can be approximated as $O(L\rho N)$. Compared with the *serial session* model, the additional computational complexity to carry out interference cancelation process is a linear function of N and L.

VI. Conclusion

In this work, we investigated the distributed network coding schemes for a *parallel session* model with network code division multiplexing scheme. In *parallel session* model, multiple relays were shared among multiple sessions. We considered the inter-session interference problem of multicasting data packets from multiple groups of source nodes to their respective destination nodes via a common wireless relay network. We proposed a NCDM scheme for the *parallel session* model to remove the inter-session interference.

In the system analysis, we found that the proposed NCDM scheme is able to minimize the inter-session interference at each destination. However, it also introduced some unstable zero elements and extra noise problems, which affect the formed code's BER performance. Based on the analysis, we investigated the relationship among the equivalent received signal vector, the number of sessions and column weight of the generator matrix. A code design criteria for the generator matrix construction was proposed. Simulation results showed that by constructing the generator matrix of the LDGM code following the proposed code design criteria, the problems caused by the NCDM scheme can be managed effectively and the BER performance can be improved significantly.

We further evaluated the proposed NCDM scheme in terms of the system throughput and complexity. From the evaluation we can see that the throughput of the *parallel session* model with the proposed NCDM scheme is $\frac{L+1}{2}$ times that of the *serial session* model. Our complexity evaluation showed that the proposed NCDM scheme for a *parallel session* model has a linear computational complexity in the number of relay nodes N and the number of sessions L.

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TABLE I
THE SUMMARY OF THE COMPLEXITY FOR THE SYSTEM MODELS APPLYING RESPECTIVE REPETITION CODES, THE *serial session* MODEL, AND THE *parallel session* MODEL WITH THE PROPOSED NCDM SCHEME.

Scheme	Relaying Complexity	Interference Cancelation Computational Complexity	Decoding Complexity
Repetition Codes	0	0	$O(N)$
Serial Session Model	$O(N)$	0	$O(N)$
Parallel Session Model	$O(N)$	$O(L\rho N)$	$O(N)$

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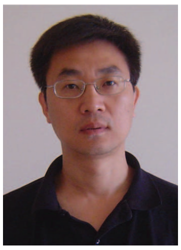

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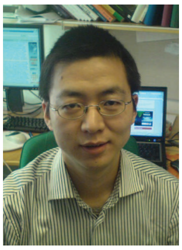

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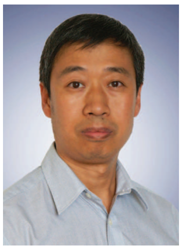

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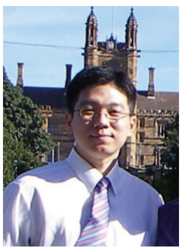

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