On The k-hop Partial Connectivity in Finite Wireless Multi-hop Networks

Zijie Zhang*, Seh Chun Ng*‡, Guoqiang Mao*‡ and Brian D. O. Anderson†‡
*School of Electrical and Information Engineering, University of Sydney, Australia
†Research School of Information Sciences and Engineering, Australian National University, Australia
‡National ICT Australia (NICTA), Australia
Email: {zijie,sehchun,guoqiang}@ee.usyd.edu.au, brian.anderson@anu.edu.au

***Abstract*—We consider wireless multi-hop networks with a finite number of (ordinary) nodes randomly deployed in a given 2D area. A finite number of gateways (infrastructure nodes) are deterministically placed in the same area. We study the connectivity between the ordinary nodes and the gateways. In real applications, it is often desirable to limit the maximum number of hops between the ordinary nodes and the gateways in order to provide reliable services. On the other hand, requiring *every* ordinary node to be connected to at least one gateway imposes strong requirement on transmission range/power or the number of gateways. Therefore it is beneficial to allow a small fraction of ordinary nodes to be disconnected from the gateways so that the network is only partially connected. Based on the above two considerations, we provide analytical results on the k-hop partial connectivity, which is the fraction of ordinary nodes that are connected to at least one gateway in at most k hops. The research provides useful guidelines on the design of wireless multi-hop networks.**

***Index Terms*—wireless multi-hop networks, partial connectivity, path length, shadowing**

I. Introduction

Wireless multi-hop networks have been actively studied in the recent decades to solve some real world problems. In this paper, we consider a wireless multi-hop network with a *finite* number of "ordinary" nodes (ONs) Poissonly and i.i.d. (independently, identically distributed) in a given 2D area. In addition, the gateways / infrastructure nodes (INs) are deterministically placed in the same area to which the other ONs should communicate. An ON can communicate with an IN, i.e. they are connected, if there is at least one (single or multi-hop) path connecting them. Examples of this type of wireless multi-hop networks include wireless mesh networks [1] and wireless sensor networks [2].

Extensive results have been obtained on the connectivity of wireless multi-hop networks in recent years. However, most results consider wireless ad hoc networks, which have been shown to be non-scalable and unreliable [3], [4]. Further, previous work has mainly focused on conditions to have *every* node in a network to communicate with *every* other node in one or more hops. In this paper, we are interested in the connectivity between ONs and INs in a network.

This work is partially supported by the Australian Research Council (ARC) under the Discovery project DP0877562. This material is based on research partially sponsored by the Air Force Research Laboratory, under agreement number FA2386-10-1-4102. The U.S. Government is authorized to reproduce and distribute reprints for Governmental purposes notwithstanding any copyright notation thereon. The views and conclusions contained herein are those of the authors and should not be interpreted as necessarily representing the official policies or endorsements, either expressed or implied, of the Air Force Research Laboratory or the U.S. Government.

The path length between ONs and INs, a metric tightly coupled to the end-to-end performance, is a major concern. A large path length is undesirable because it has been shown to have negative impact on the network performances such as bandwidth [3], packet delay and routing [5], and energy efficiency [6]. To avoid the existence of long paths, we limit the maximum allowable path length between ONs and INs. That is, an ON is said to be *connected* to an IN if there is at least one path, not longer than k hops, between them. We measure the connectivity of a network by examining the fraction of ONs which are connected to at least one IN in at most k hops, namely the *k-hop partial connectivity*. This measurement is different from the more commonly used measurement, i.e. the probability that *all* ONs are connected to at least one IN. We allow some ONs to be disconnected from the INs because in many practical scenarios it is either unnecessary or impractical to require all ONs to be connected to at least one IN [6], [7], [8]. As an example, it is acceptable for a wireless sensor network with a large number of redundant sensors to have a small number of sensors disconnected from the gateways [9]. Furthermore it has been shown in [7] that a network which allows a small fraction of nodes to be disconnected requires much lower transmission range / power than a network which requires all nodes to be connected.

