On the Information Propagation Process in Mobile Vehicular Ad Hoc Networks

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***Abstract*—In this paper, we study the information propagation process in a 1-D mobile ad hoc network formed by vehicles Poissonly distributed on a highway and traveling in the same direction at randomly distributed speeds that are independent between vehicles. Considering a model in which time is divided into time slots of equal length and each vehicle changes its speed at the beginning of each time slot, independent of its speed in other time slots, we derive analytical formulas for the fundamental properties of the information propagation process and the information propagation speed (IPS). Using the formulas, one can straightforwardly study the impact on the IPS of various parameters such as radio range, vehicular traffic density, and time variation of vehicle speed. The accuracy of the results is validated using simulations. The research provides useful guidelines on the design of vehicular ad hoc networks (VANETs).**

***Index Terms*—Information propagation speed (IPS), mobile ad hoc network, vehicular ad hoc network (VANET).**

I. Introduction

A VEHICULAR ad hoc network (VANET) is a mobile multihop network formed by vehicles traveling on the road. As a new way of communication, VANETs have attracted significant interest in not only academia but in industry as well [1]. IEEE has taken up working on new standards for VANETs, such as the IEEE 1609 Family of Standards for Wireless Access in Vehicular Environments (WAVE) [2]. Furthermore, there are many projects on VANETs such as InternetITS in Japan, Network on Wheels in Germany, and the PReVENT project in Europe [3]. In this paper, we study the expected propagation speed for a piece of information to be broadcast along the road in a VANET, which is referred to as the *information propagation speed (IPS)*. Due to the mobility of vehicles, the topology of a VANET is changing over time. Furthermore, traffic density on a road can significantly vary, depending on the time of day or day of the week. Therefore, the information propagation process in a VANET can be quite different from that in a static network.

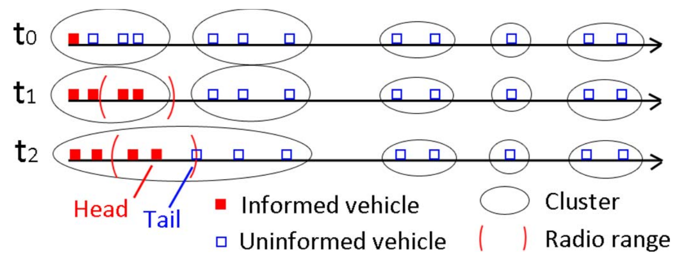

Fig. 1. Topology of a VANET at different time instances. In the figures in this paper, the positive direction of the axis is the direction of information propagation.

A VANET is usually partitioned into a number of clusters [4], [5], where a *cluster* is a maximal set of vehicles in which every pair of vehicles is connected by at least one multihop path. Due to the mobility of vehicles, the clusters are splitting and merging over time. Therefore, the information propagation in a VANET is typically based on a store-and-forward scheme, same as that in a delay-tolerant network [4]. Considering the example shown in Fig. 1, a piece of information starts to propagate from the origin toward the positive direction of the axis at time t_0. The vehicles that have received this piece of information are referred to as the *informed vehicles*, where other vehicles are *uninformed*. As indicated by the leftmost ellipse, the first informed vehicle is inside a cluster of four vehicles at time t_0. At time t_1, the message is forwarded, in a multihop manner, to the foremost vehicle in its cluster. The propagation of the message within a cluster, which begins at t_0 and ends at t_1, is called a *forwarding process*. In a forwarding process, the IPS is determined by the per-hop delay and the length of the cluster. The *per-hop delay* β is the time required for a vehicle to receive and process a message before it is available for further retransmission [6]. The value of β depends on the practical implementation, and a common assumption for the value of β reflecting typical technology is 4 ms [6]. We show that the

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per-hop delay has a significant impact on the IPS, particularly when the vehicle density is high.

Define the *head* at time t to be the informed vehicle with the largest coordinate at time t. Define the *tail* at time t to be the uninformed vehicle with the smallest coordinate at time t. Two vehicles can directly communicate with each other if and only if their Euclidean distance is smaller than or equal to the radio range r_0. Although this so-called *unit disk model* is a simplified model, it can be indicative for real-world scenarios. A realistic radio model usually takes into account statistical variations of the received signal power around its mean value [7]. It is shown in [7] that these variations can actually increase the connectivity of a network. Therefore, the analysis under the unit disk model provides a conservative estimate on the performance of a VANET. (We further explore this issue in Section VIII-C.) As shown in Fig. 1, at time t_1, the tail is outside the radio range of the head. Then, a *catch-up process* begins, during which the informed vehicles hold the information until the head catches up with the tail (at time t_2). We study both the forwarding process and the catch-up process in this paper.

The main contributions of this paper are given as follows: 1) Analytical results on the distribution of the time required for a catch-up process are provided. The impact of vehicle density, vehicle speed distribution, and vehicle speed variation over time is considered, where previous research (e.g., [5] and [8]) failed to consider the impact of the time-varying vehicle speed, which results in an unrealistic model according to traffic theory [9]. 2) A first passage phenomenon, which will be introduced later, is considered in the analysis. This first passage phenomenon is a major technical hurdle in the accurate analysis of the catch-up process when vehicle speeds are allowed to change over time. To the best of our knowledge, this work is the first attempt to consider the impact of the first passage phenomenon on the information propagation in VANETs. 3) The forwarding process is studied, taking into account the per-hop propagation delay and packet collision, which give an upper bound on the IPS that was not recognized by previous research (e.g., [5], [8], and [10]). It is shown that the per-hop delay and packet collision have significant impacts on the IPS, particularly when the vehicle density is high. Finally, a closed-form equation is derived for the distribution of the length of a cluster, where, in previous research, only numerical solutions [3] or approximate results [5] were provided. Based on the preceding results, the analytical results for the IPS are derived, including the impact of various parameters such as radio range, vehicular traffic density, and the time variation of vehicle speed. The results in this paper provide useful guidelines on the design of a mobile VANET.

The rest of this paper is organized as follows: Section II reviews the related work. Section III introduces the mobility model and network model used in this paper. The analysis on the catch-up process for a generic speed distribution is given in Section IV, followed by the analysis on the catch-up process for the Gaussian speed distribution in Section V. In Section VI, the analysis on the forwarding process is provided, including the analysis on the cluster length. Based on the preceding results, the IPS is derived in Section VII. Finally, Section IX concludes this paper.

II. Related Work

In recent years, VANETs have attracted significant interest due to their large number of potential applications [1]. In [11], Fracchia and Meo introduced the design of a warning delivery service in VANETs. They studied the propagation of a warning message in a 1-D VANET, where vehicles move in the opposite direction from the propagating direction of the warning message. However, their analysis is based on an oversimplified assumption that the topology does not change over time during the information propagation process. In [12], Camara *et al.* studied the propagation speed of public safety warning messages in a VANET when infrastructures (road side units) are destroyed by natural disasters such as flooding and earthquakes. Through simulation, they showed that using vehicles as virtual road side units can significantly speed up the warning message distribution process, compared with traditional emergency alert systems that rely on infrastructures.

The IPS is an important performance metric for VANETs, particularly for the safety messaging applications [12], [13]. In [6], Wu *et al.* studied the IPS through simulations. They used a commercial microscopic traffic simulator, i.e., CORSIM [14], to simulate the traffic on a highway. Then, the topology data from CORSIM were imported into a wireless communication simulator to study the properties of the information propagation process. They showed that the IPS significantly varies with vehicle densities.

There are analytical studies on VANETs based on the assumption that the vehicle speed does not change over time, which is referred to as the *constant speed model*. In [3], Yousefi *et al.* provided analytical results on the distribution of intervehicle distance in a 1-D VANET under the constant-speed model and the Poisson arrival model: In the *Poisson arrival model*, the number of vehicles passing an observation point on the road during any time interval follows a homogeneous Poisson process with intensity λ. By applying the results used in studying the busy period in queuing theory, they further analyzed the connectivity distance, which is a quantity similar to the cluster length to be introduced later in this paper. However, they did not provide a closed-form formula for the distribution of the cluster length. In [8], Agarwal *et al.* studied the IPS in a 1-D VANET where vehicles are Poissonly distributed and move at the same speed but in either the positive or the negative direction of the axis. They derived the upper and lower bounds for the IPS, which provided a hint on the impact of vehicle density on the IPS. However, the bounds are not tight, and many factors, e.g., time variation of speeds and propagation delay, were ignored in their analysis. In [5], Wu *et al.* considered a 1-D VANET where vehicles are Poissonly distributed and the vehicle speeds are uniformly distributed in a designated range. They provided a numerical method to compute the IPS under two special network models, i.e., when the vehicle density is either very low or very high, which are obviously oversimplified models [4], [9]. The aforementioned studies [3], [5], [8] were all based on the constant speed model. In this paper, the time variation of vehicle speed is considered and is shown to have a significant impact on the information propagation process in VANETs.

