Opportunistic Broadcast in Mobile Ad-hoc Networks Subject to Channel Randomness

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Abstract—Broadcast in mobile ad-hoc networks is a challenging and resource demanding task, due to the effects of dynamic network topology and channel randomness. In this paper, we consider 2D wireless ad-hoc networks where nodes are randomly distributed and move following a random direction mobility model. A piece of information is broadcast from an arbitrary node. Based on an in-depth analysis into the popular Susceptible-Infectious-Recovered (SIR) epidemic routing algorithm for mobile ad-hoc networks, an energy and spectrum efficient broadcast scheme is proposed, which is able to adapt to fast-changing network topology and channel randomness. Analytical results are provided to characterize the performance of the proposed scheme, including the fraction of nodes that can receive the information and the delay of information propagation. The accuracy of analytical results is verified using simulations.

Index Terms—mobile ad-hoc networks, shadowing, opportunistic routing, epidemic routing

I. INTRODUCTION

A mobile ad-hoc network (MANET) is a self-organizing network composed of mobile devices like smart phones, tablet PCs or intelligent vehicles. In a MANET, information propagation relies on local ad-hoc connections that emerge opportunistically as devices move and meet each other. Such ad-hoc connections are determined by two major factors: dynamic topology and channel randomness.

The dynamic topology of a MANET often resembles the topology of a human network, in the sense that the mobility of nodes in a MANET is not only similar to, but often governed by, the movements of their human owners. In view of this, epidemic routing algorithms [1], [2] have been proposed as a fast and reliable approach to broadcast information in MANETs. On the other hand, unlike the spreading of epidemic disease in human networks, the information propagation scheme in MANETs can often be carefully designed.

In addition to varying network topology, channel randomness also has a significant impact on information broadcast in MANETs. It has been shown that channel shadowing has negative impacts on information propagation in MANETs using traditional routing algorithms like Ad hoc On Demand Distance Vector (AODV). Further, the wireless connection between two nodes can be affected by the activity of other nodes. A typical example is the cognitive radio network [3], where secondary users communicate with each other by exploiting spectrum holes - the channels that are temporarily and locally unused by primary users. Due to the uncertainty in the presence of spectrum holes, shadowing effect and mobility of users, information propagation in mobile networks requires an adaptive and flexible scheme to conquer these challenges.

On the basis of an in-depth analysis of the epidemic algorithm, this paper proposes an opportunistic broadcast scheme particularly suited for MANETs while achieving certain goals, e.g. reducing energy or bandwidth consumption. More specifically, the following contributions are made in the paper: 1) an opportunistic broadcast scheme is proposed, motivated by the analysis of the information propagation process using an epidemic routing algorithm; 2) it is shown that in comparison with the epidemic routing algorithm, the opportunistic broadcast scheme leads to less energy and bandwidth consumption to achieve the same number of recipients, i.e. the same fraction of nodes that receive the information; 3) the opportunistic broadcast scheme is adaptive to channel randomness, such as the shadowing effects and the uncertainty in the availability of transmission opportunities like spectrum holes or assigned time slots; 4) analytical results for the performance of the opportunistic broadcast scheme are presented, including the fraction of nodes that receive the information and time delay; 5) it is shown that shadowing effects benefit the performance of the opportunistic broadcast scheme, measured by the above two metrics, which is in contrast with previous studies, e.g. [4], considering traditional routing algorithms.

The rest of this paper is organized as follows: Section II reviews related work. Section III introduces the system model. The analysis of the information propagation process is presented in Section IV. Section V validates the analysis using simulations. Finally Section VI concludes this paper and proposes possible future work.

II. RELATED WORK

Epidemic routing algorithms have been popular choices for information broadcast in MANETs. Early studies, e.g. [5], assumed that a network is connected at any time instant, where a network is connected if and only if (iff) there is at least one path connecting any pair of nodes. However, it is often costly or impractical to ensure that a network is always connected [1], due to fast-changing network topology or...
channel randomness. Hence this paper studies MANETs from percolation perspective, as described rigorously in Section IV.