The rest of this paper is organized as follows: Section II reviews the related work. Section III introduces the network model used in this paper. The analysis on the probability distribution of the path length is given in Section IV, followed by the analytical results on the k-hop partial connectivity. The accuracy of our results is verified by simulation in Section V. Finally Section VI concludes this paper.

II. Related work

The study of k-hop partial connectivity relies on the study of probability distribution of the path length between two nodes separated by a known Euclidean distance, which has been studied in the literature [6], [10], [11]. In [10], Chandler analyzed the probability that two arbitrary nodes separated by a known distance can communicate in k or less hops where nodes are uniformly distributed in a 2D area. In [11], Zorzi et al. proposed a geographic random forwarding scheme in a 2D network where nodes are placed in the coverage area of a transmitting node according to a Poisson process. They obtained an upper and a lower bound on the average path length between two nodes with a given Euclidean distance and a given average number of neighbors. However they assumed that every source-destination (S-D) pair is connected.

The aforementioned studies were however incomplete since the spatial dependence problem was ignored. The *spatial dependence* problem [5] arises because in a wireless multi-hop network the event that a randomly chosen node is k hops apart from a particular node is not independent of the event that another randomly chosen node is i hops apart from the same node for $i \leq k$. The spatial dependence problem is a major technical obstacle in the accurate analysis of path length distributions. Ignoring the spatial dependence problem, analytical results for the path length distribution is given in [12] (resp. [13]) for UDM (resp. LSM), where UDM stands for the unit disk model and LSM stands for the log-normal shadowing model. Recently, the accuracy of the path length distribution was improved in [6], for the UDM, by considering the spatial dependence of two-hop neighbors, instead of only one-hop neighbors considered in the literature, e.g. [12], [13]. In this paper, we extend the study in [6] to a more general model, i.e. LSM.

Some studies on partial connectivity can be found in the literature in the context of giant component for ad-hoc networks. A giant component is the largest cluster in the network where a cluster is a maximal set of nodes where there is a path between any two nodes in the set. Consider an ad-hoc network with n nodes uniformly and i.i.d. in a unit square, the one-hop connection between two nodes follows the UDM, Ta et al. [7] proposed an empirical formula for the minimum transmission range required for having a giant component of size pn with $0.5 < p \leq 1$. Later this work has been extended to consider the LSM in [8] where an upper bound for the minimum transmission power required for the above giant component when $n \to \infty$ was obtained. Both work showed that significant energy saving can be achieved if we require only most nodes, rather than all nodes, to be inter-connected. Note that the above results are for ad-hoc networks only.

Consider a wireless multi-hop network where ONs are uniformly and i.i.d. in a unit square; there are 4 INs, one at each corner of the square. Assuming the UDM, Bermudez and Wicker [9] investigated how the fraction of ONs that are connected to at least one IN changes as a function of the transmission range. However, only simulation results are obtained and the limit on the maximum path length between ONs and INs was not considered. In a more recent work of Ng et al. [14], they studied analytically the probability that a network is two-hop connected, i.e. all ONs in the network are at most two hops away from at least one IN under a generic channel model where UDM and LSM are its special cases, where ONs are Poissonly distributed in a unit square and 4 INs are located at the corners of the square. However, the partial connectivity problem is not considered and the limit on the maximum path length between ONs and INs is set to be two (rather than a generic k considered in this paper).