III. System Model

A. Mobility Model

A synchronized random walk mobility model is considered in this paper. Specifically, time is divided into time slots of equal length τ. Each vehicle randomly chooses its new speed at the beginning of each time slot, independent of other vehicles and its own speed in other time slots, according to a certain distribution with a mean value $E[v]$. It is shown later that the constant-speed model forms a special case in the aforementioned mobility model when $\tau \to \infty$. Due to a limited speed acceleration, the vehicle speed in the real world does not change as rapidly as in the aforementioned mobility model. Therefore, as will be shown in Section VIII-C, the results based on the aforementioned model provide a conservative estimate on the IPS, which is desirable for the safety messaging applications (e.g., [11] and [12]). Furthermore, we also discuss the impact of a nonsynchronized mobility model at the beginning of Section V.

In the preceding model, the speed of a vehicle can be considered as having a constant component $E[v]$ and a variable component with zero mean. Accordingly, the vehicular network can be decomposed into two components: 1) a network in which all vehicles travel at a constant speed and 2) a network in which vehicles travel at speeds following the same prescribed distribution $f_v(v)$ with zero mean. Our analysis focuses on the IPS in the second network component, where $f_v(v)$ is the probability density function (pdf) of the speed distribution. The first network component is separately considered and is combined into the result at the end of the analysis. Furthermore, a positive (negative) value of the speed means that the vehicle is traveling in the same (opposite) direction as the direction of information propagation. Therefore, when $E[v]$ is a positive (negative) value, our analysis provides the results for a VANET in which a message propagates in the same (opposite) direction as the direction of the vehicle traffic flow. In this paper, the analysis is first performed for a generic speed distribution. Then, detailed analytical results are given for the Gaussian speed distribution with standard deviation σ, which is commonly used for the VANETs on a highway [3], [9], [15].

The aforementioned speed-change time interval τ depends on practical conditions, e.g., a sports car may more frequently change its speed than a heavy truck. Reasonable values for the time interval can be from 1 to 25 s [16]. The vehicle mobility parameters, i.e., $E[v]$, σ, and τ, are taken from practical measurements. Typical values for $E[v]$ and σ are given in [15], where the usual record time intervals for a vehicle speed monitor are $\tau = 1$ s, 5 s [17]. We conduct our analysis in the discrete-time domain $(t = i\tau)$ to obtain closed-form analytical equations, which give better insight into the impact of different parameters on the IPS. Extension to the continuous-time domain is straightforward, following the procedure outlined in this paper.

B. Network Model

We adopt a commonly used traffic model in vehicular traffic theory [9] in which vehicles independently travel in the same direction on a 1-D infinite line and follow the Poisson arrival model with a rate λ veh/s. The Poisson arrival model is a commonly used traffic model in vehicular traffic theory based on real-world measurements [9]. Furthermore, the Poisson arrival model and the Poisson distribution of the vehicles are also commonly used traffic models in studies of VANETs [3], [8], [11].

The following lemma relates the spatial distribution of the vehicles on the road to the Poisson arrival model of the vehicles. The result on the spatial distribution of vehicles is used in the rest of this paper.

Lemma 1: If the traffic in a VANET follows the Poisson arrival model with rate λ and the speed of each vehicle changes at the beginning of each time slot, independent of other vehicles, according to $f_v(v)$, then at any time instant, the spatial distribution of the vehicles on the road follows a homogeneous Poisson process with intensity $\rho = \lambda \int_{-\infty}^{\infty} (f_v(v)/v) dv$.

Proof: It has been shown in [3] that, if the vehicle speeds do not change over time, then at any time instant, the distances between adjacent vehicles (intervehicle distance l) are independent and follow an exponential distribution with rate parameter $\rho = \lambda \int_{-\infty}^{\infty} (f_v(v)/v) dv$. It follows that the spatial distribution of the vehicles follows a homogeneous Poisson process with intensity ρ.

Next, we apply the mathematical induction to study the spatial distribution of the vehicles under our mobility model, in which vehicles are allowed to change their speeds from one time slot to another. In the first time slot $[0, \tau)$, it is straightforward to show that the spatial distribution of the vehicles follows a homogeneous Poisson process with intensity ρ, because the speed does not change during a time slot.

Assume that, in the ith time slot $[(i-1)\tau, i\tau)$, the spatial distribution of the vehicles still follows a homogeneous Poisson process with intensity ρ. When the next time slot begins, each vehicle chooses its new speed, independent of other vehicles, according to $f_v(v)$. Then, according to the random splitting property of a Poisson process [18], the spatial distribution of the vehicles traveling at the speed $v \in [v_m, v_m + dv_m)$ during the $i+1$th time slot $[i\tau, (i+1)\tau)$ follows a homogeneous Poisson subprocess with intensity $\rho f_v(v_m) dv_m$. These vehicles have the same speed so that, at any time instant during the $i+1$th time slot, the spatial distribution of these vehicles does not change. Then, according to the random coupling property of a Poisson process [18], the spatial distribution of all the vehicles in the $i+1$th time slot follows a homogeneous Poisson process, which is the sum of all the subprocesses, with the intensity $\int_{-\infty}^{\infty} \rho f_v(v_m) dv_m = \rho$. ■

Note that previous research [3] only considered that the vehicle speeds do not change over time. In Lemma 1, we consider the time variation of vehicle speeds in the analysis of vehicle distribution on the road.

IV. Catch-Up Process for a Generic Speed Distribution

In this section, we study the catch-up process in a VANET where the vehicle speeds follow a generic pdf $f_v(v)$. Without loss of generality, it is assumed that the catch-up process starts at time 0. The *displacement* x of a vehicle at time t is defined

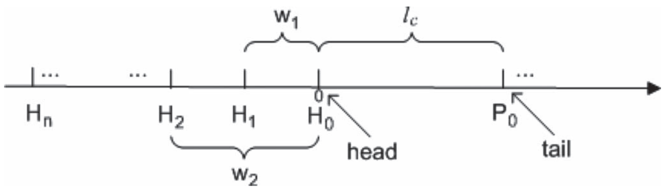

Fig. 2. VANET at the beginning of a catch-up process with gap l_c, where l_c is the Euclidean distance between the head and the tail at time 0. Hereinafter, a catch-up process where the distance between the head and the tail is l_c at time 0 is referred to as a *catch-up process with gap* l_c.

as the difference between the position of the vehicle at time 0 and its position at time t.

A. Modeling the Movement of a Single Vehicle

Denote by $p(x,t)$ the probability that the displacement of a vehicle is x at time t. Because the speed does not change during a time slot, $p(x,\tau)$ can be easily obtained from $f_v(v)$.

Due to the independence of the vehicle speeds in different time slots (hence, the displacements), we have, for $t = i\tau$

$$p(x,t) = p(x,i\tau) = \overbrace{(p * p * \cdots * p)(x,\tau)}^{\text{i−fold convolution}}. \tag{1}$$

The calculation of the aforementioned i-fold convolution can be simplified by using the Fourier and inverse Fourier transform.

B. Modeling the Movement of the Head and the Tail

Denote by H_m (P_m) the mth vehicle to the left of the head H_0 (to the right of the tail P_0) at time 0, as shown in Fig. 2. Define w_m to be the Euclidean distance between H_m and H_0 at time 0.

Let us first consider the movement of the head. Define $x_m(t)$ to be the displacement of H_m at time t. Define $y(t)$ to be the displacement of the head at time t. Note that the head vehicle at time 0 is not necessarily the head vehicle at time t because the original head may be overtaken by another informed vehicle during time $(0,t]$. It follows that

$$y(t) = \max\{x_0(t), x_1(t) - w_1, x_2(t) - w_2, \ldots, x_n(t) - w_n\} \tag{2}$$

where n is the number of vehicles to the left of the head that have the potential to overtake the head vehicle at time 0.

