Stochastic Characterization of Information Propagation Process in Vehicular Ad hoc Networks

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Abstract—This paper studies the information propagation process in wireless communication networks formed by vehicles traveling on a highway. Corresponding to different lanes of the highway and different types of vehicles, we consider that vehicles in the network can be categorized into a number of traffic streams, where the vehicles in the same traffic stream have the same speed distribution while the speed distributions of vehicles in different traffic streams are different. We analyze the information propagation process of the aforementioned vehicular network and obtain an analytical formula for the information propagation speed (IPS). Using the formula, one can straightforwardly study the impact of parameters such as radio range, vehicular traffic density, vehicular speed distribution, and the time variation of vehicular speed on the IPS. The accuracy of the analytical results is validated using simulations.

Index Terms—Information propagation speed (IPS), intelligent transport system, mobile ad hoc network, vehicular ad hoc network (VANET).

I. INTRODUCTION

RECENTLY, intelligent transportation systems have attracted significant interest in both academia and industry [1]–[3]. In an intelligent transportation system, wireless communication between vehicles is the lifeblood of intelligence. This paper studies vehicular ad hoc networks (VANETs), which are wireless communication networks formed by vehicles traveling on a highway. The information propagation process in VANETs is quite different from that in a static network where devices remain stationary over time. In a VANET, information can propagate not only through wireless links between adjacent vehicles but also through the physical movements of the

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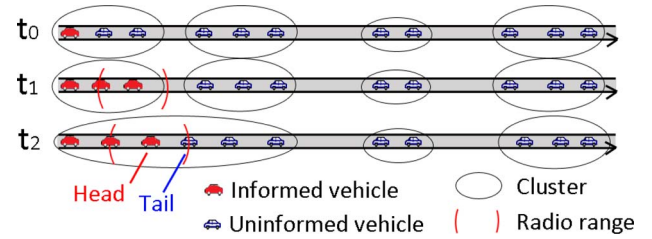Digital Object Identifier 10.1109/TITS.2013.2274274

Fig. 1. Illustration of a VANET at different time instants. We consider that information propagates from left to right in the figures hereafter.

vehicles. This paper investigates the information propagation process in VANETs in detail, isolating a number of its statistical properties. In particular, this paper analytically studies the expected propagation speed for a piece of information to propagate along the road in a VANET, which is referred to as the *information propagation speed (IPS)*.

A VANET can usually be partitioned into a number of clusters [4] and [5], where a *cluster* is a maximal set of vehicles, wherein every pair of vehicles is connected by a multihop path. Due to the mobility of vehicles, clusters are splitting and merging over time. Therefore, information propagation in a VANET is typically based on a store-carry-forward scheme [4]. Considering the example illustrated in Fig. 1, a piece of information starts to propagate from the origin toward the positive direction of the axis at time t_0. The vehicles that have received this information are referred to as the *informed vehicles*, whereas other vehicles are *uninformed*. At time t_1, the message is forwarded, in a multihop manner, from the first informed vehicle to the foremost vehicle in the cluster. The propagation of message within a cluster, which begins at t_0 and ends at t_1, is called a *forwarding process*. In a forwarding process, the IPS is determined by the per-hop delay and the length of the cluster, where the *per-hop delay* is the time required for a vehicle to receive and process a message before it is available for further relaying [6]. It is shown later in this paper that both the per-hop delay and the cluster length have significant impacts on the IPS.

Define the *head* at time t to be the informed vehicle with the largest coordinate at time t. Define the *tail* at time t to be the uninformed vehicle with the smallest coordinate at time t. Two vehicles can directly communicate with each other if and only if (iff) their Euclidean distance is smaller than or equal to the radio range r_0, i.e., we are adopting the unit disk communication model. As shown in Fig. 1, at time t_1, the tail is outside the radio range of the head. Then, a *catch-up process* begins, during

which the head holds the information until the boundary of its radio range reaches the tail (at time t_2).

Recent research has shown that downstream traffic (i.e., a set of vehicles traveling in the opposite direction of information propagation) can be exploited to improve the IPS [4], [7] and [8]. Furthermore, real-world measurements show that vehicles traveling in different lanes (e.g., bus lane or heavy truck lane) often have different speed distributions [9] and [10]. In view of these observations, this paper considers VANETs with multiple traffic streams, where a *traffic stream* is a set of vehicles following the same speed distribution. Traffic streams can be used to represent vehicles traveling in different lanes, or different types of vehicles (e.g., sports cars or heavy trucks). Interesting results are obtained by allowing vehicles in different traffic streams to have different speed distributions.

The main contributions of this paper include the following.

- First, a theoretical analysis of the information propagation process, including both the forwarding process and the catch-up process, is provided. It is shown that a small difference in the average vehicular speeds between different traffic streams can result in a significant increase in the IPS.
- Second, the impact of time-varying vehicular speed on the information propagation process is considered in the analysis. Different from previous studies (e.g., [7] and [8]), consideration of time-varying vehicular speed allows the catch-up process to occur among vehicles in the same traffic stream, which is essential for determining IPS in VANETs where most vehicles travel in the same direction (e.g., toward downtown).
- Furthermore, the impact of the so-called pseudo catch-up events (to be introduced in Section V-C) on the catch-up process is analyzed, which is shown to have a significant impact on the information propagation process, particularly when the vehicular density is low.
- Based on the analysis of the catch-up process and the forwarding process, analytical results on the IPS are derived, which are validated using simulations.
- Finally, simplifications of the analytical results on both the catch-up process and the forwarding process are provided; the simplified formula is important for readily and intuitively grasping the interactions among fundamental parameters in the information propagation process.

The rest of this paper is organized as follows: Section II reviews related work. Section III introduces the mobility and network models used in this paper. The forwarding process is studied in Section IV. The analysis of the catch-up process is given in Section V, followed by the results on the IPS in Section VI. In addition, simplified results for both the catch-up process and the forwarding process are presented in Section VII. Section VIII validates the analysis using simulations. Section IX concludes this paper and discusses future work.

II. RELATED WORK

Wireless communication is essential for a number of applications in intelligent transportation systems, such as safety messaging, traffic monitoring, in-vehicle entertainment, etc. In [11], Camera et al. studied a scenario, in which some infrastructure stations were destroyed by natural disasters, such as flooding and earthquake, while public safety warning messages can still be broadcast with the help of vehicle-to-vehicle communications (between vehicles on the road segments that are not destroyed). It is shown that without vehicle-to-vehicle communications, more than 200 infrastructure stations are required to spread a message to all the nodes in less than 1 h. With the use of vehicle-to-vehicle communications, even if there are only two infrastructure stations remaining working, it takes around 20 min to send the warning message to all the nodes in the region.

The great potential benefits of the intelligent transportation systems have motivated a large number of studies on the information propagation process in a wireless communication network formed by vehicles, i.e., a VANET. It is worth noting that the information propagation process in a VANET can be quite different from that in a static network, as introduced in Section I. Furthermore, the vehicular traffic pattern on a road can vary significantly, depending on the traffic density in different lanes or at different times of the day [9] and [10]. In [6], Wu et al. showed using simulation that the IPS varies significantly with the vehicular density.

A number of analytical studies on VANETs were based on the assumption that the vehicular speed is time invariant, i.e., vehicular speed does not change over time. Under this assumption, in [12], Yousefi et al. provided analytical results on the distribution of the intervehicle distance in a VANET. Some studies further assume that all vehicles have the same speed. In [4], Agarwal et al. studied the IPS in a VANET where vehicles are Poissonly distributed and move at the same time-invariant speed, but either in the positive direction (upstream) or the negative (downstream) direction of the axis. The upstream and downstream traffic have the same density. Later in [7], the same authors extended their previous work in [4], allowing for the situation that the upstream traffic and the downstream traffic have different vehicular densities. They derived upper and lower bounds on the IPS, with a focus on the impact of vehicular density on the IPS. Recent research of Baccelli et al. in [8] studied the IPS under the same setting as that in [7]. They showed that there exists a threshold value of the vehicular density, under which information propagates at the vehicular speed and above which information propagates dramatically faster.

Note that a number of previous studies, e.g., [4], [7], and [8], assumed that all vehicles move at the same time-invariant speed. Evidently, under such model, the catch-up event cannot occur among vehicles traveling in the same direction. On the other hand, interesting and more realistic results can be obtained by considering that vehicles have different speeds. In [5], Wu et al. considered a VANET where vehicles are Poissonly distributed on the road and the vehicular speeds are uniformly and independently distributed within a designated range. Considering one traffic stream and time-invariant vehicular speed, they studied the catch-up process between vehicles traveling in the same direction and provided analytical results on the IPS when the vehicular density is either very low or very high. In
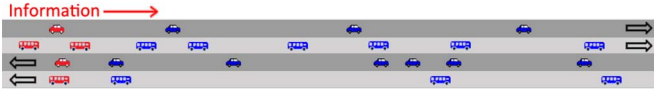
Fig. 2. Illustration of a VANET with four traffic streams. Information is propagating in the positive direction of the axis.

addition, they also considered VANETs with two traffic streams toward opposite directions. Through numerical computation, they showed that the IPS can be further increased by exploiting the vehicles traveling in the opposite direction.