III. NETWORK MODEL

We consider a wireless multi-hop network located in a square area $L \times L$, where nodes are i.i.d. according to a homogeneous Poisson process with intensity ρ. Two most widely used connection models, i.e. the unit disk model (UDM) and the log-normal shadowing model (LSM), are used in this paper. UDM is based on the path loss attenuation model, which is suitable to model the radio environment in free space without clutters [15]. Under the UDM, two nodes are directly connected iff their Euclidean distance is not larger than the transmission range r_0. In the present of clusters, LSM is usually used [15]. Under the LSM, the signal power attenuation (in dB) follows a normal distribution:

$$10 \log_{10} \frac{P(x)}{C P_t x^{-\eta}} \sim Z \tag{1}$$

where C is a constant, P_t is the transmission power, x is the Euclidean distance between the receiver and the transmitter, $P(x)$ is the received signal power at distance x, η is the path-loss exponent. The shadowing fades Z is a zero-mean Gaussian distributed random variable with standard deviation σ. When $\sigma = 0$ the LSM reduces to the UDM. As widely used in the literature, we assume that the shadowing fades between all pairs of nodes are i.i.d. and the link is symmetric.

Generally, a destination node is said to be k hops apart from a source node if the shortest path between them, measured by the number of hops, is k. However, these shortest paths are normally difficult or very costly to be discovered by routing algorithms in a real world. Hence many existing routing algorithms use the idea of greedy forwarding (GF), i.e. to forward the packets to the neighboring node that is closest to the destination [1] [16]. Despite the requirement on the location information, the GF algorithm has shown great potentials due to their low control overhead and capability of adapting to dynamic network topologies. In this paper, the number of hops from source to destination nodes, obtained from shortest paths, are referred to as *shortest path length*, whereas the number of hops obtained from GF are referred to as *feasible path length*.

IV. ANALYTICAL RESULTS

We first study the probability distribution of the path length between a S-D pair separated by a known distance. Then we use the probability to obtain the k-hop partial connectivity.

A. Probability distribution of path length

Define $\phi(k|x_0)$ to be the probability that a S-D pair is k hops apart using GF, conditioned on x_0, where hereinafter we use "conditioned on x_0" to denote "conditioned on the Euclidean distance between the source and the destination

[1] It is shown in [3] that with a sensing range two times the transmission range, the path created by a basic GF algorithm can be a good approximation to the shortest path without the complex recovery algorithms.

being x_0" for simplicity. Two nodes are directly connected iff the received signal power exceeds a given threshold $P_{\min}$. Therefore from Eq. 1, we have:

$$\begin{aligned}\Pr(P(x_0) \geq P_{\min}) &= \Pr(CP_t x_0^{-\eta} 10^{z/10} \geq P_{\min}) \\ &= \Pr(z \geq 10\log_{10}(\frac{x_0}{r_0})^{\eta}) \quad (2) \\ &= \Pr(x_0 \leq r_0 \exp(\frac{z \ln 10}{10\eta})) \quad (3)\end{aligned}$$

where $r_0 = \left(\frac{CP_t}{P_{\min}}\right)^{\frac{1}{\eta}}$. Observe that Eq. 3 reduces to $x_0 \leq r_0$ in the absence of shadowing effect, i.e. for $z = 0$. So, r_0 is the transmission range in the absence of shadowing effect.

According to Eq. 2 and 3, the event that two nodes are directly connected happens iff either of the following two conditions being satisfied: 1) Given the Euclidean distance between two nodes being x_0, they are directly connected iff the (random) shadowing fades $z \geq 10\log_{10}(\frac{x_0}{r_0})^{\eta}$. 2) Given the shadowing fades between two nodes being z, they are directly connected iff their Euclidean distance $x_0 \leq r_0 \exp(\frac{z \ln 10}{10\eta})$.

Based on the first condition, the probability of having a direct connection for a pair of S-D nodes separated by x_0 is:

$$\phi(1|x_0) = \int_{10\log_{10}(\frac{x_0}{r_0})^{\eta}}^{\infty} q(z)dz. \quad (4)$$

where $q(z) = \frac{1}{\sigma\sqrt{2\pi}} \exp(-\frac{z^2}{2\sigma^2})$ is the pdf (probability density function) of the shadowing fades.