Because the movement of a vehicle is independent of other vehicles, $x_m(t)$ and $x_j(t)$ are independent for any $m \neq j$. Therefore, the cumulative distribution function (cdf) of the displacement of the head at time t is

$$\Pr(y(t) \le y) = \prod_{m=0}^{n} \Pr(x_m(t) - w_m \le y)$$
$$= \prod_{m=0}^{n} \left(\int_{0}^{\infty} \int_{-\infty}^{y+w_m} p(x,t) f_{w_m}(w_m)\, dx\, dw_m \right) \tag{3}$$

where $p(x,t)$ is given by (1), w_m is the distance between H_m and H_0 at time 0, $w_0 = 0$, and $f_{w_m}(w_m)$ is the pdf of w_m.

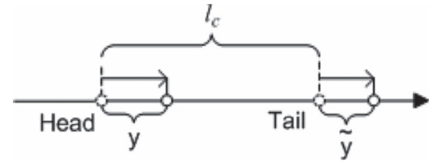

Fig. 3. Displacements of the head and the tail at time t in a catch-up process with gap l_c. The reduction of distance is $z = y - \tilde{y}$.

As an easy consequence of the Poisson distribution of the vehicles (proved in Lemma 1), the intervehicle distance follows an exponential distribution. Therefore, we have

$$f_{w_m}(w_m) = \frac{\rho e^{-\rho w_m} (\rho w_m)^{m-1}}{(m-1)!}, \text{ for } m \ge 1. \tag{4}$$

Define $p_h(y,t)$ to be the probability that the displacement of the head is y at time t. Then

$$p_h(y,t) = \frac{\partial \Pr(y(t) \le y)}{\partial y}. \tag{5}$$

The calculation is, however, tedious for a generic speed distribution. Therefore, only the methodology for the analysis of the catch-up process is shown in this section. A detailed analytical result for the catch-up process under the Gaussian speed distribution is shown in Section V.

Denote by $p_g(\tilde{y},t)$ the probability that the displacement of the tail is $\tilde{y}$ at time t. The analysis for the movement of the tail is similar to that for the head and is, therefore, omitted.

C. Catch-Up Delay

As shown in Fig. 2, consider a catch-up process where the Euclidean distance between the head and the tail is l_c at the beginning of the catch-up process (time 0), which is referred to as *a catch-up process with gap* l_c. Define the *catch-up delay* t_c to be the time taken from time 0 until the time when the head and the tail move into the radio range of each other for the first time, i.e., $t_2 - t_1$ in Fig. 1. We do not consider the rare event that the distance between the head and the tail becomes larger than r_0 again during $(t_c, t_c + \beta)$, which may cause the transmission of a packet being interrupted, because the per-hop delay β (e.g., 4 ms [6]) is usually much smaller than the time interval for a vehicle to change speed (typically longer than a second [16]). It is worth noting that there is a first passage phenomenon in the catch-up process, i.e., the catch-up process finishes as soon as the head and the tail move into the radio range of each other. Therefore, the catch-up delay is t_c *if* and *only if* (iff) the distance between the head and the tail reduces from l_c at time 0 to r_0 *for the first time* at time t_c. This first passage phenomenon is essential for the analysis of the catch-up process.

Denote by $p_H(z,t)$ the probability that the reduction of the Euclidean distance between the head and the tail is z at time t, with regard to their original distance at time 0. As shown in Fig. 3, it can be shown that

$$p_H(z,t) = \int_{-\infty}^{\infty} p_h(y,t) p_g(y - z, t) dy. \tag{6}$$

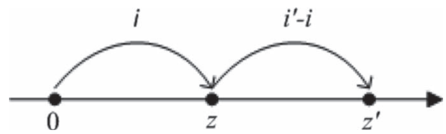

Fig. 4. Cass of random walks taking i' time slots to walk from 0 to z' through an intermediate point z.

Note that the aforementioned equation can be converted to convolution if $p_g(\tilde{y}, t) = p_g(-\tilde{y}, t)$, which is the case to be introduced in the next section when the vehicle speed follows a Gaussian distribution.

Denote by $h(z, i)$ the probability that the reduction in the distance between the head and the tail reaches z in the ith time slot $[(i-1)\tau, i\tau)$. Therefore

$$h(z, i) = \int_{(i-1)\tau}^{i\tau} p_H(z, t) dt. \tag{7}$$

To obtain a closed-form result, the probability of the reduction in the distance between the head and the tail being z at time $t \in [(i-1)\tau, i\tau)$ is considered to be approximately equal to the probability of the reduction of the distance between the head and the tail being z at time $t = i\tau$. This approximation provides a fairly accurate result when τ is small (e.g., $\tau = 1$ s, 5 s), as shown in Sections VIII-A and C. Therefore, from (7)

$$h(z, i) = \int_{(i-1)\tau}^{i\tau} p_H(z, t) dt \approx \tau p_H(z, i\tau). \tag{8}$$

Define $\xi(z, i)$ to be the first passage probability [19] of $h(z, i)$, viz., the probability that the reduction in the distance between the head and the tail reaches z in the ith time slot $[(i-1)\tau, i\tau)$ *for the first time* since time 0. The relationship between $\xi(z, i)$ and $h(z, i)$ can be studied as the first passage time in a stochastic process [20]. The first passage time of a diffusing particle or a random walker is the time at which the particle or the random walker *first* reaches a specified site.

A standard procedure is applied to determine the first passage probability $\xi(z, i)$ [19], [20]. As shown in Fig. 4, consider a class of random walks starting at time 0, and walking from point 0 to z' must proceed by going through a point z. The transition from 0 to z' can be decomposed into two independent stages: in the first stage, an agent walks from 0 to z for the first time in the ith time slot; in the second stage, the agent walks from z to z' in the $i' - i$th time slot, not necessarily for the first time. Then, we can obtain the following [19], [20]:

$$h(z', i') = \sum_{i=1}^{i'} \xi(z, i) h(z' - z, i' - i). \tag{9}$$

The convolution can be simplified by the Z-transform with regard to i, which is denoted by the operator $\mathcal{Z}$. According to the convolution theorem, (9) becomes

$$(\mathcal{Z}h)(z', s) = (\mathcal{Z}\xi)(z, s)(\mathcal{Z}h)(z' - z, s) \tag{10}$$

and thus
$$(\mathcal{Z}\xi)(z, s) = \frac{(\mathcal{Z}h)(z', s)}{(\mathcal{Z}h)(z' - z, s)}. \tag{11}$$

Then, by inverse Z-transform, we can obtain $\xi(z, i)$. Denote by $F_\xi(z, i)$ the cdf of $\xi(z, i)$ with regard to i, i.e., the probability that the reduction in the distance between the head and the tail has reached z during time $(0, i\tau]$. It follows that $F_\xi(l_c - r_0, i)$ is the probability that the head and the tail have moved into the radio range of each other during time $(0, i\tau]$. Therefore, the expected catch-up delay (t_c) for a catch-up process with gap l_c is

$$E[t_c | l_c] = \tau \sum_{i=1}^{\infty} \left(1 - F_\xi(l_c - r_0, i)\right). \tag{12}$$

D. Distribution of the Gaps l_c

Denote by $f_l(l)$ the pdf of the Euclidean distance between any two adjacent vehicles. Due to the Poisson distribution of vehicles, it is evident that $f_l(l) = \rho e^{-\rho l}$. Denote by $f_{l_c}(l_c)$ the pdf of the Euclidean distance between any two adjacent but disconnected vehicles. It is straightforward that, for $l_c > r_0$

$$f_{l_c}(l_c) = \frac{f_l(l_c)}{1 - \int_0^{r_0} f_l(l) dl} = \rho e^{-\rho(l_c - r_0)}. \tag{13}$$

V. Catch-Up Process for a Gaussian Speed Distribution

In this section, we provide detailed analytical results on the catch-up process under the Gaussian speed distributions, which is a commonly used assumption for the VANETs on a highway [3], [9], [15]. The procedure of the analysis is the same as that introduced in the previous section, except for some adjustments to obtain a simpler result.