In [6], Chen et al. studied the information propagation process using a Susceptible-Infectious (SI) epidemic algorithm, which though reliable is a costly scheme due to lack of a proper mechanism to stop the transmission. Considering a Susceptible-Infectious-Recovered (SIR) epidemic algorithm, our previous work [2] studied the information propagation process in a MANET under the same setting as this paper. The SIR algorithm postulates that nodes need to keep transmitting for a prescribed time period. A drawback is that a long and uninterrupted transmitting period can be difficult to allocate in some practical networks, e.g. a cognitive radio network. This paper takes a further step toward developing a broadcast scheme addressing the difficulty in information propagation in MANETs subject to channel randomness.

There are other broadcast schemes for MANETs beside epidemic algorithms. Friedman et al. [7] reviewed some gossip-based algorithms that can be suitable for information broadcast in MANETs. They pointed out that achieving high energy and spectrum efficiencies are challenging and open problems. Wu et al. [8] studied the use of ferries, viz. nodes with controllable mobility, to improve information propagation in ad-hoc networks. In contrast, this paper focuses on MANETs where the nodes move uncontrollably.

III. System model

A. Network model

Consider a MANET where at some initial time nodes are independently and identically distributed (i.i.d.) on a torus \((0, L)^2\) following a homogeneous Poisson point process with intensity \(\lambda\). It follows that the expected number of nodes in the network is \(N=\lambda L^2\).

A commonly-used radio propagation model is the unit disk model (UDM), under which two nodes are directly connected if the distance between them is not larger than the radio range \(r_0\). Specifically, under UDM, the received signal strength (RSS) at a receiver separated by distance \(x\) from the transmitter is \(P_r(x)=CP_t x^{-\eta}\), where \(C\) is a constant, \(P_t\) is the transmission power common to all nodes and \(\eta\) is the path loss exponent, which can vary from 2 in free space to 6 in urban areas [9]. A transmission is successful if the RSS exceeds a given threshold \(P_{\text{min}}\). Therefore, the required transmission power \(P_t\) allowing a radio range \(r_0\) is \(P_t = \frac{P_{\text{min}}}{\eta}\).

In reality, the actual RSS may have significant variations around the mean value; this is typically taken into account in the log-normal shadowing model (LSM) [9]. Under LSM, the RSS attenuation (in dB) follows a Gaussian distribution: \(10 \log_{10}(P_r(x)/CP_t x^{-\eta}) \sim \mathcal{N}(0, \sigma^2)\), where \(P_r(x)\) is the RSS under LSM and \(Z\) is a zero-mean Gaussian distributed random variable with standard deviation \(\sigma\). When \(\sigma=0\), the model reduces to the UDM. In practice, the value of \(\sigma\) is often computed from measurement data and can be as large as 12 [9]. Denote by \(q(z)\) the probability density function (pdf) of the shadowing fades \(Z\); then: \(q(z) = \frac{1}{\sqrt{2\pi}\sigma^2} \exp(-\frac{z^2}{2\sigma^2})\).

As widely assumed in the literature [9], we consider that the shadowing fades \(Z\) between all pairs of transmitter and receiver are i.i.d. and the link is symmetric.

Regarding mobility, we adopt the random direction model (RDM) [10]. Specifically, each node chooses its direction independently and uniformly in \([0, 2\pi]\), and then move there-after at a constant speed \(V\). The information propagation process under other mobility models, e.g. those incorporating time variations of speed and direction, can be studied through a similar technique as that shown in the paper, which however requires a more complex analysis and is left as future work.

Note that under the aforementioned model, at any time instant, the spatial distribution of nodes is stationary and follows a homogeneous Poisson process with intensity \(\lambda\) [11].

B. Broadcast scheme

Suppose that a piece of information is broadcast from an arbitrary node. Once a node receives the information for the first time, it becomes infectious. The infectious node holds the information for a fixed amount of time \(\tau_s\) (called the sleeping time interval) followed by a random but bounded amount of time \(\tau_r\) (to be described in the next paragraph), then re-transmits the information (to all nodes directly connected to the infectious node) for a prescribed amount of time \(\tau_d\) (called the transmitting time interval). Such a sleep-active cycle repeats for a fixed number of times, denoted by a positive integer \(\beta\), after which the node recovers. A recovered node stops transmitting the information and will ignore all future transmissions of the same information that it receives. Therefore, the total transmitting time of an infectious node is \(\beta\tau_d\). The information propagation process naturally stops (i.e. reaches the steady state) when there is no infectious node in the network; and the nodes that have received the information are referred to as the informed nodes. An animation of the information propagation process is available on [12].