To derive $\phi(k|x_0)$ for $k > 1$, we analyze all possible locations of the relay nodes between the source and destination using the second condition, i.e. Eq. 3. Define $r_N(z_S)$ to be the transmission range of a transmitter (S) conditioned on the shadowing fades being z_S:

$$r_N(z_S) = r_0 \exp(\frac{z_S \ln 10}{10\eta}). \quad (5)$$

Therefore any node, whose received signal power from the transmitter (S) has shadowing fades $Z_S \in [z_S, z_S + dz_S]$, is directly connected to S iff its Euclidean distance to the transmitter is smaller than or equal to $r_N(z_S)$.

Denote by $A(x, r_1, r_2)$ the intersectional area of two circles with distance x between centers and radii r_1 and r_2 respectively. The size of the area can be easily calculated [6].

Define x_k to be the remaining Euclidean distance between the k^{th} hop node (S_k) and the destination (D). Then $A_1 = A(x_{k-1}, r_N(z_1), x_k)$ is the intersectional area of the circles $C(S_{k-1}, r_N(z_1))$ and $C(D, x_k)$. Similarly we have $A_2 = A(x_{k-2}, r_N(z_2), x_k)$.

Define $f(x_k|x_0)$ to be the pdf of the remaining Euclidean distance to the destination from S_k being x_k, conditioned on x_0. Because of the first type of the spatial dependence problem, $f(x_k|x_0)$ depends on the remaining distances of previous hop nodes, i.e. $x_{k-1}, x_{k-2}, \cdots, x_0$. An accurate calculation of $f(x_k|x_0)$ requires all previous hops to be considered, but the calculation is complicated. Previous research, e.g. [11], only considered the dependence on previous one hop, which leads to a large error. Our technique can be used to compute $f(x_k|x_0)$ considering the impact of the locations of all previous hops. However as shown in Section V, considering only previous two hops provides a fairly accurate result while not causing a sharp increase in computational complexity.

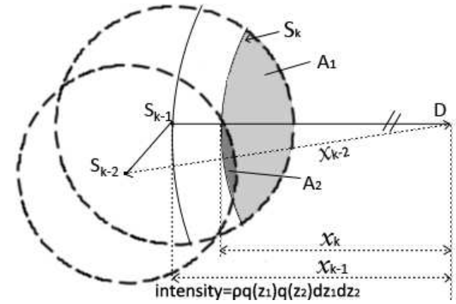

Fig. 1. Possible positions for S_k, which is the k^{th} hop node, are located on the arc. Consider the nodes whose received signal power from S_{k-1} and S_{k-2} have shadowing fades $Z_1 \in [z_1, z_1 + dz_1]$ and $Z_2 \in [z_2, z_2 + dz_2]$ respectively. The dashed-line circles represent the transmission range of S_{k-1} (resp. S_{k-2}) conditioned on the above values of shadowing fades. A_1, A_2 and x_k, x_{k-1}, x_{k-2} are described in the following text.

Define $g(x_k|x_{k-1}, x_{k-2})$ to be the pdf of the remaining Euclidean distance to the destination from S_k being x_k, conditioned on $\mathcal{D}$, where $\mathcal{D}$ is the event that the remaining distances at S_{k-1} and S_{k-2} are x_{k-1} and x_{k-2} respectively. Accordingly define the cdf (cumulative distribution function) of the remaining distance at the k^{th} hop node to be $\Pr(X_k \leq x_k|x_{k-1}, x_{k-2})$. We will derive this cdf by studying the following two events: denote by $\mathcal{B}$ the event that there is at least one node whose Euclidean distance to the destination is smaller than x_k *and* has a direct connection to S_{k-1} *and* has no direct connection to S_{k-m} for $m \in [2, k]$; denote by $\mathcal{C}$ the event that the node S_{k-1} is not directly connected to the destination. Events $\mathcal{B}$ and $\mathcal{C}$ are independent because the shadowing fades are i.i.d. Therefore:

$$\Pr(X_k \leq x_k|x_{k-1}, x_{k-2}) = \Pr(\mathcal{B}|\mathcal{D}) \times \Pr(\mathcal{C}|\mathcal{D}). \quad (6)$$