For a zero-mean Gaussian speed distribution with standard deviation σ, the pdf of the vehicle speed is

$$f_v(v) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-v^2}{2\sigma^2}\right). \tag{14}$$

At the end of the first time slot, i.e., $t = \tau$, it is straightforward to show that

$$p(x, \tau) = \frac{1}{\sigma\tau\sqrt{2\pi}} \exp\left(\frac{-x^2}{2(\sigma\tau)^2}\right). \tag{15}$$

Furthermore, because the convolution of two Gaussian functions is another Gaussian function [21], using (1), we can obtain

$$p(y, i\tau) = \frac{1}{\sigma_i\sqrt{2\pi}} \exp\left(\frac{-y^2}{2\sigma_i^2}\right) \tag{16}$$

where $\sigma_i^2 = i(\sigma\tau)^2$.

Next, we consider the situation in which vehicles are allowed to change speed at different time instants. Without loss of generality, consider that a vehicle changes its speed at time τ_0 for the first time since time 0, where τ_0 is uniformly distributed in $(0, \tau]$. Therefore, for $t = i\tau$, (1) becomes

$$\begin{aligned} p(x, t) &= p(x, i\tau) \\ &= p(x, \tau_0) * \overbrace{(p * p * \cdots * p)(x, \tau)}^{\text{(i−1)−fold convolution}} * p(x, \tau - \tau_0) \\ &= \frac{1}{\sigma_i\sqrt{2\pi}} \exp\left(\frac{-x^2}{2\sigma_i^2}\right) \end{aligned} \tag{17}$$

where $\sigma_i^2 = (i-1)\sigma^2\tau^2 + \sigma^2\tau_0^2 + \sigma^2(\tau-\tau_0)^2 = i\sigma^2\tau^2 + 2\sigma^2\tau_0^2 - 2\sigma^2\tau\tau_0$.

Compared with the result using a synchronized mobility model [i.e., (16)], the additional terms are $2\sigma^2\tau_0^2 - 2\sigma^2\tau\tau_0$. To simplify the analysis, we ignore the additional terms and consider the synchronized mobility model only. The error caused by ignoring these additional terms and assuming a synchronized mobility model is given by $\int_0^\tau (2\sigma^2\tau_0^2 - 2\sigma^2\tau\tau_0)(1/\tau)d\tau_0 = -\sigma^2\tau^2/3$. It is obvious that the error is small, compared with the dominant term $i\sigma^2\tau^2$, particularly when i is large. Furthermore, the accuracy of this approximation is verified in Section VIII-C.

A. Catch-Up Delay in a Basic Catch-Up Process

In this section, we temporarily ignore the possibility of overtaking, i.e., we consider *a basic catch-up process* involving only the vehicle, which is the head at time 0, catching up with the vehicle, which is the tail at time 0. We have the following lemma for the basic catch-up process:

Lemma 2: In a basic catch-up process where vehicle speed follows a zero-mean Gaussian distribution with standard deviation σ, the probability that the reduction in the distance between the head and the tail is z *for the first time* during the ith time slot is

$$\xi(z,i) = \frac{z}{2i\tau\sigma\sqrt{\pi i}} \exp\left(-\frac{z^2}{4i\tau^2\sigma^2}\right). \tag{18}$$

Proof: Because the Gaussian speed distribution is symmetric with respect to the mean, for the displacement of the tail, we have $p_g(\tilde{y},t) = p(\tilde{y},t) = p(-\tilde{y},t)$. Therefore, using (6), it can be shown that

$$\begin{aligned} p_H(z,i\tau) &= \int_{-\infty}^{\infty} p(y,i\tau)p_g(z-y,i\tau)dy \\ &= (p*p)(z,i\tau) = \frac{1}{\tilde{\sigma}_i\sqrt{2\pi}} \exp\left(\frac{-z^2}{2\tilde{\sigma}_i^2}\right) \end{aligned} \tag{19}$$

where $\tilde{\sigma}_i^2 = 2i(\sigma\tau)^2$. Therefore

$$h(z,i) = \tau p_H(z,i\tau) = \frac{\tau}{\tilde{\sigma}_i\sqrt{2\pi}} \exp\left(\frac{-z^2}{2\tilde{\sigma}_i^2}\right). \tag{20}$$

Inspired by [20, 6.4], $h(z,i)$ in (20) can be rewritten as the following to calculate the Z-transform:

$$h(z,i) = \frac{\tau}{2\pi}\int_{-\infty}^{\infty} \exp\left(-jz\alpha - \frac{\tilde{\sigma}_i^2}{2}\alpha^2\right) d\alpha \tag{21}$$

where j denotes $\sqrt{-1}$.

Then, perform the Z-transform on (21) with regard to i, i.e.,

$$\begin{aligned} (\mathcal{Z}h)(z,s) &= \sum_{i=1}^{\infty} e^{-si}h(z,i) \\ &= \frac{\tau}{2\pi}\int_{-\infty}^{\infty} \exp(-jz\alpha)\sum_{i=1}^{\infty}\exp(-si)\exp\left(-\frac{\tilde{\sigma}_i^2}{2}\alpha^2\right)d\alpha. \end{aligned} \tag{22}$$

With $\tilde{\sigma}_i^2 = 2i(\sigma\tau)^2$ holds.

$$\begin{aligned} (\mathcal{Z}h)(z,s) &= \frac{\tau}{2\pi}\int_{-\infty}^{\infty} \exp(-jz\alpha) \\ &\quad \times \sum_{i=1}^{\infty}\exp(-si)\exp\left(-i(\sigma\tau)^2\alpha^2\right)d\alpha \\ &= \frac{\tau}{2\pi}\int_{-\infty}^{\infty} \exp(-jz\alpha)(s+\sigma^2\tau^2\alpha^2)^{-1}d\alpha \\ &= \frac{\tau}{2}\frac{\exp\left(-z\sqrt{s/(\sigma^2\tau^2)}\right)}{\sqrt{s\sigma^2\tau^2}}. \end{aligned} \tag{23}$$

Then, according to (11), we have

$$(\mathcal{Z}\xi)(z,s) = \frac{(\mathcal{Z}h)(z',s)}{(\mathcal{Z}h)(z'-z,s)} \tag{24}$$

$$\begin{aligned} &= \frac{\exp\left(-z'\sqrt{s/(\sigma^2\tau^2)}\right)}{\exp\left(-(z'-z)\sqrt{s/(\sigma^2\tau^2)}\right)} \\ &= \exp\left(-z\sqrt{s/(\sigma^2\tau^2)}\right). \end{aligned} \tag{25}$$

Finally, using the inverse Z-transform, it can be obtained that

$$\xi(z,i) = \frac{z}{2i\tau\sigma\sqrt{\pi i}} \exp\left(-\frac{z^2}{4i\tau^2\sigma^2}\right). \tag{26}$$

■

We say that one vehicle *catches up* with another vehicle iff the Euclidean distance between them reduces to the radio range r_0. Then, using Lemma 2, one can readily obtain the following result:

Theorem 1: Consider two vehicles separated by Euclidean distance l_c at time 0. The probability that one vehicle catches up with the other for the first time in the ith time slot is $(l_c - r_0/2i\tau\sigma\sqrt{\pi i})\exp(-(l_c-r_0)^2/4i\tau^2\sigma^2)$.

Proof: It is straightforward that the reduction in distance is $z = l_c - r_0$. Then, using Lemma 2, the theorem is readily proved. ■

B. Catch-Up Delay With Overtaking Permitted

In the previous section, the possibility of overtaking is not included in the calculation of the first passage probability, to obtain a closed-form result in (26). In this section, the possibility of overtaking is considered to provide a more accurate result on the catch-up delay. Recall that H_m is the mth vehicle to the left of the head H_0 at time 0 and that $P_{m'}$ is the m'th vehicle to the right of the tail P_0 at time 0. Note that all the vehicles H_m ($P_{m'}$) for $m, m' \in [1,\infty)$ can possibly overtake the head H_0 (tail P_0).