It is worth noting that we require the random time interval \(\tau_s+\tau_r\) after each transmission to accomplish three objectives: firstly, \(\tau_s\) is chosen to allow sufficient time (e.g. \(V\tau_s \geq 2r_0\)) for a node to move away from the location of its previous broadcast, to reduce repeating transmission to the same set of nodes; secondly, \(\tau_r\) introduces randomness in the transmitting time instants, which can reduce collisions and contention between nodes caused by simultaneous transmissions; thirdly, \(\tau_r\) provides flexibility in determining the transmitting time of a node, so that a node can transmit at its convenience (e.g. at assigned time slots or to exploit locally available spectrum holes) in a decentralized manner while the performance of a broadcast in the whole network (e.g. certain fraction of informed nodes) is still guaranteed. These features are valuable for a MANET subject to dynamic topology and channel randomness.

Denote by \(p_r(\tau_r)\) the pdf of \(\tau_r\), which is determined by practical implementation, such as the choice of media access control protocols (e.g. CSMA or TDMA). Using CSMA for example, if a node finds the channel busy after its sleeping time interval \(\tau_s\), then a random back-off time is often used, whose distribution determines the distribution of \(\tau_r\).

The above scheme is referred to as a general broadcast scheme. Note that when \(\beta=1\), \(\tau_r=0\) and \(\tau_s\) takes a constant value \(\tau_0\), the general broadcast scheme becomes a traditional SIR epidemic algorithm [1], [2].

Further, an opportunistic broadcast scheme in Section IV, where \(\tau_0\) is set to the minimum time required to transmit a single packet as motivated by the analysis, so that a secondary user can take the opportunity of a short spectrum hole and transmit its data.
The network introduced in this section is denoted by $G$. Further, we impose the requirement of a sufficiently large network, i.e., $L > \beta(\tau_s + \max(\tau_i) + \tau_a)V$, so that a node will not be wrapped through its motion in the torus back to the point where it became infected, at least not before it recovers.

IV. ANALYSIS OF THE INFORMATION PROPAGATION PROCESS

A. The effective node degree - definition

We first propose a single metric to summarize the impact of different parameters, i.e., $\lambda, V, r_0, \beta, \tau_i, \tau_r$ and $\tau_a$, on the information propagation process of a MANET. As an extension of our previous work in [2], the metric is defined as follows.

**Definition 1.** The effective node degree $R_0$ of an infectious node is the expected number of nodes that are directly connected to the infectious node during its total transmitting time, whose length is $\beta r_a$.

Note that $R_0$ is the same for all nodes due to the stationarity and homogeneity of node distribution on the polygon. Assuming the knowledge of $R_0$, whose evaluation is studied later, the next sub-section reviews some fundamental properties of the information propagation process.

B. Fraction of informed nodes

We first study the fraction of informed nodes from the percolation perspective asymptotically, i.e. we increase the network area to infinity (i.e. let $L \to \infty$) while keeping other parameters (i.e. $\lambda, V, r_0, \beta, \tau_i, \tau_r$ and $\tau_a$) unchanged.

**Definition 2.** The percolation probability $p_c$ of a MANET is the probability that a piece of information broadcast from an arbitrary node can be received by a non-vanishingly-small fraction of nodes asymptotically.

Define $z_0$ as the expected fraction of informed nodes in the steady state. Then we report the following two results:

**Theorem 1.** Consider a network $G$, whose effective node degree is $R_0$. The percolation probability satisfies $p_c \leq 1 + \frac{1}{R_0} W(-R_0 e^{-R_0})$, where $W(.)$ is the Lambert W Function.

**Theorem 2.** Consider a network $G$, whose effective node degree is $R_0$. The expected fraction of informed nodes in the steady state satisfies $z_0 \leq (1 + \frac{1}{R_0} W(-R_0 e^{-R_0}))^2$, where $W(.)$ is the Lambert W Function.