In our earlier work [13], we considered a time-varying vehicular speed model and showed that the time variation of vehicular speed has a significant impact on the IPS. However, in [13], only one traffic stream was considered, i.e., all vehicles travel in the same direction and follow the same speed distribution. In this paper, we extend the earlier work by considering multiple traffic streams and evaluating the impact of vehicular densities and speed distributions in different traffic streams on the information propagation process in VANETs.

III. SYSTEM MODEL

Consider a VANET with a total of N traffic streams (see Fig. 2). As mentioned earlier, we consider the widely used unit disk communication model, using which two vehicles can directly communicate with each other iff their Euclidean distance is not larger than the radio range r_0. Furthermore, the time required for a vehicle to receive and process a message before it is available for further relaying is also considered in this paper, which is called the per-hop delay β. The value of β reflecting typical technology is 4 ms [6].

A *synchronized random walk mobility model* is considered. Specifically, time is divided into time slots with equal length τ, where the ith time slot is $t \in ((i-1)\tau, i\tau]$. Consider a VANET with a total of N traffic streams. Each vehicle in the nth $(n \in \{1, \ldots, N\})$ traffic stream chooses its speed randomly at the beginning of each time slot, independent of the speeds of other vehicles and its own speeds in other time slots, according to a probability density function (pdf) $f_n(v)$. A nonsynchronized mobility model (i.e., each vehicle changes its speed at different time instants) is evaluated by simulations later in Section VIII and shown to have slight impact on the IPS. In this paper, we consider the Gaussian speed distribution, i.e., $f_n(v) \sim \mathcal{N}(\mu_n, \sigma_n^2)$, where μ_n (resp. σ_n^2) is the mean speed (resp. variance) of the nth traffic stream. The Gaussian speed distribution has been widely used for modeling vehicles on a freeway [5], [14], and [15]. Although the mobility model allows arbitrary values of μ_n, in the real world [15], typical values of μ_n can be ± 25 m/s or ± 10 m/s, corresponding to the speeds of vehicles traveling in fast lane or slow lane, respectively. Note that a positive (resp. negative) value of the vehicular speed means that the vehicle is moving in the positive (resp. negative) direction of the axis. When a vehicle changes its speed from a positive value to a negative one, which may happen frequently on a slow-traffic road with a small mean vehicular speed (e.g., 10 m/s), it means that a vehicle changes its moving direction. Furthermore, another interesting scenario is that a traffic stream

with $\mu_n = \sigma_n = 0$, corresponding to the vehicles stopped at roadside or some roadside stations without (expensive) wired connection; these roadside units are shown later in this paper to have great potential of improving the IPS. An animation of the information propagation process in a VANET with either one-direction or two-direction traffic is available on the web page [16].

At the beginning of the information propagation process, the vehicles in the nth traffic stream follow a homogeneous Poisson spatial distribution with density ρ_n. Then, the vehicles start to move according to the aforementioned random walk mobility model. Furthermore, it can be shown that the spatial distribution of vehicles in the nth traffic stream remains a homogeneous Poisson distribution with density ρ_n at any time instant [13]. Then, according to the superposition property of Poisson processes [17], at any time instant, the spatial distribution of all the vehicles on the road follows a homogeneous Poisson process with intensity $\rho = \sum_{n=1}^{N} \rho_n$. This means that the total vehicle density is ρ vehicles per meter. Note that the assumption of Poisson distribution of vehicles is only suitable for free-flow traffic [12], i.e., the movements of vehicles are independent. Therefore, the model is applicable to the VANET scenarios where the vehicular density is low, such as the VANET on a freeway.

In the following three sections, we study the information propagation process by separately analyzing its two major constituent subprocesses: the forwarding process and the catch-up process. On the basis of these studied processes, simplified results are provided in Section VII for both the forwarding process and the catch-up process.

IV. FORWARDING PROCESS

Define the *forwarding delay* as the time required for a packet to be forwarded from the leftmost vehicle in a cluster to the rightmost vehicle in the cluster. Note that we consider that the radio propagation speed is infinite in a forwarding process, because the information propagation delay is usually small. For example, a typical value of per-hop delay in vehicle-to-vehicle communication is $\beta = 4$ ms [6], which means that the radio propagation speed can be as large as $250/0.004 = 62500$ m/s, which is much faster than the moving speed of vehicles (typical speed is 25 m/s [15]). Therefore, it is assumed that a cluster does not become disconnected during the forwarding process. Then, the expected forwarding delay in a cluster with length x is $\mathbb{E}[t_f|x] = \beta \mathbb{E}[k|x]$, where $\mathbb{E}[k|x]$ (given in [18]) is the expected number of hops between two vehicles separated by distance x.

Denote by $X(\rho)$ the length of an arbitrary cluster in a VANET with density ρ, i.e., it is the Euclidean distance between the leftmost vehicle and the rightmost vehicle in a cluster. Then, the pdf of the cluster length is [13]

$$\Pr\left(X(\rho) = x\right) = \frac{\rho}{(e^{\rho r_0} - 1)} \sum_{m=0}^{\lfloor x/r_0 \rfloor} \left[\frac{(-\rho(x - mr_0))^{m-1}}{-m!} \right.$$

$$\left. \times \left(\rho(x - mr_0) + m\right) e^{-\rho mr_0} \right]. \quad (1)$$

It follows that the cumulative distribution function (cdf) of cluster length is

$$\Pr\left(X(\rho) \leq x\right) = \int_0^x \Pr\left(X(\rho) = x_0\right) \mathrm{d}x_0. \tag{2}$$

V. CATCH-UP PROCESS

We study the catch-up process in this section. Without loss of generality, it is assumed that the catch-up process starts at time 0. The *displacement* γ of a vehicle at time t is defined to be the distance between the vehicular position at time t and its position at time 0. Note that $\gamma \in (-\infty, \infty)$, where a positive (resp. negative) value of γ means that the position of the vehicle at time t is on the right (resp. left) of its position at time 0.

This section is organized as follows: in Lemma 1, we study the movement of a single vehicle. Then, the result on the distance between two vehicles at a given time is summarized in Lemmas 2 and 3, followed by the study on the so-called pseudo catch-up process in Lemma 4. After considering the overtaking of vehicles in Section V-D, we obtain the delay of a catch-up process in Section V-E.

A. Modeling the Movement of a Single Vehicle

Consider a single vehicle first. Suppose that a vehicle is located at the origin ($\gamma = 0$) at time 0. Denote by $p_n(\gamma, t)$ the pdf that the displacement of a vehicle in the nth traffic stream is γ at time t. We have the following result:

Lemma 1: Suppose that a vehicle in the nth traffic stream follows a Gaussian speed distribution with mean μ_n and variance σ_n^2. Under the synchronized random walk mobility model, there holds

$$p_n(\gamma, t) = \frac{1}{\tilde{\sigma}_i \sqrt{2\pi}} \exp\left(\frac{-(\gamma - \mu_n i\tau - \mu_n(t - i\tau))^2}{2\tilde{\sigma}_i^2}\right) \tag{3}$$

where $\tilde{\sigma}_i^2 = i(\sigma_n\tau)^2 + \sigma_n^2(t - i\tau)^2$, and $i = \lfloor t/\tau \rfloor$.

Proof: According to the Gaussian speed distribution model, the pdf of the speed of a vehicle in the nth traffic stream is

$$f_n(v) = \frac{1}{\sigma_n\sqrt{2\pi}} \exp\left(\frac{-(v - \mu_n)^2}{2\sigma_n^2}\right). \tag{4}$$

Because the speed does not change during a time slot, it is straightforward that $p_n(\gamma, \tau)$ is also a Gaussian function

$$p_n(\gamma, \tau) = \frac{1}{\sigma_n\tau\sqrt{2\pi}} \exp\left(\frac{-(\gamma - \mu_n\tau)^2}{2(\sigma_n\tau)^2}\right). \tag{5}$$

Recall that we consider that the speed of a vehicle at a time slot is independent of that in another time slot. Hence, the displacements of a vehicle are independent across time slots. Then, there holds

$$p_n(\gamma, t) = \overbrace{(p_n * p_n * \ldots * p_n)(\gamma, \tau)}^{i\text{ - fold convolution}} * p_n(\gamma, t - i\tau) \tag{6}$$

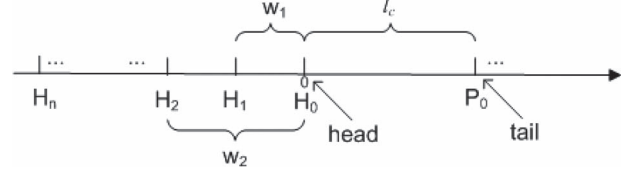where $i = \lfloor t/\tau \rfloor$.

Fig. 3. Illustration of a VANET at the beginning of a catch-up process.