Lemma 3: Denote by $q_{mm'}(i|l_c)$ the probability that H_m catches up with $P_{m'}$ ($m, m' \in [0,\infty)$) for the first time in the ith time slot, in a catch-up process with gap l_c. Then, using Lemma 2, we have

$$q_{mm'}(i|l_c) \approx \xi(l_c - r_0 + m/\rho + m'/\rho, i). \tag{27}$$

Proof: Recall that w_m is the distance between H_m and H_0 at time 0, which follows an exponential distribution. Therefore, the expected distance between H_m and H_0 at time 0 is $\int_0^\infty w_m f_{w_m}(w_m) dw_m = m/\rho$, where $f_{w_m}(w_m)$ is given by (4). Similarly, the expected distance between $P_{m'}$ and P_0 at time 0 is m'/ρ. Then, in a catch-up process with gap l_c, the expected distance between H_m and $P_{m'}$ at time 0 is $l_c + m/\rho + m'/\rho$. In order for H_m to catch up with $P_{m'}$, the reduction in distance should be $z = l_c - r_0 + m/\rho + m'/\rho$. ■

Remark 1: In Lemma 3, only the mean value of the distance between vehicles is required. This provides us with the flexibility to extend the analysis from the Poisson distribution model to another vehicle distribution model, i.e., we only need to replace m/ρ in (27) by the corresponding average intervehicle distance if a different vehicle distribution model is used. The rest of the analysis on the catch-up process does not depend on the particular vehicle distribution model being used. However, the accuracy of using mean value approximation for another vehicle distribution needs to be validated.

Denote by $H(i|l_c)$ the probability that none of the $H_m - P_{m'}$ pairs $(m, m' \in [0, \infty))$ catches up in the ith time slot, in a catch-up process with gap l_c. Due to the independence of the movements of vehicles, we have

$$H(i|l_c) = \prod_{m,m' \in [0,\infty)} (1 - q_{mm'}(i|l_c)) \tag{28}$$

where $q_{mm'}(i|l_c)$ is given by Lemma 3. During numerical evaluation, finite values of m, m' can provide fairly accurate results, which are discussed later in Section VIII-A.

Denote by $h(i|l_c)$ the probability that at least one pair of $H_m - P_{m'}$ catches up in the ith time slot and none of them has caught up before the ith time slot, in a catch-up process with gap l_c. It is straightforward that

$$h(i_c|l_c) = (1 - H(i_c|l_c)) \prod_{i=1}^{i_c - 1} H(i|l_c). \tag{29}$$

Finally, the expected delay for a catch-up process with gap l_c is

$$E[t_c|l_c] = \sum_{i=1}^{\infty} i\tau h(i|l_c). \tag{30}$$

VI. Analysis on the Forwarding Process

In a forwarding process, the packet is forwarded in a multi-hop manner between vehicles inside a cluster. We start with the analysis on the distribution of the length of the cluster.

A. Cluster Length

Define the cluster length x_0 to be the diameter of a cluster, which is the Euclidean distance between the vehicles at the two ends of a cluster. Define $f_{x_0}(x_0)$ to be the pdf of the cluster length, which can be studied as the pdf of the busy period in queuing theory. In previous research, only numerical solutions [3] or approximate results [5] were provided. In this section, we provide a closed-form formula for the pdf of the cluster length using a different method inspired by the study on the connectivity of random interval graph [22] and theory of coverage processes [23].

Theorem 2: In a VANET where the spatial distribution of the vehicles follows a homogeneous Poisson process with intensity ρ, the pdf of the cluster length is

$$f_{x_0}(x_0) = \frac{\rho}{(e^{\rho r_0} - 1)} \sum_{m=0}^{\lfloor x_0/r_0 \rfloor} \frac{(-\rho(x_0 - m r_0))^{m-1}}{-m!} \times (\rho(x_0 - m r_0) + m) e^{-\rho m r_0} \tag{31}$$

where m is an integer, and $\lfloor . \rfloor$ is the floor function.

The proof is shown in the Appendix.

Theorem 2 shows a closed-form formula for the pdf of the cluster length, which is essential for analytical study on the performance of VANETs.

B. Hop Count Statistics in a Cluster

To study the IPS in the forwarding process, the number of hops between the leftmost vehicle and the rightmost vehicle in the cluster needs to be calculated. Two vehicles are said to be k hops apart if the shortest path between them, which is measured by the number of hops, is k. Define $\phi_k(x_0)$ to be the probability that two vehicles separated by Euclidean distance x_0 are k hops apart. It is assumed that the positions of the vehicles do not change during the forwarding process since the forwarding delay is relatively small, which is also confirmed in Fig. 7. Therefore, the probability $\phi_k(x_0)$ can be calculated by the result introduced in [24] for a static 1-D multi-hop network.

Define $\Pr_s(x_0)$ to be the probability of successful transmissions between any pair of vehicles separated by Euclidean distance x_0. An end-to-end packet transmission is successful if a packet can reach the destination by any number of hops. Therefore, $\Pr_s(x_0) = \sum_{k=1}^{\infty} \phi_k(x_0)$.

Define $\phi_{ks}(x_0)$ to be the conditional probability that a packet reaches its destination at the k_sth hop, conditioned on the transmission being successful and the Euclidean distance between source and destination being x_0. It is trivial to see that $\phi_k(x_0) = \phi_{ks}(x_0) \Pr_s(x_0)$. Therefore, the expected number of hops between two vehicles separated by distance x_0, given that they are connected, is

$$E_s[k_s|x_0] = \frac{\sum_{k=1}^{\infty} k\phi_k(x_0)}{\Pr_s(x_0)}. \tag{32}$$

Define the *forwarding delay* to be the time required for a packet to be forwarded from the leftmost vehicle to the rightmost vehicle in the cluster, which is $t_1 - t_0$ in the example shown in Fig. 1. Then, the expected forwarding delay in a cluster with length x_0 is $E[t_f|x_0] = \beta E_s[k_s|x_0]$.

Remark 2: The hop count statistics for a 1-D network with inhomogeneous Poisson distribution of nodes is studied in [25], which provides us with the required methodology for extending our analysis on the forwarding process from the Poisson distribution model to another vehicle distribution model.

VII. INFORMATION PROPAGATION SPEED

The entire information propagation process can be considered as a renewal reward process [26], where each cycle consists of a catch-up process, followed by a forwarding process, and the reward is the information propagation distance during each cycle. As mentioned in Section III, $E[v]$ is the constant component of the vehicle speed. It can be shown that [4], [5]

$$\begin{aligned} E[v_{ip}] &\approx \frac{\text{expected length of one cycle}}{\text{expected time duration of one cycle}} + E[v] \\ &= \frac{\int_{r_0}^{\infty} l_c f_{l_c}(l_c) dl_c + \int_0^{\infty} x_0 f_{x_0}(x_0) dx_0}{\int_{r_0}^{\infty} E[t_c|l_c] f_{l_c}(l_c) dl_c + \beta + \frac{1}{1-p_c} \int_0^{\infty} E[t_f|x_0] f_{x_0}(x_0) dx_0} \\ &\quad + E[v] \end{aligned} \tag{33}$$

where $p_c = 2W_{\min}N_b/(W_{\min}+1)^2 + 2W_{\min}N_b$ is the probability of collision given in [27], $W_{\min}$ is the minimum contention window size, and $N_b = \rho 2\pi r_0$ is the average node degree. Packet collision can be shown to have negative impact on the forwarding process, i.e., reducing the IPS, when the vehicle density is high. To illustrate this effect, we conducted more simulations using the parameters shown in [28], i.e., $W_{\min} = 32$.

VIII. SIMULATION RESULTS

In this section, we report on simulations to validate the accuracy of the analytical results for the catch-up process. The simulations are conducted in a VANET simulator written in C++. Each point shown in the figures is the average value from 2000 simulations. The confidence interval is too small to be distinguishable and, hence, is ignored in the following plots. The radio range is $r_0 = 250$ m [5]. The mobility parameters are $E[v] = 25$ m/s and $\sigma = 7.5$ m/s [15]. To distinguish the impact on the IPS of packet collision and other parameters, we let $p_c = 0$, except in Fig. 11.

A. Catch-Up Process

As mentioned earlier, we use $\tau = 1$ s, 5 s in this paper. Only the results for $\tau = 5$ s are shown in this section since the results for $\tau = 1$ s have a similar (and slightly better) accuracy. The traffic density is $\lambda = 0.3$ veh/s. It follows that the spatial distribution of the vehicles follows a homogeneous Poisson process with intensity $\rho = \int_0^{\infty}(f_v(v)/v)dv = 0.012$ veh/m, which is a low traffic density, resulting in a large number of catch-up processes. The results for other densities are quite similar and, hence, are not shown in this section.