Using the same technique as that shown in [2], the above two theorems can be readily proved for the broadcast schemes considered in this paper. The proof is therefore omitted and it is further verified by simulation in Section V that the analytical bound is close to the actual value.

It can be seen that the effective node degree determines the performance of the broadcast scheme. We next present further analysis into the effective node degree.

C. The effective node degree - analysis

It is clear that the consumptions of bandwidth and energy are determined by the transmitting time. Therefore, we next study the choice of the transmitting time interval $\tau_a$.

Denote $R_{\tau_a}$ as the effective node degree associated with a sleep-active cycle with transmitting time $\tau_a$. Then, we propose a metric to gauge the spectrum and energy efficiencies.

**Definition 3.** The transmission efficiency $E(\tau_a) \triangleq \frac{r_a}{\tau_a}$ is the effective node degree per unit of transmitting time in a sleep-active cycle of an arbitrary infectious node.

**Lemma 1.** Consider a network $G$, where all parameters are fixed except $\tau_a$. To maximize the transmission efficiency, $\tau_a$ needs to be minimized.

**Proof:** In a large network (i.e., $L > \beta(\tau_s + \max(\tau_i) + \tau_a)V$) under UDM, we have

$$E(\tau_a) \triangleq \frac{R_{\tau_a}}{\tau_a} = \frac{8\theta V \tau_a}{\pi} + \frac{\pi R_0^2 A}{\tau_a} \triangleq \frac{8\theta_0 V \lambda}{\pi} + \frac{\pi R_0^2 A}{\tau_a}. \tag{1}$$

It is clear that $E(\tau_a)$ is a decreasing function of $\tau_a$. The same conclusion can be obtained using the same method for the LSM, which is not included here due to page limits.

**Remark 1.** According to Lemma 1, in order to maximize the transmission efficiency, $\tau_a$ should be set to the minimum time required to transmit a piece of information. Considering that broadcast is usually used for small pieces of information, such as advertisements or emergency alerts, the transmission time of a packet is usually in the order of milliseconds. Further, because the typical moving speed is 1.5m/s for human or 10m/s for vehicles [13], the displacement of a node during a time $\tau_a$ is in the order of millimeters, which is negligible (hence ignored hereafter) compared to the radio range (usually in the order of meters).

**Remark 2.** The reduction of $\tau_a$ can not only save energy and bandwidth, but also facilitate opportunistic broadcasting. For example, it can be difficult for the secondary users in a cognitive radio network to find a spectrum hole with a long period, say 100 seconds [2]. On the other hand, using the opportunistic broadcast scheme, a secondary user can take the opportunity of a short spectrum hole and transmit its data.

Next, we propose the following metric to facilitate the choice of sleeping time interval $\tau_a$, which determines when a node transmits.

**Definition 4.** The clustering factor $\phi(\tau_a)$ is the expected number of nodes that are directly connected to an infectious node in both a given transmission and the following one.

In order to incorporate channel randomness into the analysis, we need the following lemma.

**Lemma 2** ([14]). Suppose the wireless channel between a transmitter (S) and a receiver has shadowing fadings $z_S$. Then these two nodes are connected iff their distance $x$ satisfies:

$$x \leq r_0 \exp(\frac{z_S \ln 10}{10\eta}) \triangleq r_N(z_S). \tag{2}$$

Using Lemma 2, we have the following results.

**Lemma 3.** Consider a network $G$. The clustering factor satisfies

$$\phi(\tau_a) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} |A_p(\theta, r_N(z_1), r_N(z_2))| \times \frac{1}{\pi} \int_{\tau_a}^A \tau_r(p_r(\tau_a q(z_1) q(z_2)) d\tau_r d\theta d z_1 d z_2. \tag{3}$$

1For example, the time required to transmit a packet of size 256 bytes via a 10Mbps link is $\frac{256 \text{bits}}{10 \text{Mbps}} = 204.8\mu$s.
where \( \theta_m = 2 \arcsin \frac{r_s(z_1) r_s(z_2)}{2V(r_s + r_1)} \) and \(|A_p(\theta, r_1, r_2)|\) is given by Eq. 4.

Proof: Denote by \( \Theta \) the angle measured counterclockwise from the moving direction of an infectious node to the moving direction of an arbitrary node. According to the mobility model introduced in Section III, it is straightforward that \( \Theta \) is uniformly distributed in \([0, 2\pi)\).