According to the property that the convolution of two Gaussian functions is a Gaussian function [19], the Gaussian function after i-fold convolution has mean $\mu_n i\tau$ and variance $i(\sigma_n\tau)^2$. Then, the lemma is readily proved. ∎

B. Distance Between a Pair of Vehicles

As shown in Fig. 3, denote by H_ζ (resp. P_ζ) the ζth vehicle to the left of the head H_0 (resp. to the right of the tail P_0) at the beginning of a catch-up process (viz., at time 0). If H_ζ happens to be in the nth traffic stream, then an additional label n (e.g., H_ζ^n) is used to indicate that the vehicle is in the nth traffic stream. In this subsection, we study the Euclidean distance between H_ζ^n and P_η^m, where $n, m \in \{1, \ldots, N\}$; n may or may not be equal to m; and ζ, η are nonnegative integers. Note that we need to consider arbitrary nonnegative integer values of ζ, η rather than $\zeta = \eta = 0$, due to the fact that, at time $t > 0$, the head vehicle H_0 (resp. tail vehicle P_0) can be overtaken by another informed vehicle (resp. uninformed vehicle); consequently, the information may be forwarded from H_ζ to P_ζ for any nonnegative integer values of ζ and η.

The information can be forwarded from H_ζ^n to P_η^m as soon as their distance reduces to the radio range r_0. Next, we characterize the reduction in the distance between H_ζ^n and P_η^m compared with their initial distance at the beginning of the catch-up process.

Lemma 2: Denote by $g_{\zeta\eta}^{nm}(z, t)$ the pdf of the reduction in the distance between H_ζ^n and P_η^m being z at time t, with regard to their distance at time 0 (viz., the start time of the catch-up process). Then, there holds

$$g_{\zeta\eta}^{nm}(z, t) = \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left(\frac{-(z - \mu_t)^2}{2\sigma_t^2}\right) \tag{7}$$

where $\mu_t = \mu_n i\tau + \mu_n(t - i\tau) - \mu_m i\tau - \mu_m(t - i\tau)$, $\sigma_t^2 = i\sigma_n^2\tau^2 + \sigma_n^2(t - i\tau)^2 + i\sigma_m^2\tau^2 + \sigma_m^2(t - i\tau)^2$, and $i = \lfloor t/\tau \rfloor$.

Proof: The proof is straightforward using the same technique as that in Lemma 1; hence, it is omitted here. ∎

Note that vehicle H_ζ^n *catches up* another vehicle P_η^m when their Euclidean distance reduces to radio range r_0 *for the first time*. However, during a sufficiently long time interval, the reduction in distance between H_ζ^n and P_η^m can reach a given value z at several time instants with nonzero probabilities. We are only interested in the event that the reduction in the distance between H_ζ^n and P_η^m reaches z *for the first time*, i.e., the first passage phenomenon. A thorough understanding of the first passage phenomenon is essential for the analysis of the catch-up process.

Lemma 3: Denote by $G_{\zeta\eta}^{nm}(z, i)$ the pdf of the event that the reduction in the distance between H_ζ^n and P_η^m, with regard to

their distance at time 0, reaches $z(z > 0)$ *for the first time* in the ith $(i \geq 1)$ time slot. Then

(a) When $\mu_n = \mu_m$

$$G^{nm}_{\zeta\eta}(z,i) = \frac{z}{i\sqrt{2\pi i \tau^2 (\sigma_n^2 + \sigma_m^2)}} \exp\left(\frac{-z^2}{2i\tau^2(\sigma_n^2 + \sigma_m^2)}\right).$$
(8)

(b) When $\mu_n \neq \mu_m$ and $i = 1$

$$G^{nm}_{\zeta\eta}(z,i) \leq \frac{1}{2}\left(1 - \mathrm{erf}\left(\frac{z - \hat{\mu}}{\sqrt{2\sigma_i^2}}\right)\right).$$
(9)

(c) When $\mu_n \neq \mu_m$ and $i \geq 2$

$$G^{nm}_{\zeta\eta}(z,i) \leq \frac{1}{4}\left(1 + \mathrm{erf}\left(\frac{z - \mu_{i-1}}{\sqrt{2\sigma_{i-1}^2}}\right)\right)\left(1 - \mathrm{erf}\left(\frac{z - \hat{\mu}}{\sqrt{2\sigma_i^2}}\right)\right)$$
(10)

where $\hat{\mu} = \mu_n i\tau - \mu_m i\tau$, $\sigma_i^2 = (\sigma_n^2 + \sigma_m^2)\tau^2 i$, and $\mathrm{erf}(.)$ is the error function.

Proof: We first consider the case that two traffic streams have the same mean speed, i.e., $\mu_n = \mu_m$. In this case, we have $\mu_t = 0$ in (7).

It is straightforward that, in the ith time slot, the probability that the reduction in the distance between H^n_ζ and P^m_η reaches z is $\int_{(i-1)\tau}^{i\tau} g^{nm}_{\zeta\eta}(z,t)\mathrm{d}t$. According to the mean value theorem, there exists $\hat{t} \in ((i-1)\tau, i\tau]$, such that $\int_{(i-1)\tau}^{i\tau} g^{nm}_{\zeta\eta}(z,t)\mathrm{d}t = \tau g^{nm}_{\zeta\eta}(z,\hat{t})$. To facilitate the first passage analysis, we approximate $\hat{t}$ by $i\tau$ and apply a standard procedure, as shown in [20], to determine the first passage probability $G^{nm}_{\zeta\eta}(z,i)$. Then, there holds

$$\tau g^{nm}_{\zeta\eta}(z', i'\tau) = \tau \sum_{i=1}^{i'} G^{nm}_{\zeta\eta}(z,i)g^{nm}_{\zeta\eta}(z' - z, (i' - i)\tau).$$
(11)

The convolution can be simplified by performing the Z-transform on $g^{nm}_{\zeta\eta}(z, i\tau)$, with regard to i, i.e.,

$$(\mathcal{Z}g)(z,s) = \sum_{i=1}^{\infty} e^{-si} g^{nm}_{\zeta\eta}(z, i\tau)$$
(12)

$$= \frac{1}{2}\frac{\exp\left(-z\sqrt{2s/((\sigma_n^2 + \sigma_m^2)\tau^2)}\right)}{\sqrt{s(\sigma_n^2 + \sigma_m^2)\tau^2/2}}.$$
(13)

The derivation from (12) to (13) is similar to the proof of Lemma 2 in [13]; hence, it is omitted here. Then, according to (11), there holds

$$(\mathcal{Z}G)(z,s) = \frac{(\mathcal{Z}g)(z',s)}{(\mathcal{Z}g)(z' - z, s)}$$
$$= \exp\left(-z\sqrt{2s/((\sigma_n^2 + \sigma_m^2)\tau^2)}\right).$$

Finally, using the inverse Z-transform operation, it is straightforward to obtain that, for $\mu_n = \mu_m$

$$G^{nm}_{\zeta\eta}(z,i) = \frac{z}{i\sqrt{2\pi i \tau^2 (\sigma_n^2 + \sigma_m^2)}} \exp\left(\frac{-z^2}{2i\tau^2(\sigma_n^2 + \sigma_m^2)}\right).$$
(14)

When $\mu_n \neq \mu_m$, we use a method different from the complicated first passage analysis to calculate $G^{nm}_{\zeta\eta}(z,i)$ from $g^{nm}_{\zeta\eta}(z,t)$. Define $\mathcal{A}$ to be the event that the reduction in distance between H^n_ζ and P^m_η is smaller than z at time $(i-1)\tau$, with regard to time 0. Define $\mathcal{B}$ to be the event that the reduction in distance between H^n_ζ and P^m_η is larger than z at time $i\tau$, with regard to time 0. Then, for $\mu_n \neq \mu_m$ and $i \geq 2$

$$
\begin{aligned}
&G^{nm}_{\zeta\eta}(z,i)\\
&\leq \Pr(\mathcal{A})\Pr(\mathcal{B}|\mathcal{A}) \leq \Pr(\mathcal{A})\Pr(\mathcal{B})
\end{aligned}
$$
(15)

$$= \int_0^z g^{nm}_{\zeta\eta}(z_0, (i-1)\tau)\,\mathrm{d}z_0 \int_z^{\infty} g^{nm}_{\zeta\eta}(z_0, i\tau)\mathrm{d}z_0$$
(16)

$$= \frac{1}{2}\left(1 + \mathrm{erf}\left(\frac{z - \mu_{i-1}}{\sqrt{2\sigma_{i-1}^2}}\right)\right)\frac{1}{2}\left(1 - \mathrm{erf}\left(\frac{z - \hat{\mu}}{\sqrt{2\sigma_i^2}}\right)\right)$$
(17)

where $\hat{\mu} = \mu_n i\tau - \mu_m i\tau$, $\sigma_i^2 = (\sigma_n^2 + \sigma_m^2)\tau^2 i$. The first inequality is due to the fact that we do not consider the first passage phenomenon. Hence, $\Pr(\mathcal{A})\Pr(\mathcal{B}|\mathcal{A})$ is larger than or equal to the probability that the reduction in distance between H^n_ζ and P^m_η reaches z *for the first time* in the ith time slot. When $\mu_n \neq \mu_m$, $g^{nm}_{\zeta\eta}(z,t)$ becomes a Gaussian function with a nonzero mean according to Lemma 2. It follows that the mean value of the distance between H^n_ζ and P^m_η is either a decreasing (when $\mu_n < \mu_m$) or an increasing (when $\mu_n > \mu_m$) function of t. Therefore, this bound is tight when the difference between μ_n and μ_m is large, which is usually satisfied in practical situations that different traffic streams are formed by vehicles traveling in fast and slow lanes or even the vehicles stopped at roadside. Second, the inequality $\Pr(\mathcal{B}|\mathcal{A}) \leq \Pr(\mathcal{B})$ in (15) is due to the observation that $\mathcal{A}$ and $\mathcal{B}$ are negatively correlated. Events $\mathcal{A}$ and $\mathcal{B}$ are negatively correlated because conditioned on the occurrence of event $\mathcal{A}$, that the reduction in distance is less than z at time $(i-1)\tau$, the occurrence of event $\mathcal{B}$, that the reduction in distance is larger than z at time $i\tau$, is less likely due to the bounded value of vehicular speed.