Fig. 5(a) shows the probability that a catch-up process with gap $l_c = 400$ finishes within time t. It can be seen that, when $m = m' = 4$, the analytical result gives a good approximation. Moreover, considering more vehicles in the overtake process, e.g., $m = 6$ or $m = 8$, has marginal impact on the results. This is because, as the distance between vehicles increases, the probability of overtaking rapidly decreases, which can be seen in Fig. 5(b). Fig. 5(b) shows the simulation result of the probability that a randomly chosen vehicle overtakes another vehicle within time $t = i\tau$, where their distance is z at time 0. Because

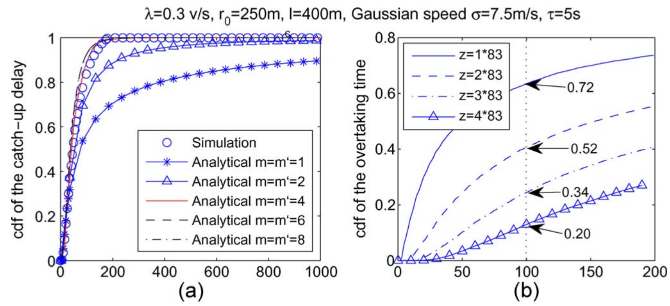

Fig. 5. (a) CDF of the catch-up delay for a catch-up process with gap $l_c = 400$. (b) Simulation results on the probability that a randomly chosen vehicle overtakes another vehicle within time t, where their initial Euclidean distance is $z = m \times 83$ at time 0.

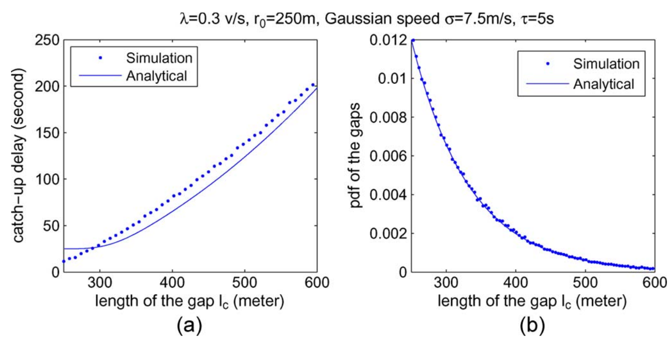

Fig. 6. (a) Expected catch-up delay. (b) PDF of the length of the gap (l_c).

the average intervehicle distance is $1/\rho = 1/0.012 \approx 83$, the curve of $z = m \times 83$ in Fig. 5(b) approximately illustrates the probability that vehicle H_m overtakes H_0 before time t. As can be seen in the figure, the probability that H_4 overtakes H_0 within 100 s while none of H_1, H_2, and H_3 overtakes H_0 within 100 s is approximately given by $0.2(1-0.72)(1-0.52)(1-0.34) = 0.01774$. It can be understood that the probability that H_0 is overtaken by another vehicle, e.g., H_5, $H_6 \ldots$, is very small. Therefore, considering $m = m' = 4$ can provide a good approximation.

Fig. 6(a) shows the expected catch-up delay for a catch-up process with gap l_c. It can be seen that the analytical result, which considers $m = m' = 4$, provides a good approximation. The discrepancy between the simulation result and the analytical result is caused by the approximations used during the analysis. Specifically, the first passage analysis is only applied to the analysis of the catch-up process between a pair of vehicles, which are the head and the tail at the start of the catch-up process. However, the first passage analysis does not consider the possibility that the head (the tail) may be overtaken by other vehicles during the catch-up process. Furthermore, Fig. 6(b) also verifies that the intervehicle distance, under our network model and the Gaussian speed distribution, still follows an exponential distribution with $\rho = 0.012$. This property is also expected to hold in some other speed distributions, which is an issue that is left as future work.

B. Forwarding Process

In addition to the simulation settings introduced earlier, the per-hop delay is $\beta = 4$ ms [5]. Fig. 7(a) shows the expected

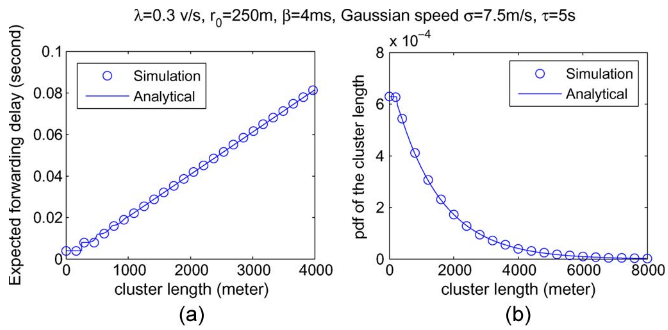

Fig. 7. Expected forwarding delay and the pdf of the cluster length. (a) Expected forwarding delay. (b) PDF of the cluster length.

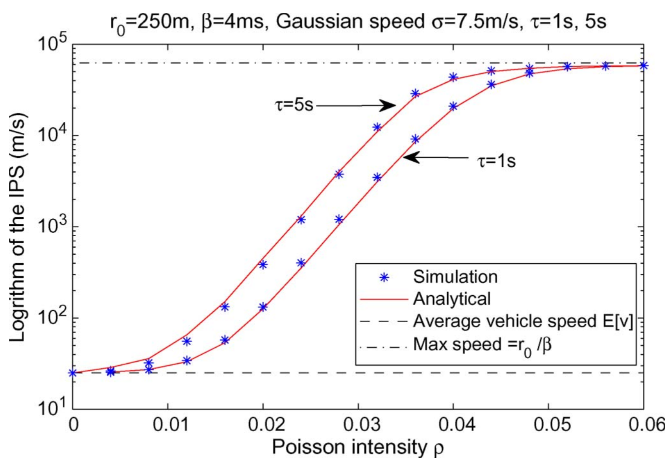

Fig. 8. Expected IPS for $\tau = 1$ s, 5 s.

forwarding delay in a cluster with a given length. Fig. 7(b) shows the pdf of the cluster length. It can be seen that the analytical results match the simulation results very well. The results for other values of the parameters have a similar accuracy and are thus omitted. Furthermore, it is interesting to note that, in Fig. 7(b), the pdf of the cluster length is a constant for $x_0 \in [0, 250]$. This is because, within the radio range ($r_0 = 250$) of the first vehicle, $\Pr(\mathcal{E}_4) = 1$ [(36)], and the cluster length is x_0 if and only if there is a vehicle in $[x_0, x_0 + dx_0)$, and there is no vehicle in $[x_0 + dx, x_0 + r_0)$. It follows that the pdf of the cluster length is a constant for $x_0 \in [0, r_0]$.

C. IPS

In addition to the simulation settings introduced earlier, the Poisson arrival rate λ is varied from 0 to 1.5 veh/s. With $E[v] = 25$, the spatial distribution of the vehicles follows a homogeneous Poisson process with intensity ρ ranging from 0 to 0.06. For completeness of the plot, $\rho = 0$ is included, which means that there is only one vehicle on the road. Therefore, the average number of neighbors (average node degree) varies from 0 to 30, which represents a large range of traffic densities.

Fig. 8 shows the expected IPS for $\tau = 1$ s, 5 s. It can be seen that, when the vehicle density is low, the IPS is determined by vehicle speeds because there is little packet forwarding in the network. When the vehicle density increases, small clusters are formed, and the IPS is determined by the catch-up delay, which is further determined by the mobility of the vehicles. It can be

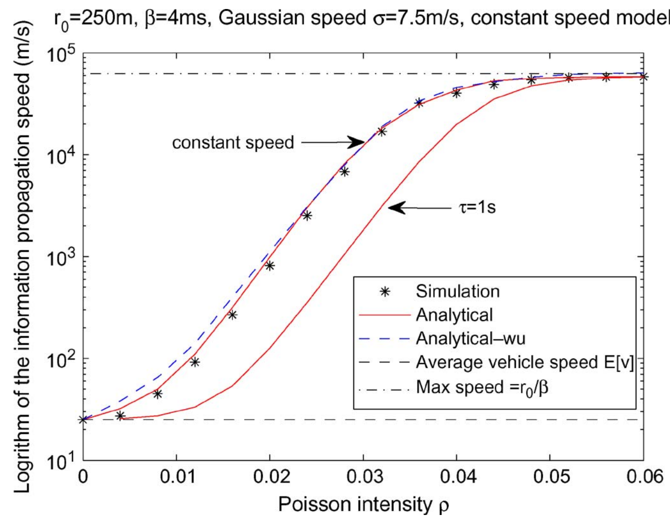

Fig. 9. Expected IPS under the constant speed model, together with the curve for $\tau = 1$ for comparison. The result *Analytical-wu* is calculated based on [5].

seen that the more frequently the speed changes, the slower the information propagates. This is mainly because changing speed has the potential to interrupt the catch-up process, i.e., during a catch-up process, the tail may speed up and the head may slow down. An intuitive explanation can be provided by considering an extreme case. In the extreme case that the speed-change time interval tends to 0, it can be shown using the central limit theorem that the average vehicular speed in any specified time interval converges to the mean speed $E[v]$. Hence, the network topology becomes static, and the expected IPS equals the mean speed $E[v]$ because there is no catch-up process to bridge the gaps. Finally, as the vehicle density further increases, clusters become larger, and the forwarding process starts to dominate. Therefore, the IPS increases until it reaches the maximum value, which is determined by the per-hop delay in the forwarding process. The maximum IPS is obviously equal to r_0/β, where β is the per-hop delay.