Suppose that an infectious node transmits once at point \( S_1 \), then it moves by distance \((\tau_s + \tau_t)V \) to point \( S_2 \) and transmits again, as shown in Fig. 1.

Next, we focus on a subset of nodes that fulfill the following three conditions: 1) they move in direction \( \Theta \in (\theta, \theta + d\theta) \); 2) their RSS from the infectious node has shadowing fades \( Z_1 \in [z_1, z_1 + dz_1] \) when the infectious node transmits at \( S_1 \); 3) their RSS from the infectious node has shadowing fades \( Z_2 \in [z_2, z_2 + dz_2] \) when the infectious node transmits at \( S_2 \). Due to the independence of shadowing fades and the thinning property of Poisson processes, these nodes are distributed following a homogeneous Poisson process with intensity \( \frac{1}{2\pi} q(z_1) q(z_2) dz_1 dz_2 \). Among this subset of nodes, the nodes that are connected to the infectious node in the first transmission are in the disk centered at point \( S_1 \) with radius \( r_N(z_1) \), which is denoted by \( C(S_1, r_N(z_1)) \). Further, when the infectious node transmits at \( S_2 \), these nodes move by distance \((\tau_s + \tau_t)V\) from being contained in \( C(S_1, r_N(z_1)) \) to being contained in a new disk \( C(B, r_N(z_1 + dz_2)) \) as shown in Fig. 1. Then, the nodes connected to the infectious node in both two transmissions are in the area \( C(S_2, r_N(z_2)) \cap C(B, r_N(z_1 + dz_2)) \), as shown in Fig. 1, whose size can be calculated using the following formula:

\[
|A_p(\theta, r_1, r_2)| = \left[ \begin{array}{l}
\min(\pi r_1^2, \pi r_2^2), \\
\pi r_1^2 \arccos(\frac{\phi(\theta) + r_2^2 - r_1^2}{2 r_1 r_2 \cos(\theta)}) + \pi r_2^2 \arccos(\frac{\phi(\theta) + r_1^2 - r_2^2}{2 r_1 r_2 \cos(\theta)}) \end{array} \right]
\]

\[
\] \( \psi(\theta) = 2(\tau_s + \tau_t)V \sin \frac{\theta}{2} \) is the length of \( BS_2 \).

Next we consider all subsets of nodes. Note that the intersection area only exists when \( |\theta| < 2 \arcsin \frac{\phi(\theta) + r_2^2 - r_1^2}{2 r_1 r_2 \cos(\theta)} \) \( \theta_m \); and only the cases for \( \theta \in (0, \theta_m) \) need to be calculated due to symmetry. Therefore, the clustering factor satisfies

\[
\phi(\tau_s) = \lambda E[|A_p(\theta, r_N(z_1), r_N(z_2))|] \le \lambda \int_{-\infty}^{\infty} \int_{\infty}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} |A_p(\theta, r_N(z_1), r_N(z_2))| \times \frac{1}{2\pi} p_{\tau_1}(\tau_s) q(z_1) q(z_2) d\tau_1 d\theta dz_1 dz_2.
\]

Finally, we have the following theorem for the value of \( R_0 \).

Theorem 3. The effective node degree of a network \( G \) using the proposed opportunistic broadcast scheme satisfies

\[
R_0 \le \beta \lambda \pi \eta \exp \left( \frac{(\sigma \ln 10)^2}{50 \pi^2} \right) - (\beta - 1) \phi(\tau_s).
\]

Proof: We first consider two consecutive transmissions. Along the same lines as Lemma 3, define \( \phi(\tau_s, z_1, z_2) \) to be

\[
\]

Fig. 1. An illustration to the nodes connected to an infectious node in both two consecutive transmissions. Symbols are defined in Lemma 3.