We start with the analysis of event $\mathcal{B}$. In this paragraph, we only consider the subset of nodes whose received signal power from S_{k-1} and S_{k-2} have shadowing fades $Z_1 \in [z_1, z_1+dz_1]$ and $Z_2 \in [z_2, z_2 + dz_2]$ respectively. Because the shadowing fades are i.i.d., these nodes are Poissonly distributed with intensity $\rho q(z_1)q(z_2)dz_1dz_2$. Denote by $\mathcal{E}$ the event that the shadowing fades $Z_1 \in [z_1, z_1 + dz_1]$ and $Z_2 \in [z_2, z_2 + dz_2]$. Ignore the boundary effect, then $\Pr(\mathcal{B}|\mathcal{D}, \mathcal{E})$ is equal to the probability that there is at least one node in area $A_1 \setminus A_2$ as shown in Fig. 1. The area A_2 needs to be excluded because if there is a node in this area, that node will be closer to the destination than S_{k-1}, which violates the condition that S_{k-1} is the $(k-1)^{th}$ hop node using GF. Since it is not necessarily the case that $A_2 \subset A_1$, we approximate the size of the overlapping area $A_1 \setminus A_2$ by $(A_1 - A_2)^+$, where $(A_1 - A_2)^+ = \max\{0, A_1 - A_2\}$. Therefore $1 - \Pr(\mathcal{B}|\mathcal{D}, \mathcal{E})$ is equal to $\exp(-(A_1 - A_2)^+ \rho q(z_1)q(z_2)dz_1dz_2)$, which is the probability that there is no node in area $A_1 \setminus A_2$. Note that area A_1 and A_2 depend on z_1 and z_2 respectively.

Then considering all subset of nodes, we have

$$\Pr(\mathcal{B}|\mathcal{D}) = 1 - \prod_{z_1,z_2\in(-\infty,+\infty)} \exp(-(A_1 - A_2)^+\rho q(z_1)q(z_2)dz_1dz_2$$
$$= 1 - \exp(-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(A_1 - A_2)^+\rho q(z_1)q(z_2)dz_1dz_2).$$

It is evident that event $\mathcal{C}$ only depends on x_{k-1}. Therefore:

$$\Pr(\mathcal{C}|\mathcal{D}) = \Pr(\mathcal{C}|x_{k-1}) = 1 - \phi(1|x_{k-1})$$

From the above two equations and Eq. 6, we have:

$$\Pr(X_k \le x_k|x_{k-1}, x_{k-2}) = (1 - \phi(1|x_{k-1}))\times \tag{7}$$
$$(1 - \exp(-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(A_1 - A_2)^+\rho q(z_1)q(z_2)dz_1dz_2))$$

By Leibniz integral rule:

$$g(x_k|x_{k-1}, x_{k-2}) = \frac{\partial \Pr(X_k \le x_k|x_{k-1}, x_{k-2})}{\partial x_k}$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\partial(A_1 - A_2)^+}{\partial x_k}\rho q(z_1)q(z_2)dz_1dz_2$$
$$\times \exp(-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(A_1 - A_2)^+\rho q(z_1)q(z_2)dz_1dz_2)$$
$$\times(1 - \phi(1|x_{k-1})) \tag{8}$$

where the calculations of $\partial A_1/\partial x_k$ and $\partial A_2/\partial x_k$ can be found in [6].