Fig. 9 shows the expected IPS under the constant speed model, i.e., the vehicle speed does not change over time. The result *Analytical-wu* is calculated based on [5] for comparison. The constant speed model is a special case of the mobility model used in this paper, i.e., when $\tau \to \infty$. In Fig. 9, we choose a fairly large value of τ to obtain analytical result under our mobility model and use the result as an approximation of the result under the constant speed model. The result *Analytical-wu*, which does not consider first passage phenomenon, provides a good match with the simulations. This is because the first passage phenomenon does not have a significant impact when the vehicle speed does not change over time. Finally, it can be seen that the constant speed model used in previous research (e.g., [5] and [8]) causes serious overestimation of the IPS by almost an order of magnitude. Therefore, time variation of vehicle speed is an important factor affecting the IPS.

Fig. 10 shows the expected IPS under nonsynchronized mobility models. In the analysis, we choose the synchronized random walk mobility model to study the impact of the time variation of vehicular speed on IPS. To verify the general applicability of the analytical study based on the simplified

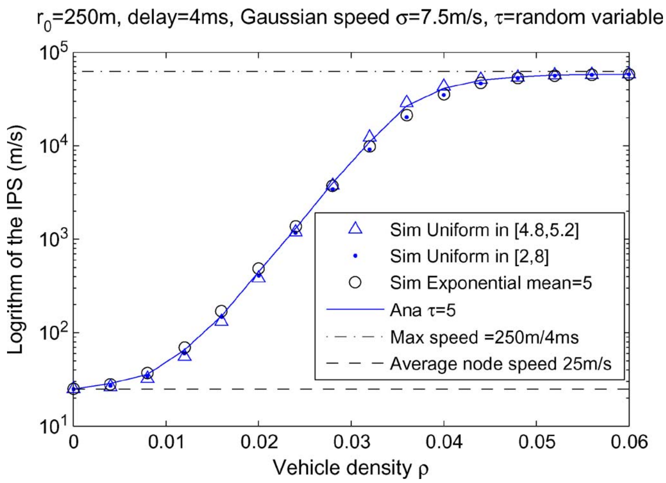

Fig. 10. Expected IPS under nonsynchronized mobility models.

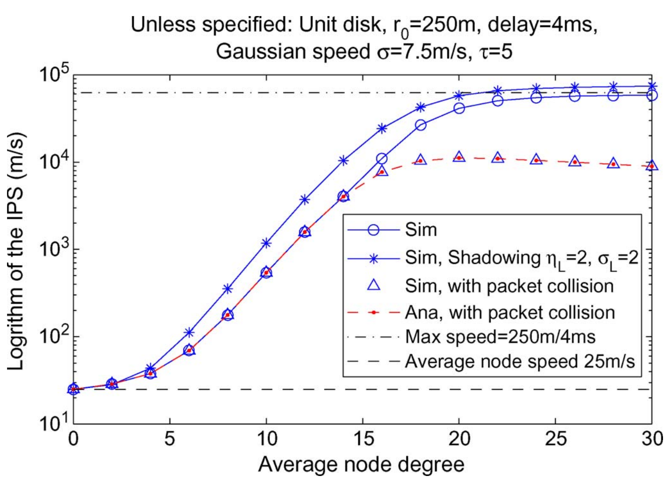

Fig. 11. Expected IPS in a VANET subject to log-normal shadowing and packet collision, where η_L is the path-loss exponent, and σ_L is the standard deviation of the log-normal shadowing. Furthermore, when a head cannot transmit a packet to any uninformed vehicle to the right of itself, the head keeps trying to retransmit the packet after every 0.9-s time delay. The value is chosen according to the real-world measurement that the channel coherent time of a VANET on the freeway is about 0.3–1.5 s [29].

mobility model, more simulations are conducted. As shown in Fig. 10, three different mobility models are evaluated, i.e., the speed-change time interval τ of each vehicle is uniformly selected from [4.8, 5.2] (i.e., about 5) or from [2, 8] (i.e., within a larger range around 5) and for τ following an exponential distribution with mean 5. Under all three mobility models, vehicles change speed at different time instants (nonsynchronized), and the average speed-change time interval is 5 s. It can be seen that the IPSs under nonsynchronized mobility models are very close (almost indistinguishable) to each other, and our analysis using the simplified (synchronized) mobility model provides a good estimation on the IPS.

We consider the unit disk model in the analysis. The unit disk communication model is constructed based on the path-loss attenuation model, which is suitable to model the radio environment in free space without clutters [30]. Therefore, the unit disk model is suitable for the VANET on the freeway. To study the impact of clutters such as road-side buildings, simulation results of the expected IPS under the log-normal shadowing model [30] are shown in Fig. 11. The results are compared under the condition that the average node degrees (i.e., the average number of neighbors per node) under the log-normal model and under the unit disk model are the same. In the log-normal shadowing model, the received signal strength (RSS) attenuation (in decibels) follows a normal distribution with a mean value equal to the RSS under the path-loss attenuation model. This random variation on the RSS attenuation provides a higher chance for a node to find a next-hop neighbor. Hence, even with the same average node degree, the IPS under log-normal shadowing model is faster than that under unit disk model. A similar observation is obtained in the study of network connectivity in [7] and [30, Th. 2.5.2]. Therefore, the IPS under the unit disk model can be considered as a lower bound on the IPS of a VANET in the real world.

In addition, the third and fourth curves in Fig. 11 show the IPS subject to packet collision with collision probability p_c given in Section VII. It can be seen that the packet collision has a significant impact on the IPS, particularly when the vehicle density is high.

IX. Conclusion and Future Work

In this paper, analytical models have been created for the information propagation process in a mobile ad hoc network formed by vehicles moving on a highway. Analytical results have been provided for the expected delay in catch-up process, expected delay in forwarding process, and the distribution of the cluster length. Based on the aforementioned results, the IPS has been derived. It has been shown that various parameters, such as vehicle density and time variation of vehicle speed, can have a significant impact on the IPS. By taking the real-world measurements such as λ, $E[v]$, σ, and τ, our results can provide a quick estimation of the IPS with good accuracy. The research provides useful guidelines on the design of mobile VANETs.

The analysis in this paper is conducted in a 1-D network. A straightforward extension to 2-D networks is to consider a Manhattan model [32], i.e., grid topology. In grid topology, each street (edge) can be treated as a 1-D roadway, which can be studied using the results obtained in this paper. Together with existing results on how a message passes through a road intersection area [33], our results can be adapted for 2-D grid networks. Furthermore, for unconstrained mobility in 2-D or 3-D mobile ad-hoc networks, the methodology developed in this paper for the analysis of the catch-up process and forwarding process can also be applicable.

Appendix
Proof of Theorem 2

Let the origin of the axis be the position of the leftmost vehicle of a cluster. Let N be the random variable representing the number of vehicles in the cluster. The cluster length lies in $[x_0, x_0 + dx_0)$, and there are n vehicles in this cluster if and only if the conditions given here hold.

$\mathcal{E}_1$: There is a vehicle in $[x_0, x_0 + dx_0)$.
$\mathcal{E}_2$: There is no vehicle in $[x_0 + dx_0, x_0 + r_0)$.
$\mathcal{E}_3$: There are $n - 2$ vehicles in $(0, x_0)$.