Furthermore, when $\mu_n \neq \mu_m$ and $i = 1$, it is straightforward that

$$G^{nm}_{\zeta\eta}(z,i) \leq \Pr(\mathcal{B}) = \frac{1}{2}\left(1 - \mathrm{erf}\left(\frac{z - \hat{\mu}}{\sqrt{2\sigma_i^2}}\right)\right).$$
(18)

∎

The results on $G^{nm}_{\zeta\eta}(z,i)$ are important steps in the analysis of the catch-up delay and will be used in the following subsections.

C. Pseudo Catch-Up Events

Another major challenge in the analysis of the catch-up process in a VANET with multiple traffic streams is the study of the *pseudo catch-up event*, denoted by Ψ. The pseudo catch-up event occurs when a packet is forwarded from a head (e.g., H_0) to a tail (P_0) that is in another traffic stream different from the head, but H_0 overtakes P_0 later (and all other informed vehicles) and becomes the head again, owing to the mobility of
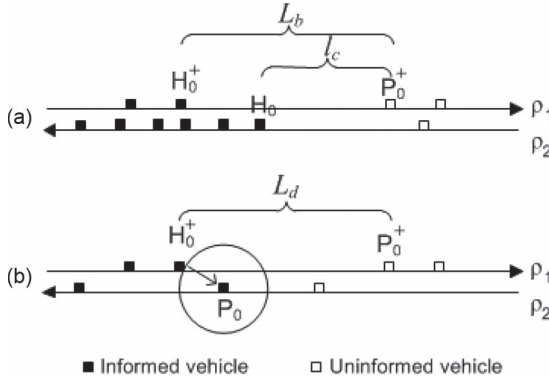
Fig. 4. Illustration of a pseudo catch-up event in a VANET with two traffic streams in opposite directions. (a) Suppose that, at time 0, the distance between the head (H_0) and the tail (P_0^+) is l_c and the distance between H_0^+ and P_0^+ is L_b. (b) Suppose that, at time $t > 0$, the distance between H_0^+ and P_0^+ becomes L_d. Furthermore, suppose that the information is forwarded from H_0^+ to P_0 at time t. However, due to the higher speed of H_0^+ compared with P_0, H_0^+ may overtake P_0 later and become the head again, resulting in a pseudo catch-up event.

vehicles. Such event often occurs when the head and the tail are in two traffic streams with very different mean vehicular speeds, e.g., two traffic streams in opposite directions.

An example of the pseudo catch-up event is illustrated in Fig. 4. Suppose that the distance between the head (H_0) and the tail (P_0^+) is l_c at time 0, as shown in Fig. 4(a). After some time, the information is forwarded from H_0^+ to an uninformed vehicle P_0, as shown in Fig. 4(b), i.e., a catch-up event occurs. However, due to the higher speed of H_0^+ compared with P_0, H_0^+ may overtake P_0 later and all vehicles in the cluster containing P_0 and become the head again. In this case, the previous catch-up event becomes a pseudo catch-up event, in the sense that the previous catch-up does not advance the propagation of the information. The pseudo catch-up event was first analyzed in detail in [8] and has been ignored in a number of previous works (e.g., [5] and [13]). Indeed, as shown later, under certain conditions, the pseudo catch-up event can have significant impact on the information propagation process. This paper quantitatively characterizes the impact of the pseudo catch-up events on the IPS and provides a more accurate estimate on the IPS, as shown in Section VIII.

Next, we present an analytical result on the probability that a pseudo catch-up event occurs. For simplicity, hereafter, a catch-up process, where the initial distance between the head H_0 and the tail P_0 is l_c at time 0, is referred to as a *catch-up process with gap l_c*. Denote by $f_c(l_c)$ the pdf of l_c. Because of the Poisson distribution of vehicles, it is straightforward that, for $l_c > r_0$

$$f_c(l_c) = \frac{\rho \exp(-\rho l_c)}{\int_{r_0}^{\infty} \rho \exp(-\rho l) dl} = \rho \exp\left(-\rho(l_c - r_0)\right). \quad (19)$$

Lemma 4: Denote by $\Pr(\Psi)$ the probability that a pseudo catch-up event occurs. There holds

$$\Pr(\Psi) \leq \int_{r_0}^{\infty} f_b(l_b) \Pr\left(X\left(\rho - \rho_{n_{\max}}\right) \leq l_b - 2r_0\right) dl_b \quad (20)$$

where $n_{\max} \triangleq \arg\max_n\{\mu_n\}$, $f_b(.)$ is given by (23), and $\Pr(X(\rho) \leq x)$ is given by (2).

Proof: To characterize pseudo catch-up events, we first consider the distance between the head and the tail. Consider a catch-up process with gap l_c, i.e., the Euclidean distance between the head and the tail is l_c at the beginning of the catch-up process. Denote by H_0^+ (resp. P_0^+) the rightmost informed (resp. leftmost uninformed) vehicle in the $n_{\max}^{th}$ traffic stream at the beginning of a catch-up process, where the $n_{\max}^{th}$ traffic stream is the traffic stream with the largest mean speed, i.e., $n_{\max} = \arg\max_n\{\mu_n\}$. As illustrated in Fig. 4, denote by L_b (resp. L_d) the Euclidean distance between H_0^+ and P_0^+ at the beginning (resp. at time $t > 0$) of a catch-up process. Suppose that, at time $t > 0$, a message is forwarded from H_0^+ to P_0. If the length of the cluster following P_0 is shorter than $L_d - 2r_0$, then the information cannot be forwarded to P_0^+ via multihop forwarding. Furthermore, as mentioned earlier, H_0^+ and P_0 are in different traffic streams and have a nonzero mean relative speed; hence, H_0^+ may overtake P_0 and all vehicles in cluster containing P_0 and become the head again. Therefore, a sufficient condition, which gives an upper bound on $\Pr(\Psi)$, for a catch-up process to be successful is that the cluster containing P_0 is long enough to bridge the gap L_d, such that the catch-up process moves the message from H_0^+ to P_0^+.

Furthermore, there holds $L_b \succeq L_d$ in stochastic ordering,[1] due to the fact that the distance between H_0^+ and P_0^+ tends to reduce over time, because vehicles with fast (resp. slow) speed may overtake other vehicles and become the new head H_0^+ (resp. new tail P_0^+) after a certain time period. In view of the analytical tractability of L_b, the following analysis considers L_b, instead of L_d, to obtain an upper bound on $\Pr(\Psi)$.

Define $f_l(l_b|l_c)$ as the pdf of L_b given a catch-up process with gap l_c. Because the distribution of vehicles in each traffic stream follows a Poisson process independent of other traffic streams, there holds

$$f_l(l_b|l_c) = \rho_{n_{\max}} \exp\left(-\rho_{n_{\max}}(l_b - l_c)\right). \quad (21)$$

Then, using the total probability theorem, there holds

$$
\begin{aligned}
&f_l(l_b) \\
&= \int_{r_0}^{l_b} f_l(l_b|l_c) \frac{\rho \exp(-\rho l_c)}{\int_{r_0}^{l_b} \rho \exp(-\rho l) dl} dl_c \\
&= \int_{r_0}^{l_b} \rho_{n_{\max}} \exp\left(-\rho_{n_{\max}}(l_b - l_c)\right) \frac{\rho \exp(-\rho l_c)}{\int_{r_0}^{l_b} \rho \exp(-\rho l) dl} dl_c \\
&= \frac{\rho_{n_{\max}} \rho \exp\left(-\rho_{n_{\max}} l_b\right)}{\exp(-\rho r_0) - \exp(-\rho l_b)} \int_{r_0}^{l_b} \exp(\rho_{n_{\max}} l_c - \rho l_c) dl_c \\
&= \frac{\rho \rho_{n_{\max}}}{\rho - \rho_{n_{\max}}} \frac{\exp\left(-\rho_{n_{\max}} l_b + (\rho_{n_{\max}} - \rho) r_0\right) - \exp(-\rho l_b)}{\exp(-\rho r_0) - \exp(-\rho l_b)}.
\end{aligned}
\tag{22}
$$

---

[1] Using stochastic ordering, we say $L_b \succeq L_d$ iff $\Pr(L_b > c) \geq \Pr(L_b > c)$ for any number $c \in (-\infty, \infty)$.
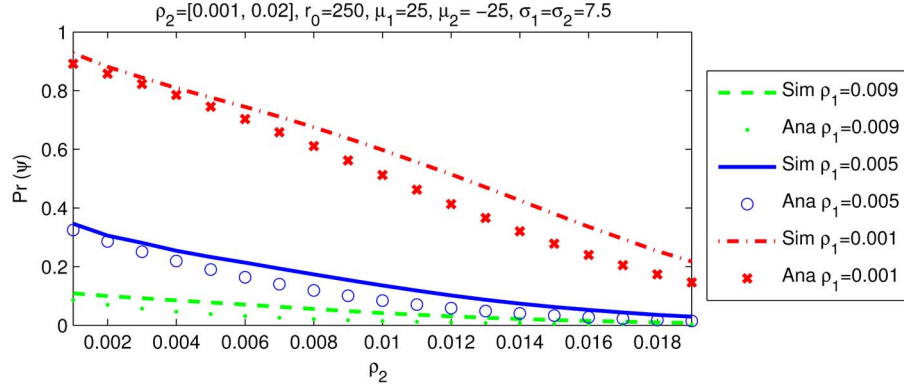
Fig. 5. Simulation (Sim) and analytical (Ana) results on the probability of pseudo catch-up events, viz., $\Pr(\Psi)$, in a VANET with two traffic streams. Detailed simulation parameters are introduced in Section VIII.