Define $h(x_k, x_{k-1}|x_0)$ to be the joint pdf of the remaining Euclidean distances at S_k and S_{k-1} being x_k and x_{k-1} respectively, conditioned on x_0. We can derive $f(x_k|x_0)$ from $h(x_k, x_{k-1}|x_0)$. For $k = 1$, the calculation only depends on the distance between S-D. Therefore:

$$f(x_1|x_0) = \int_{-\infty}^{\infty}\frac{\partial A(x_0, r_N(z_1), x_1)}{\partial x_1}\rho q(z_1)dz_1 \tag{9}$$
$$\times \exp(-\int_{-\infty}^{\infty}A(x_0, r_N(z_1), x_1)\rho q(z_1)dz_1)(1 - \phi(1|x_0)).$$

For $k = 2$, it is straightforward that:

$$h(x_2, x_1|x_0) = g(x_2|x_1, x_0)f(x_1|x_0). \tag{10}$$

For $k > 2$, $h(x_k, x_{k-1}|x_0)$ can be calculated recursively:

$$h(x_k, x_{k-1}|x_0) = \int_0^{x_0} g(x_k|x_{k-1}, x_{k-2})h(x_{k-1}, x_{k-2}|x_0)dx_{k-2}.$$

Finally for $k > 1$, we have:

$$f(x_k|x_0) = \int_0^{x_0} h(x_k, x_{k-1}|x_0)dx_{k-1}. \tag{11}$$

Under LSM, the destination may be possibly reached in a single hop no matter how far the remaining distance at that hop is. Therefore, we have for $k \ge 2$:

$$\phi(k|x_0) = \int_0^{x_0}\phi(1|x_{k-1})f(x_{k-1}|x_0)dx_{k-1}. \tag{12}$$

With Eq. 4, the derivation of $\phi(k|x_0)$ is completed for $k \ge 1$.

Define $\Phi(k|x_0)$ to be the probability that a pair of S-D nodes is connected by a path with at most k hops, conditioned on x_0. It is straightforward that:

$$\Phi(k|x_0) = \sum_{m=1}^{k}\phi(m|x_0) \tag{13}$$

B. *k-hop partial connectivity*

In this sub-section, we derive the k-hop partial connectivity for wireless multi-hop networks with M IN at known locations. Examples of the wireless multi-hop networks with one IN or four INs are illustrated in Fig. 2. The following results do not depend on the particular placement strategy of INs and it remains our future work to find the optimal placement of INs which maximize the k-hop partial connectivity.

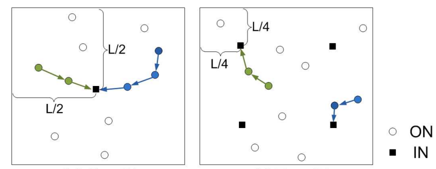

Fig. 2. Examples of the information transmission from ONs to IN(s).

Define $p(k)$ to be the fraction of ONs which are connected to at least one IN by a path in at most k hops. Ignore the spatial dependence problem, then $p(k)$ is approximately equal to the probability that an arbitrary ON is connected to at least one IN in at most k hops. The accuracy of this approximation is verified in Section V. Consider a network area A of size $\|A\|$, where $\|A\| = L^2$ for our case. Denote by $\mathbf{B}_i$ the location of the i^{th} IN. It is straightforward that

$$p(k) = \frac{1}{\|A\|}\int_A\left[1 - \prod_{i=1}^{M}(1 - \Phi(m\|\mathbf{Y} - \mathbf{B}_i|))\right]d\mathbf{Y} \tag{14}$$

where $|.|$ denotes the Euclidean norm. The product term inside the integral in Eq. 14 is the probability that an ON at location $\mathbf{Y}$ is not connected to the INs located at $\mathbf{B}_i$ in at most k hops.

V. Simulation results

In this section, we report on simulations to validate the accuracy of the analytical results. The simulations are conducted using a wireless network simulator written in C++. ONs are randomly deployed in a 400×400 square area following a homogeneous Poisson process with intensity $\rho = 0.003$. Simulations are also conducted for other network sizes (from 70×70 to 1800×1800) which showed similar results. Due to the page limit, we omit the results for other network sizes. For UDM, r_0 is varied from 10 to 80, which results in the average node degree varying from around 1 to 60. For LSM, several values of η and σ are used in our simulations, but only the results for $\eta = \sigma = 4$ are shown in this paper and results for other values of σ showed similar trend. Every point shown in the simulation result is the average value from 2000 simulations. As the number of instances of random networks used in the simulation is large, the confidence interval is too small to be distinguishable and hence is ignored in the figures. In each plot, the transmission ranges required to achieve 0.9 k-hop partial connectivity, i.e. 90% of ONs are connected to at least one IN in at most k hops, are shown for comparison.