$\mathcal{E}_4$: The intervehicle distance between any two adjacent vehicles for those n vehicles in $[0, x_0 + dx_0)$ is smaller than or equal to r_0.

Denote by $\Pr(\mathcal{E}_m)$ the probability of event $\mathcal{E}_m$ in the preceding list. Due to the Poisson distribution of vehicles, it is straightforward to show that

$$\Pr(\mathcal{E}_1) = \rho dx_0, \quad \Pr(\mathcal{E}_2) = e^{-\rho r_0} \tag{34}$$

$$\Pr(\mathcal{E}_3) = \frac{(\rho x_0)^{n-2} e^{-\rho x_0}}{(n-2)!}. \tag{35}$$

Furthermore, $\Pr(\mathcal{E}_4)$ can be studied using [22, Lemma 1], which provides result on the connectivity of random interval graph. In [22], vertices are uniformly distributed on a unit interval. Due to the Poisson distribution of the vehicles in our case, given that there are n vehicles in a cluster with length x_0, these vehicles also follow a uniform distribution. Therefore, by scaling the cluster length x_0 to 1 and, consequently, the radio range to r_0/x_0, we have the following equation from Lemma 1 in [22]:

$$\Pr(\mathcal{E}_4) = \sum_{m=0}^{\min\{n-1, \lfloor x_0/r_0 \rfloor\}} \binom{n-1}{m} (-1)^m \left(1 - m\frac{r_0}{x_0}\right)^{n-2} \tag{36}$$

where m is an integer, and $\lfloor . \rfloor$ is the floor function. For the convenience of the following calculation, let $\binom{n-1}{m} = 0$ for $m > n-1$. Thus, the preceding summation is from $m = 0$ to $\lfloor x_0/r_0 \rfloor$.

Events $\mathcal{E}_1$, $\mathcal{E}_2$, and $\mathcal{E}_3$ are independent of each other. Event $\mathcal{E}_4$, which is conditioned on event $\mathcal{E}_3$, is independent of events $\mathcal{E}_1$ and $\mathcal{E}_2$. Define $f(x_0, N = n)$ to be the probability that the cluster length lies in $[x_0, x_0 + dx_0)$, and there are n vehicles in this cluster. It is evident that $f(x_0, N = n) = \Pr(\mathcal{E}_1)\Pr(\mathcal{E}_2)\Pr(\mathcal{E}_3)\Pr(\mathcal{E}_4)$. Next, we derive $f_{x_0}(x_0)$ using $f(x_0, N = n)$.

If the cluster only consists of one vehicle, then the cluster length is 0, and the probability of this event is $e^{-\rho r_0}$. If the cluster consists of more than one vehicle, then the pdf of cluster length x_0 is

$$\begin{aligned} f_{x_0}(x_0) &= \frac{\sum_{n=2}^{\infty} f(x_0, N = n)}{\Pr(N \geq 2)} \\ &= \sum_{n=2}^{\infty} \frac{\rho e^{-\rho r_0} (\rho x_0)^{n-2} e^{-\rho x_0}}{(n-2)!(1 - e^{-\rho r_0})} \\ &\quad \times \sum_{m=0}^{\lfloor x_0/r_0 \rfloor} \binom{n-1}{m} (-1)^m \left(1 - m\frac{r_0}{x_0}\right)^{n-2} \\ &= \frac{\rho e^{-\rho r_0} e^{-\rho x_0}}{(1 - e^{-\rho r_0})} \sum_{n=2}^{\infty} \frac{(\rho x_0)^{n-2}}{(n-2)!} \\ &\quad \times \sum_{m=0}^{\lfloor x_0/r_0 \rfloor} \binom{n-1}{m} (-1)^m \left(1 - m\frac{r_0}{x_0}\right)^{n-2}. \end{aligned} \tag{37}$$

Using $\binom{n}{m} = n!/m!(n-m)!$, the summation terms in (37) can be simplified as

$$\begin{aligned} &\sum_{n=2}^{\infty} \frac{(\rho x_0)^{n-2}}{(n-2)!} \sum_{m=0}^{\lfloor x_0/r_0 \rfloor} \binom{n-1}{m} (-1)^m \left(1 - m\frac{r_0}{x_0}\right)^{n-2} \\ &= \sum_{m=0}^{\lfloor x_0/r_0 \rfloor} \frac{(-1)^m}{m!} \sum_{n=2}^{\infty} \frac{(n-1)!}{(n-2)!} \frac{\left(\rho x_0 \left(1 - m\frac{r_0}{x_0}\right)\right)^{n-2}}{(n-m-1)!} \\ &= \sum_{m=0}^{\lfloor x_0/r_0 \rfloor} \frac{(-1)^m}{m!} \sum_{n=2}^{\infty} \frac{(n-1)\left(\rho(x_0 - mr_0)\right)^{n-2}}{(n-m-1)!} \\ &= \sum_{m=0}^{\lfloor x_0/r_0 \rfloor} \frac{(-1)^m}{m!} \left(\rho(x_0 - mr_0)\right)^{m-1} \\ &\quad \times \sum_{n=2}^{\infty} \frac{(n-1)\left(\rho(x_0 - mr_0)\right)^{n-m-1}}{(n-m-1)!} \\ &= \sum_{m=0}^{\lfloor x_0/r_0 \rfloor} \frac{(-1)^m}{m!} \left(\rho(x_0 - mr_0)\right)^{m-1} \\ &\quad \times \sum_{n=m+1}^{\infty} \frac{(n-1)\left(\rho(x_0 - mr_0)\right)^{n-m-1}}{(n-m-1)!} \end{aligned} \tag{38}$$

$$\begin{aligned} &= \sum_{m=0}^{\lfloor x_0/r_0 \rfloor} \frac{(-1)^m}{m!} \left(\rho(x_0 - mr_0)\right)^{m-1} \\ &\quad \times \sum_{\alpha=0}^{\infty} \frac{(\alpha + m)\left(\rho(x_0 - mr_0)\right)^{\alpha}}{\alpha!}. \end{aligned} \tag{39}$$

Using $xe^x = \sum_{\alpha=0}^{\infty} (\alpha x^\alpha / \alpha!)$, (39) can be written as

$$\begin{aligned} &\sum_{m=0}^{\lfloor x_0/r_0 \rfloor} \frac{(-1)^m}{m!} \left(\rho(x_0 - mr_0)\right)^{m-1} \\ &\qquad \times \left(\rho(x_0 - mr_0) + m\right) e^{\rho(x_0 - mr_0)}. \end{aligned} \tag{40}$$

Substitute the preceding equation into (37), one can obtain

$$\begin{aligned} f_{x_0}(x_0) &= \frac{\rho e^{-\rho r_0} e^{-\rho x_0}}{(1 - e^{-\rho r_0})} \sum_{m=0}^{\lfloor x_0/r_0 \rfloor} \frac{(-1)^m}{m!} \left(\rho(x_0 - mr_0)\right)^{m-1} \\ &\quad \times \left(\rho(x_0 - mr_0) + m\right) e^{\rho(x_0 - mr_0)} \\ &= \frac{\rho e^{-\rho r_0}}{(1 - e^{-\rho r_0})} \sum_{m=0}^{\lfloor x_0/r_0 \rfloor} \frac{(-1)^m}{m!} \left(\rho(x_0 - mr_0)\right)^{m-1} \\ &\quad \times \left(\rho(x_0 - mr_0) + m\right) e^{-\rho m r_0} \\ &= \frac{\rho}{(e^{\rho r_0} - 1)} \sum_{m=0}^{\lfloor x_0/r_0 \rfloor} \frac{(-1)^m}{m!} \left(\rho(x_0 - mr_0)\right)^{m-1} \\ &\quad \times \left(\rho(x_0 - mr_0) + m\right) e^{-\rho m r_0} \\ &= \frac{\rho}{(e^{\rho r_0} - 1)} \sum_{m=0}^{\lfloor x_0/r_0 \rfloor} \frac{\left(-\rho(x_0 - mr_0)\right)^{m-1}}{-m!} \\ &\quad \times \left(\rho(x_0 - mr_0) + m\right) e^{-\rho m r_0}. \end{aligned} \tag{41}$$

■

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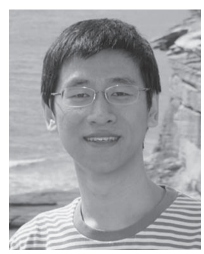

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