Define $f_b(l_b)$ as the pdf of l_b. Then, through normalization, there holds

$$f_b(l_b) = \frac{f_l(l_b)}{\int\limits_{r_0}^{\infty} f_l(l_b)\mathrm{d}l_b}. \tag{23}$$

Given the gap l_b, we next study the pseudo catch-up event. As discussed at the beginning of this lemma, to obtain an upper bound on $\Pr(\Psi)$, we consider that the pseudo catch-up event occurs if the cluster containing P_0 is shorter than $l_b - 2r_0$, which happens with probability $\Pr(X(\rho - \rho_{n_{\max}}) \le l_b - 2r_0)$, where $\Pr(X(\rho) \le x)$ is the cdf of the cluster length given by (2).

Then, the probability that a pseudo catch-up event occurs satisfies

$$\Pr(\Psi) \le \int\limits_{r_0}^{\infty} f_b(l_b) \Pr\left(X\left(\rho - \rho_{n_{\max}}\right) \le l_b - 2r_0\right) \mathrm{d}l_b.$$

■

Fig. 5 shows the probability of pseudo catch-up events, viz., $\Pr(\Psi)$. It can be seen that the probability of pseudo catch-up events is affected by the vehicular densities in both traffic streams. When the density of traffic stream $n_{\max}$ is small (e.g., 0.001 veh/m), a large percentage of catch-up processes are pseudo catch-up events. On the other hand, when the density of the traffic stream $n_{\max}$ is large (e.g., 0.01 veh/m), another traffic stream with a small density (e.g., 0.001 veh/m) can significantly help the catch-up process, in the sense of bridging the gap in the $n_{\max}^{th}$ traffic stream. Note that the analysis provides a valid and fairly tight bound on $\Pr(\Psi)$.

D. Catch-Up Process Between H_ζ and P_η

Recall that H_ζ (resp. P_ζ) is the ζth vehicle to the left of the head H_0 (resp. to the right of the tail P_0) at the beginning of a catch-up process (viz., at time 0). It is worth noting that the head vehicle at time $t > 0$ is not necessarily the head vehicle at time 0, because the previous head vehicle may be overtaken by another informed vehicle (either in the same traffic stream or in a different one) during time $(0, t]$. In view of this, we

study the catch-up process between vehicles H_ζ and P_η in this subsection.

Lemma 5: Denote by $q_{\zeta\eta}(i|l_c)$ the probability that H_ζ catches up P_η (where ζ and η are nonnegative integers) for the first time in the ith time slot, in a catch-up process with gap l_c. Then, there holds

$$q_{\zeta\eta}(i|l_c)$$

$$\approx \sum_{n=m\in\{1,\dots,N\}} \mathcal{G}_{\zeta\eta}^{nm}(z_{nm}, i)\frac{\rho_n \rho_m}{\rho^2}$$

$$+ \sum_{n,m\in\{1,\dots,N\}\,\text{for}\,n\ne m} \mathcal{G}_{\zeta\eta}^{nm}(z_{nm}, i)\frac{\rho_n \rho_m}{\rho^2}$$

$$\times \int\limits_{l_c}^{\infty} [1 - \Pr\left(X(\rho - \rho_{n_{\max}}) \le l_b - 2r_0\right)] f_l(l_b|l_c, \rho_{n_{\max}})\mathrm{d}l_b$$

$$\tag{24}$$

where $n_{\max} \stackrel{\Delta}{=} \arg\max_n\{\mu_n\}$, $z_{nm} = l_c - r_0 + \zeta/\rho_n + \eta/\rho_m$, $f_b(l_b|l_c, \rho_{n_{\max}}) = \rho_{n_{\max}}\exp(-\rho_{n_{\max}}(l_b - l_c))$, $\Pr(X(\rho) \le x)$ is given by (2), and $\mathcal{G}_{\zeta\eta}^{nm}$ is given by (25).

Proof: Denote by ϕ_{nm} the probability that a randomly chosen pair of vehicles belong to the nth and the mth traffic stream, respectively. It is straightforward that $\phi_{nm} = \rho_n \rho_m/\rho^2$, owing to the Poisson distribution of vehicles. Furthermore, due to the effect of pseudo catch-up processes studied in the previous subsection, the probability that the head and tail vehicles of a catch-up process belong to the nth and the mth traffic stream, respectively, can be different from $\phi_{nm} = \rho_n \rho_m/\rho^2$. This is one of the major challenges in the study of the catch-up process in a VANET with multiple traffic streams. In the following, we take this effect into account.

Let $z_{nm} = l_c - r_0 + w_\zeta^n + w_\eta^m$, where w_ζ^n (resp. w_η^m) is the distance between H_ζ^n and H_0 (resp. P_η^m and P_0) at time 0. Due to Poisson distribution of vehicles, the intervehicle distance in the nth traffic stream follows an exponential distribution with mean $1/\rho_n$. Denote by $f(w_\zeta^n)$ the pdf of w_ζ^n, then $f(w_\zeta^n) = \mathrm{Erlang}(\zeta, \rho_n)$ is an Erlang function because the sum of ζ independent exponentially distributed random variables follows an Erlang distribution. Denote by $\mathcal{G}_{\zeta\eta}^{nm}(i|l_c)$ the probability that

H_ζ^n catches up P_η^m for the first time in the ith time slot, in a catch-up process with gap l_c. Then it is straightforward that

$$\mathcal{G}_{\zeta\eta}^{nm}(i|l_c) = \int_0^\infty \int_0^\infty G_{\zeta\eta}^{nm}(z_{nm}, i) f\left(w_\zeta^n\right) f\left(w_\eta^m\right) \mathrm{d}w_\zeta^n \mathrm{d}w_\eta^m \tag{25}$$

where $G_{\zeta\eta}^{nm}(z_{nm}, i)$ is given by Lemma 3, which is the probability that the distance between H_ζ^n and P_η^m reduces from z_{nm} to r_0 for the first time in the ith time slot. Furthermore, denote by $\psi(l_c)$ the probability that a catch-up event between H_ζ and P_η is *not* a pseudo catch-up event, in a catch-up process with gap l_c. Then, according to the total probability theorem, it is evident that

$$q_{\zeta\eta}(i|l_c) = \sum_{n,m\in\{1,\dots,N\}} \mathcal{G}_{\zeta\eta}^{nm}(i|l_c)\psi(l_c)\phi_{nm}. \tag{26}$$

Note that $\psi(l_c)$ can be seen as a weighting factor for ϕ_{nm} that addresses the bias of ϕ_{nm} introduced at the beginning of this proof. Next, we calculate $\psi(l_c)$. To take the effect of pseudo catch-up events into account, without making the calculation too complicated, we approximately consider that a catch-up event between two vehicles is a pseudo catch-up event if these vehicles are in different traffic streams and the cluster containing the tail is shorter than $L_b - 2r_0$, as shown in Fig. 4. The accuracy of this approximation has been discussed in the previous subsection and verified by simulations. Then, (26) becomes

$$q_{\zeta\eta}(i|l_c) \approx \sum_{n=m\in\{1,\dots,N\}} \mathcal{G}_{\zeta\eta}^{nm}(i|l_c)\phi_{nm}$$
$$+ \sum_{n,m\in\{1,\dots,N\} \text{ for } n\neq m} \mathcal{G}_{\zeta\eta}^{nm}(i|l_c)\phi_{nm}$$
$$\times \int_{l_c}^\infty \left[1 - \Pr\left(X(\rho - \rho_{n_{\max}}) \leq l_b - 2r_0\right)\right]$$
$$\times f_b(l_b|l_c, \rho_{n_{\max}})\mathrm{d}l_b \tag{27}$$

where $f_b(l_b|l_c, \rho_{n_{\max}})$ is the pdf of L_b given that the distance between the head and the tail at time 0 is l_c. Because the vehicles in each traffic stream follow a homogeneous Poisson distribution independent of other traffic streams, there holds

$$f_b(l_b|l_c, \rho_{n_{\max}}) = \rho_{n_{\max}} \exp\left(-\rho_{n_{\max}}(l_b - l_c)\right). \tag{28}$$

E. Delay of a Catch-Up Process

Define the *catch-up delay* t_c to be the time interval from the beginning of a catch-up process until the time when the head and the tail move into the radio range of each other for the first time, i.e., $t_2 - t_1$ in Fig. 1. This subsection studies the catch-up delay. Note that we do not consider the rare event that the distance between the head and the tail becomes larger than r_0 again during the transmission between head and tail (from time t_c to $t_c + \beta$) due to changes of vehicular speeds, which may

cause the transmission to be interrupted, because the per-hop delay β (e.g., 4 ms [6]) is usually much smaller than the time interval for a vehicle to change speed (typically longer than a second [21]).