Fig. 3 shows the k-hop partial connectivity under UDM and LSM, for $k = 5$ and $k = \infty$. The feasible path length

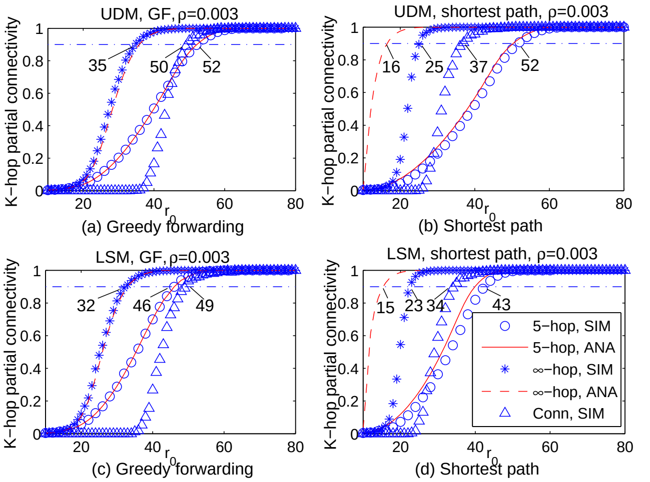

Fig. 3. The k-hop partial connectivity. SIM represents simulation result. ANA represents analytical result. Conn represents the probability that all ONs are connected to at least one IN, with no limit on the maximum number of hops.

distributions are based on the analysis introduced in this paper. The shortest path length distribution under UDM (resp. LSM) are based on the results in [12] (resp. [13]).

Firstly, it can be seen that the analytical results of k-hop partial connectivity using GF is more accurate than the analytical results using shortest path. This is because in our analysis the impact of the spatial dependence problem is carefully evaluated while the problem is ignored in previous research, e.g. [12], [13]. Secondly, one can observe a significant reduction on the transmission range required for partially connected network compared with that required for its connected counterpart, where a network is (fully) connected if all ONs are connected to at least one IN. Thirdly, the limit on the maximum number of hops between S-D nodes has a significant impact on the minimum transmission power required to meet connectivity requirement. For example, Fig. 3(a) shows that the transmission range required for the 5-hop partial connectivity to be 0.9 is $\frac{52-35}{35}$ $\approx$49% more than the transmission range required for the ∞-hop partial connectivity to be 0.9. Hence, the limitation of the maximum path length shall be carefully decided during network deployment.

Further, as shown in Fig. 4, the transmission ranges required for the k-hop partial connectivity reduces as the number of INs increases, for both $k = 5$ and $k = \infty$. Therefore a small number of INs can provide improvements in terms of throughput, energy consumption and connectivity. Finally, the k-hop partial connectivity under the LSM is always higher than that under UDM, given the same value of transmission range (i.e. same transmission power P_t). This is because the variation on the received signal strength introduced by the shadowing results in a larger number of average node degree. A similar observation is also obtained in the study of connectivity [17].