Fig. 2. An illustration of the impact of the clustering factor on the effective node degree, the curves show the ratio(\( R_0 \)) of the effective node degree at \( t = 0 \) when the fraction of informed nodes reaches \( \frac{Nz}{L} \), for \( 0 < z \le 1 \). Then, for \( N = \lambda L^2 \),

\[
T(z) \ge \tau_s + \frac{\ln Nz}{\ln(1 + \frac{R_0}{\beta})}.
\]

Proof: Recall that each infectious node has \( \beta \) transmissions separating by a random time interval \( \tau_s \). In this proof, we obtain a lower bound on the delay \( T(z) \) by considering that the time between any two consecutive transmissions is \( \tau_s \) and the infectious nodes never recover.

\[
\]

D. Information propagation delay

Lemma 4. Consider a network \( G \), whose effective node degree is \( R_0 \). A piece of information is broadcast from an arbitrary node at time \( t = 0 \) using the opportunistic broadcast scheme. Let \( T(z) \) be the expected time when the fraction of informed nodes reaches \( z \), for \( 0 < z \le 1 \). Then, for \( N = \lambda L^2 \),

\[
T(z) \ge \tau_s + \frac{\ln Nz}{\ln(1 + \frac{R_0}{\beta})}.
\]
Then, all infectious nodes transmit simultaneously at time $\tau_r, 2\tau_r, 3\tau_r, \ldots$.

Further, recall that $R_0$ is the expected number of nodes (including both informed and uninformed nodes) that have been directly connected to an infectious node during all its transmissions. However, some of the nodes connected to an infectious node may have already received the information from other infectious nodes. It follows that the expected number of new infectious nodes created by each infectious node at each time slot is not larger than $R_0/\beta \geq q$.

Let $a_k$ be the total number of infectious nodes at time $kt_r$, for $k \geq 0$. There holds $a_k \leq a_{k-1} + a_{k-1}q = a_{k-1}(1+q)$. Because $a_0 = 1$, it can be shown that $a_k \leq (1+q)^k$. Then

$$T(z) \geq \tau_r \arg \max_k (a_k \leq Nz) \quad (10)$$

$$\geq \tau_r \arg \max_k \left(1 + \frac{R_0}{\beta} \right)^k \leq Nz \quad (11)$$

$$= \tau_r \left[1 + \frac{\ln Nz}{\ln(1 + \frac{R_0}{\beta})}\right] \quad (12)$$

V. Simulation results

In this section, we report on simulations to verify the accuracy of analytical results. The simulations are conducted using a MANET simulator written in C++. Nodes are deployed on a torus $(0, 800)^2$ following a Poisson process with intensity $\lambda = 0.002$. The nodal speed $V$ is set to 10m/s (typical vehicle moving speed [13]) and $s_{\tau_r}$ is considered to be uniformly chosen from $[0, 1]$. Other distributions of $\tau_r$ show a similar trend hence not shown here.

Fig. 3 shows the percolation probability and the expected fraction of informed nodes. It can be seen that both metrics improve as either of $\tau_r$ or $\sigma$ increases, owing to the reduction of the clustering factor. Our analytical bounds are close to the simulation results as shown in Fig. 3(a), while the discrepancy in Fig. 3(b) is caused by the approximations used in deriving the fraction of informed nodes in [2], which requires another non-trivial analysis to correct.

Fig. 4 shows the time delay for a piece of information to be received by 50% of nodes. It can be seen that the length of the sleeping time interval has a great impact on the delay and our analytical result provides a valid lower bound.

Further, Fig. 3 and Fig. 4 suggest that shadowing effects benefit the information propagation process in terms of percolation probability, expected fraction of informed nodes and time delay, because an increase in $\sigma$ leads to an increase in $R_0$ as shown by Theorem 3. This is in sharp contrast with previous conclusions, e.g. [4], because traditional routing algorithms like AODV need to establish a route before sending data, whilst the route is unstable due to dynamic topology and channel randomness.

VI. Conclusion and future work

This paper proposed a spectrum and energy efficient opportunistic broadcast scheme for MANETs subject to channel randomness. The proposed scheme is decentralized and simple to implement. Further, the performance of the network using the proposed scheme, measured by the percolation probability, expected fraction of informed nodes and time delay, is analytically studied. The accuracy of the analytical results was verified by simulations.

In the future, we are going to consider the effects of fast fading such as the Rayleigh fading [9]. Further, we are interested in investigating the broadcast scheme where the distribution of $\tau_r$ is determined by certain CSMA protocols.

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