Denote by $H(i|l_c)$ the probability that none of the $H_\zeta - P_\eta$ vehicle pairs for $\zeta, \eta \in \{0, 1, 2, \dots\}$ catches up in the ith time slot, in a catch-up process with gap l_c. We next derive $H(i|l_c)$. Denote by $\Xi_{\zeta\eta}$ the event that a pair of vehicles $H_\zeta - P_\eta$ does *not* catch up in the ith time slot. It is worth noting that the events $\Xi_{\zeta\eta}$, for $\zeta, \eta \in \{0, 1, 2, \dots\}$, are *not* independent of each other. Specifically, the event $\Xi_{0\eta}$ is positively correlated [22, Sec. 1.2] with events $\Xi_{\zeta\eta}$, for any given $\eta \in \{0, 1, 2, \dots\}$ and all $\zeta \in \{1, 2, \dots\}$, because the occurrence of event $\Xi_{0\eta}$ means that the occurrences of events $\Xi_{\zeta\eta}$ are more likely. In other words, if the rightmost informed vehicle, viz., the head H_0, as shown in Fig. 3, does not catch up to an uninformed vehicle, other things being equal, then other informed vehicles are less likely to catch up to the uninformed vehicle. Similarly, the event $\Xi_{\zeta 0}$ is positively correlated with events $\Xi_{\zeta\eta}$, for any given $\zeta \in \{0, 1, 2, \dots\}$ and all $\eta \in \{1, 2, \dots\}$, because if the leftmost uninformed vehicle, viz., P_0, was not caught up by an informed vehicle, then other uninformed vehicles are less likely to be caught up by the informed vehicle. Then, according to the Fortuin–Kasteleyn–Ginibre inequality [22, Sec. 1.2], there holds

$$H(i|l_c) = \Pr\left(\bigcap_{\zeta,\eta\in\{0,1,2,\dots\}} \Xi_{\zeta\eta}\right)$$
$$\geq \prod_{\zeta,\eta\in\{0,1,2,\dots\}} (1 - q_{\zeta\eta}(i|l_c)) \tag{29}$$

where $q_{\zeta\eta}(i|l_c)$ is given by Lemma 5. Furthermore, the aforementioned inequality is tight in VANETs with low density, where the distance between vehicle pair $H_\zeta - P_\eta$ increases rapidly as either ζ or η increases, so that the effect of the aforementioned correlation issue reduces rapidly as either ζ or η increases. This is exactly the situation where this analysis (on the catch-up process) focuses on.

Denote by $h(i|l_c)$ the probability that one vehicle pair (H_ζ and P_η) catches up in the ith time slot and no vehicle pair has caught up before the ith time slot, in a catch-up process with gap l_c. Assume that the catch-up event in the ith time slot is independent of those in the jth time slots for $0 < j < i$, which is an accurate approximation when the duration of a time slot is large, e.g., $\tau = 1$ s or 5 s, as shown in Section VIII. Then

$$h(i_c|l_c) = (1 - H(i_c|l_c)) \prod_{i=1}^{i_c - 1} H(i|l_c). \tag{30}$$

Finally, the expected delay of a catch-up process with gap l_c is $\mathbb{E}[t_c|l_c] = \sum_{i=1}^\infty i\tau h(i|l_c)$.

VI. INFORMATION PROPAGATION SPEED

In a VANET where vehicles follow a homogeneous Poisson distribution, the information propagation process can be considered as a renewal reward process [5]. Each cycle in
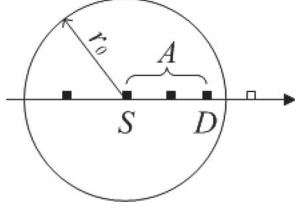
Fig. 6. Illustration of the per-hop progress, marked by A, which is the distance between a node (e.g., S) and the rightmost node in the radio range r_0 of S, which is node D.

the renewal reward process starts at the time when a catch-up process starts; consequently, each cycle consists of a catch-up process followed by a forwarding process. The reward of each cycle is the distance traveled by a packet in the catch-up process and the forwarding process during the cycle. Then, according to [23, Ch. 7.4], the expected IPS ($\mathbb{E}[v_{ip}]$) is

$$\mathbb{E}[v_{ip}]$$

$$= \frac{\text{expected length of one cycle}}{\text{expected time duration of one cycle}}$$

$$\approx \frac{\mathbb{E}[l_c] + \mathbb{E}[X(\rho)]}{\mathbb{E}[t_c] + \beta + \mathbb{E}[t_f]} + \mu_{n_{\max}}$$

$$= \frac{\int_{r_0}^{\infty} l_c f_c(l_c)\mathrm{d}l_c + \int_0^{\infty} x \Pr(X(\rho)=x)\,\mathrm{d}x}{\int_{r_0}^{\infty} \mathbb{E}[t_c|l_c] f_c(l_c)\mathrm{d}l_c + \beta + \int_0^{\infty} \mathbb{E}[t_f|x] \Pr(X(\rho)=x)\,\mathrm{d}x}$$
$$+ \mu_{n_{\max}} \qquad (31)$$

where the term $\mathbb{E}[l_c] + \mathbb{E}[X(\rho)]$ is the expected distance traveled by a packet during a catch-up process and a forwarding process, without considering the distance traveled when the packet is carried by the head vehicle during the catch-up process. The contribution of the movement of the head vehicle to the IPS is considered separately by the term $\mu_{n_{\max}}$. Furthermore, the term $\mathbb{E}[t_c] + \beta + \mathbb{E}[t_f]$ is the expected time duration of a catch-up process and a forwarding process.

VII. SIMPLIFIED RESULTS CHARACTERIZING THE INFORMATION PROPAGATION PROCESS

In the previous three sections, we study in detail the information propagation process of a VANET. To provide a more insightful understanding of the information propagation process, this section presents some simplified results to reveal more clearly the main interaction among the fundamental parameters that determine the performance of the information propagation process. Here, we first provide simplified results on the number of hops traveled by a packet in a cluster and the length of a cluster, followed by the results on the catch-up delay and IPS.

A. Simplified Hop Count

Denote by A (where $0 < A \leq r_0$) the *per-hop progress*, which is the Euclidean distance between a randomly chosen vehicle and its rightmost neighbor at a randomly chosen time instant, as illustrated in Fig. 6. Denote by Ψ_1 the event that there is no node in a given road segment with length

$r_0 - a$, where $0 < a \leq r_0$. Because vehicles follow a Poisson distribution with intensity ρ at any time instant, it is evident that $\Pr(\Psi_1) = \exp(-\rho(r_0 - a))$. Furthermore, denote by Ψ_2 the event that there is at least one node in a given road segment with length a. It is straightforward that $\Pr(\Psi_2) = 1 - \exp(-\rho a)$. Denote by C_r the event that a progress is made at a given node (e.g., S), i.e., there is at least one node to the right of node S within its radio range r_0, as illustrated in Fig. 6. Furthermore, denote by $\Pr(A \leq a, C_r)$ the probability that a progress is made at a given node and the progress is not larger than a. It is evident that $\Pr(A \leq a, C_r) = \Pr(\Psi_1) \Pr(\Psi_2)$. Finally, denote by $\Pr(A \leq a|C_r)$ the cdf of per-hop progress A, conditioned on the event that a progress is made. It is evident that $\Pr(A \leq a|C_r) = \Pr(A \leq a, C_r)/\Pr(C_r)$, where $\Pr(C_r) = 1 - \exp(-\rho r_0)$. Then, the expected per-hop progress is

$$\bar{a} \triangleq \mathbb{E}[A|C_r] = \int_0^{r_0} (1 - \Pr(A \leq a|C_r))\,\mathrm{d}a$$

$$= \int_0^{r_0} \left(1 - \frac{(1 - \exp(-\rho a)) \exp(-\rho(r_0 - a))}{1 - \exp(-\rho r_0)} \right) \mathrm{d}a$$

$$= r_0 - \frac{1}{\rho} + \frac{r_0 \exp(-\rho r_0)}{1 - \exp(-\rho r_0)}. \qquad (32)$$

Furthermore, it is straightforward that the expected number of hops from the leftmost vehicle to the rightmost vehicle in a cluster with length x can be approximated by

$$\mathbb{E}[k|x] \approx \frac{x}{\bar{a}}. \qquad (33)$$

Fig. 7(a) shows the expected number of hops from the leftmost vehicle to the rightmost vehicle in clusters with different lengths. The exact analytical result (denoted by *Ana*) is given by [18], and the approximate analytical result (denoted by *Ana-simple*) is given by (33). It is interesting to observe that (33), which is much simpler than the recursive formula presented in [18], provides a fairly accurate estimate on the hop count in VANETs. The accuracy of the approximation reduces as the cluster length increases (e.g., longer than 6000 m); however, as shown in Fig. 7(b) and studied in the next subsection, the probability of forming a long cluster decreases exponentially.