VI. Conclusions and Future Work

In this paper, we proposed the concept of k-hop partial connectivity based on the observations that in real networks, it suffices to provide reliable services to most nodes (but not

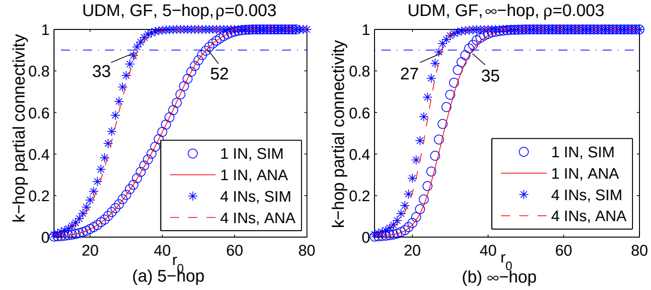

Fig. 4. The k-hop partial connectivity with one IN and four INs.

necessarily all nodes) in the network. Therefore it is desirable to limit the maximum number of hops between ONs and their associated INs. Analytical results are given for the k-hop partial connectivity which is validated by simulations. It remains to find the optimal placements of INs and the benefits of limiting the maximum number of hops on throughput, which form our future work.

References

[1] I. F. Akyildiz, X. Wang, and W. Wang, "Wireless mesh networks: a survey," *Computer Networks*, vol. 47, no. 4, pp. 445–487, 2005.

[2] C.-Y. Chong and S. P. Kumar, "Sensor networks: evolution, opportunities, and challenges," *Proceedings of the IEEE*, vol. 91, no. 8, pp. 1247–1256, 2003.

[3] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Transactions on Information Theory*, vol. 46, no. 2, pp. 388–404, 2000.

[4] M. Penrose, *Random Geometric Graphs*. Oxford University Press, 2003.

[5] X. Ta, G. Mao, and B. D. O. Anderson, "Evaluation of the probability of k-hop connection in homogeneous wireless sensor networks," in *IEEE GLOBECOM*, 2007, pp. 1279–1284.

[6] Z. Zhang, G. Mao, and B. D. Anderson, "On the effective energy consumption in wireless sensor networks," in *IEEE WCNC*, 2010, pp. 1–6.

[7] X. Ta, G. Mao, and B. D. O. Anderson, "On the giant component in wireless multi-hop networks," in *IEEE WCNC*, 2009, pp. 1–6.

[8] X. Ta, G. Mao, and B. D. Anderson, "On the giant component of wireless multi-hop networks in the presence of shadowing," *IEEE Transactions on Vehicular Technology*, vol. 58, no. 9, pp. 5152–1563, 2009.

[9] S. A. Bermudez and s. B. Wicker, "Partial connectivity of multi-hop two-dimensional finite hybrid wireless networks," in *IEEE ICC*, 2010, pp. 1–5.

[10] S. A. G. Chandler, "Calculation of number of hops required in randomly located radio network," *Electronics Letters*, vol. 25, pp. 1669–1671, 1989.

[11] M. Zorzi and R. R. Rao, "Geographic random forwarding (GeRaF) for ad hoc and sensor networks: Energy and latency performance," *IEEE Transactions on Mobile Computing*, vol. 2, no. 4, pp. 337–348, 2003.

[12] X. Ta, G. Mao, and B. D. O. Anderson, "On the probability of k-hop connection in wireless sensor networks," *IEEE Communications Letters*, vol. 11, no. 8, pp. 662–664, 2007.

[13] S. Mukherjee and D. Avidor, "Connectivity and transmit-energy considerations between any pair of nodes in a wireless ad hoc network subject to fading," *IEEE Transactions on Vehicular Technology*, vol. 57, no. 2, pp. 1226–1242, 2008.

[14] S. C. Ng and G. Mao, "Analysis of k-hop connectivity probability in 2-d wireless networks with infrastructure support," to appear in IEEE GLOBECOM 2010.

[15] T. S. Rappaport, *Wireless Communications: Principles and Practice, 2nd Edition*. Prentice Hall, 2001.

[16] M. Mauve, J. Widmer, and H. Hartenstein, "Survey on position-based routing in mobile ad hoc networks," *IEEE Networks*, vol. 15, no. 6, pp. 30–39, 2001.

[17] R. Hekmat and P. V. Mieghem, "Connectivity in wireless ad-hoc networks with a log-normal radio model," *Mobile Networks and Applications*, vol. 11, no. 3, pp. 351–360, 2006.