B. Simplified Cluster Length

In the previous subsection, we have shown that the expected per-hop progress $\bar{a}$ is a performance-determining factor, which can be used to characterize the forwarding process. Next, we consider that a multihop forwarding process can continue as long as there is at least one vehicle within distance $\bar{a}$ to the right of an arbitrary vehicle, which happens with probability $p_s = 1 - \exp(-\rho \bar{a})$, because of the Poisson distribution of vehicles. Then, the number of successful multihop forwarding progresses, denoted by κ, follows a geometric distribution with the following pdf:

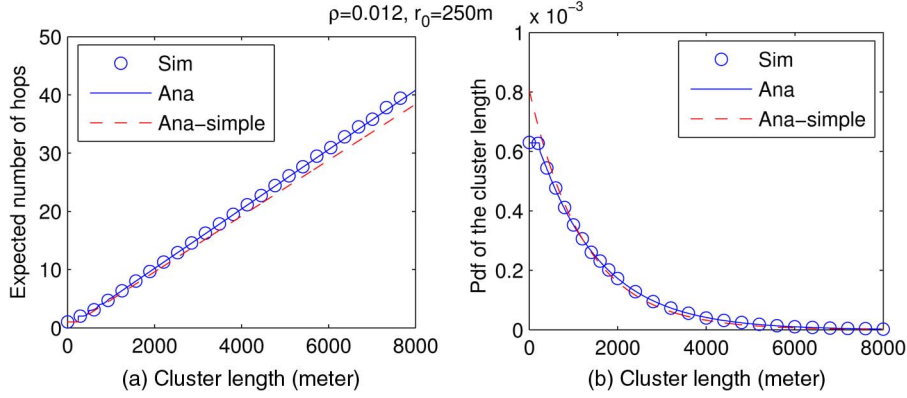$$f_{\kappa}(\kappa) = (1 - p_s) p_s^{\kappa-1}. \qquad (34)$$

Fig. 7. Simulation (Sim) and analytical (Ana) results for (a) the expected number of hops from the leftmost vehicle to the rightmost vehicle in a cluster and (b) the pdf of the cluster length.

Then, the expected time delay of a forwarding process is

$$\mathbb{E}[t_f] \approx \beta \mathbb{E}[\kappa] = \frac{\beta}{1 - p_s} = \frac{\beta}{\exp(-\rho\bar{a})}. \quad (35)$$

Furthermore, the expected cluster length in a VANET with vehicular density ρ is

$$\mathbb{E}[X(\rho)] \approx \bar{a}\mathbb{E}[\kappa] = \frac{\bar{a}}{1 - p_s} = \frac{\bar{a}}{\exp(-\rho\bar{a})}. \quad (36)$$

Next, we present an approximation of the distribution of cluster length. Note that κ is a discrete random variable, whereas the cluster length x is a continuous random variable. To facilitate the analysis, let κ_c be the continuous analog of κ, where κ_c follows an exponential distribution with parameter p_c. Then, the probability of having at least κ consecutive successful progresses can be expressed as $p_s^\kappa = \exp(-p_c\kappa)$, from which we can obtain $p_c = -\ln(p_s)$. Therefore, the pdf of cluster length can be approximated by

$$\Pr(X(\rho) = x)$$
$$\approx \frac{p_c \exp\left(-p_c\frac{x}{\bar{a}}\right)}{\int_0^\infty p_c \exp\left(-p_c\frac{x_0}{\bar{a}}\right) dx_0}$$
$$= \frac{p_c \exp\left(-p_c\frac{x}{\bar{a}}\right)}{\bar{a}}$$
$$= -\frac{\ln\left(1 - \exp(-\rho\bar{a})\right)}{\bar{a}} \exp\left(\frac{x\ln\left(1 - \exp(-\rho\bar{a})\right)}{\bar{a}}\right). \quad (37)$$

Furthermore, its cdf is

$$\Pr(X(\rho) \leq x) \approx \frac{1 - \exp\left(-\frac{p_c x}{\bar{a}}\right)}{\bar{a}}$$
$$= \frac{1 - \exp\left(\frac{\ln(1-\exp(-\rho\bar{a}))}{\bar{a}}x\right)}{\bar{a}}. \quad (38)$$

Fig. 7(b) shows the pdf of cluster length. The exact analytical result is given by (1), and the approximate analytical result is given by (37). It is interesting to note that the accuracy of the approximation is good, while at the same time, the analytical

formula of the approximate result is much simpler than that for the exact result.

C. Simplified Catch-Up Delay

Recall that the $n_{\max}^{th}$ traffic stream is the traffic stream with the largest mean speed, i.e., $n_{\max} = \arg\max_n\{\mu_n\}$. Furthermore, let the $n_{\min}^{th}$ traffic stream be the traffic stream with the smallest mean speed, i.e., $n_{\min} = \arg\min_n\{\mu_n\}$. Define t_ζ to be the expected time interval from the beginning of an arbitrary catch-up process to the time when a vehicle in traffic streams $n \in \{1, \dots, N\} \setminus n_{\max}$ comes into the radio range of a vehicle in traffic stream $n_{\max}$. Then, it is straightforward that $t_m \approx D_e(\rho - \rho_{n_{\max}})/\mu_{n_{\max}} - \mu_{n_{\min}}$, where $D_e(\rho)$ is the expected distance between two adjacent but disconnected vehicles. Because vehicles follow a homogeneous Poisson distribution with intensity ρ, there holds

$$D_e(\rho) = \frac{\int_{r_0}^\infty l_c\rho\exp(-\rho l_c)dl_c}{\int_{r_0}^\infty \rho\exp(-\rho l_c)dl_c}$$
$$= \frac{\exp(-\rho r_0)(\rho r_0 + 1)}{\exp(-\rho r_0)\rho}$$
$$= \frac{(\rho r_0 + 1)}{\rho}. \quad (39)$$

Next, we take the pseudo catch-up events into consideration. Define Υ as the event that a vehicle in traffic streams $n \in \{1, \dots, N\} \setminus n_{\max}$ comes into the radio range of a vehicle in traffic stream $n_{\max}$, in an arbitrary catch-up process. Recall that Υ is a pseudo catch-up event if the length of the cluster containing the tail vehicle is shorter than $L_b - 2r_0$, where L_b is the Euclidean distance between H_0^+ and P_0^+. Denote by p_p the probability that Υ is a pseudo catch-up event. Then

$$p_p = \mathbb{E}\left[\Pr\left(X(\rho - \rho_{n_{\max}}) \leq L_b - 2r_0\right]\right. \quad (40)$$
$$\approx \Pr\left(X(\rho - \rho_{n_{\max}}) \leq \mathbb{E}[L_b - 2r_0]\right) \quad (41)$$

where

$$\mathbb{E}[L_b - 2r_0] = \int_{2r_0}^\infty (l_b - 2r_0)\rho_{n_{\max}}\exp(-\rho_{n_{\max}} l_b)dl_b$$
$$= \frac{\exp(-\rho_{n_{\max}} 2r_0)}{\rho_{n_{\max}}}. \quad (42)$$

Recognizing that an approximation is involved, assume that the catch-up events are independent of each other; then, the number of pseudo catch-up events before a successful one follows a geometric distribution with mean $1/1 - p_p$. Therefore, the expected catch-up delay is

$$\mathbb{E}[t_c] \approx \frac{t_m}{1 - p_p} \approx \frac{(\rho - \rho_{n_{\max}})r_0 + 1}{(\mu_{n_{\max}} - \mu_{n_{\min}})(\rho - \rho_{n_{\max}})(1 - p_p)}. \quad (43)$$

D. Simplified IPS

According to (31) and the results obtained in the previous subsections, the expected IPS satisfies

$$
\begin{aligned}
\mathbb{E}[v_{ip}] &= \frac{\text{expected length of one cycle}}{\text{expected time duration of one cycle}} \\
&\approx \frac{r_0 + \mathbb{E}[X(\rho)]}{\mathbb{E}[t_c] + \mathbb{E}[t_f]} + \mu_{n_{\max}} \\
&\approx \frac{r_0 + \frac{\bar{a}}{\exp(-\rho\bar{a})}}{\mathbb{E}[t_c] + \frac{\beta}{\exp(-\rho\bar{a})}} + \mu_{n_{\max}} \\
&= \frac{r_0 + \frac{\rho r_0 - 1}{\rho\exp(1-\rho r_0)}}{\mathbb{E}[t_c] + \frac{\beta}{\exp(1-\rho r_0)}} + \mu_{n_{\max}} \quad (44)
\end{aligned}
$$

where $\mathbb{E}[t_c]$ is given by (43).

Note that the result is much simpler than that given by (31) and in [13], where the accuracy and applicability of the simplified result are discussed in the next section. Furthermore, the impact of fundamental parameters on the performance of the information propagation process is clearly shown. For example, a larger difference between mean vehicular speeds $[\mu_{n_{\max}} - \mu_{n_{\min}}$ in (43)] leads to a faster IPS. More observations dealing with the impact of the fundamental parameters are discussed in the next section.

VIII. SIMULATION RESULTS

Here, we report on simulations to validate the accuracy of the analytical results. The simulations are conducted in a VANET simulator written in C++. The simulator only simulates packet transmissions in the network layer, which allows us to separate the impacts of factors from other layers (e.g., modulation schemes or coding methods) on the information propagation process and to focus on the factors of interest in this paper, i.e., vehicular density, mobility, and radio range. Hence, the simulator can better validate the accuracy of the analytical results obtained in the paper, as compared with standard simulators such as OMNET or NS2/3. The VANET simulator is available on the web page [16]. Each point shown in the figures is the averaged value from 1000 simulations. The confidence interval is too small to be distinguishable and, hence, is ignored in the following plots. The radio range is $r_0 = 250$ m [5]. The typical values of the mean and standard deviation of vehicular speed are 25 and 7.5 m/s, respectively [15]. The vehicle mobility parameters, i.e., μ_n, σ_n, and τ, are taken from practical measurements, where the usual record time intervals for a vehicular speed monitor are $\tau = 1$ s, 5 s [24]. The traffic density is varied,
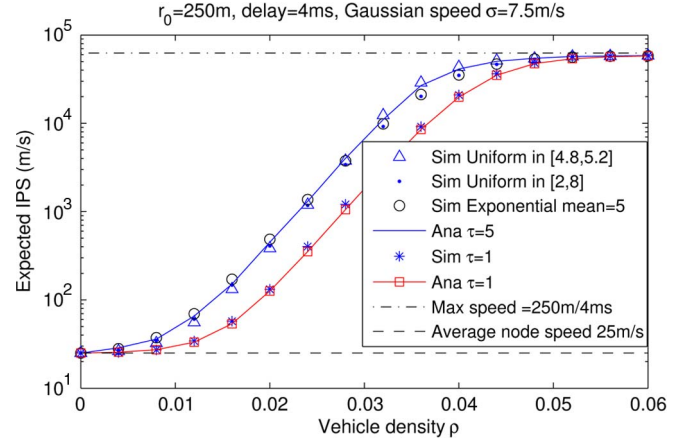

Fig. 8. Simulation (Sim) and analytical (Ana) results of the expected IPS in a VANET with one traffic stream ($N = 1$).

so that the intensity ρ of the homogeneous Poisson process (governing the spatial distribution of the vehicles) varies from 0 to 0.06 veh/m. For completeness of the plot, $\rho = 0$ is included, which means that there is only one vehicle in each traffic stream. Consequently, the average number of neighbors varies from 0 to 30, which represents a large range of traffic densities.

First, Fig. 8 validates our mobility model. The analytical results are given by (31), which is derived under the synchronized random walk model, whereas the simulation results show the expected IPS under nonsynchronized mobility models. Three different mobility models are evaluated: the speed-change time interval τ of each vehicle is uniformly selected from [4.8, 5.2] (i.e., around 5 s) or from [2, 8] (i.e., within a larger range around 5 s) and for τ following an exponential distribution with mean 5 s. Under all three mobility models, vehicles change speed at different time instants (nonsynchronized) and the average speed-change time interval is 5 s. It can be seen that the IPSs under nonsynchronized mobility models are very close (almost indistinguishable) to each other and our analysis using the simplified (synchronized) mobility model provides a good estimation on the IPS.

Moreover, it can be observed in Fig. 8 that, when the vehicular density is low, the IPS is close to vehicular moving speed because there is less packet forwarding in the network. Furthermore, when the vehicular density increases, small clusters are formed and the IPS increases. As the vehicular density further increases, clusters become larger and the forwarding process starts to dominate. Therefore, the IPS increases until it reaches the maximum value, which is determined by the per-hop delay in the forwarding process. The maximum IPS is obviously equal to r_0/β, where β is the per-hop delay. When the vehicular density is moderate, the IPS is determined by the catch-up delay, which is further determined by the mobility of the vehicles. It is shown in Fig. 8 that the more frequently the speed changes, the slower the information propagates. This is mainly because changing speed has the potential to interrupt the catch-up process, i.e., during a catch-up process, the tail may speed up and the head may slow down.

Moreover, vehicular speed distribution also has a significant impact on the catch-up process, hence, the IPS. Fig. 9 shows the IPS in a VANET with two traffic streams with equal vehicular
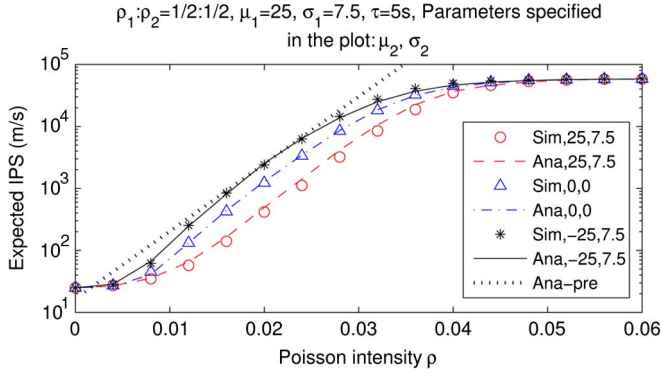
Fig. 9. Expected IPS in a VANET with different vehicular speeds in two traffic streams ($N = 2$). The plot *Ana-pre* shows the IPS derived in [8] for a VANET with vehicular speed 25 and -25 in two streams respectively. (a) Poisson intensity ρ. (b) Radio range (in meters).
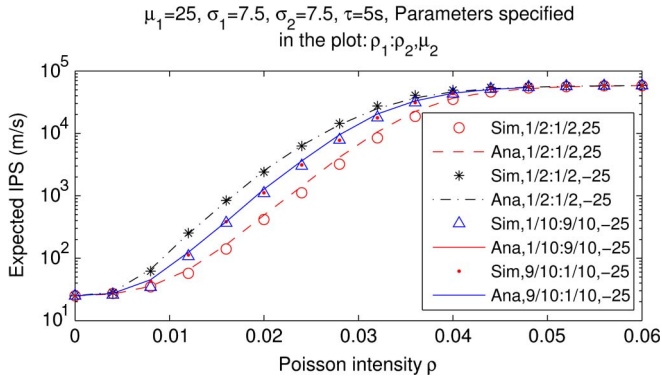

Fig. 10. Expected IPS in a VANET with different vehicular densities in two traffic streams. The first four curves are kept in Fig. 9 for comparison.

Fig. 11. Expected IPS in a VANET with four traffic streams.

density but different vehicular speed distributions. Specifically, the mean speed of one traffic stream is fixed at 25, and the mean speed of another traffic stream is varied. It can be seen that our analytical result has a better match with the simulation results than that in [8]. An interesting observation is that the IPS increases when the average speed of the vehicles in one of the traffic streams is reduced from 25 to 0 m/s. Furthermore, an even faster IPS is observed when the average speed of the vehicles in one of the traffic streams is -25 m/s, i.e., two traffic streams head in opposite directions. The reason behind this interesting observation is that a larger relative speed between vehicles results in faster catch-up processes, hence, a faster IPS. This can be also seen from the analytical results of $G_{\zeta\eta}^{nm}(z, i)$, e.g., applying the fact that the error function is an increasing function to (9). This observation tells us that making use of the vehicles traveling in the opposite direction of information propagation can increase the IPS. Moreover, some (stationary) roadside units without (expensive) wired connections can also significantly increase the IPS in a VANET. The optimum placement of the roadside units is not the focus of this paper. However, inspired by [25], we conjecture that equidistant placement could be the optimum solution.

Fig. 10 shows the expected IPS in a VANET with different vehicular densities in two traffic streams. Specifically, the total traffic density ρ is fixed, whereas the fractions of traffic density in two traffic streams are varied to examine the impact of traffic
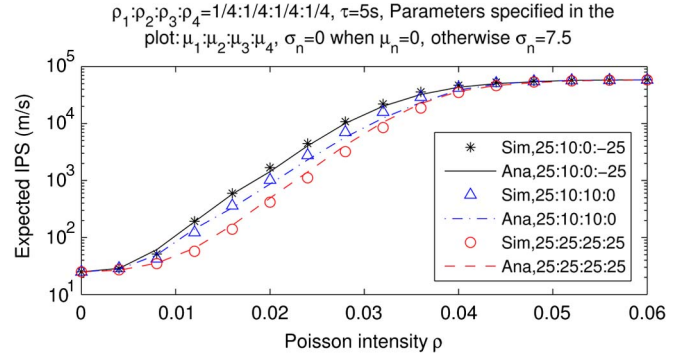
densities in different traffic streams on the IPS. First, it can be seen that an uneven distribution of the vehicular densities between traffic streams (e.g., $\rho_1 = \rho/10, \rho_2 = 9\rho/10$) results in a slower IPS compared with the IPS in a VANET with evenly distributed traffic densities (e.g., $\rho_1 = \rho_2 = \rho/2$). This is because an uneven distribution of the vehicular densities between traffic streams results in a smaller number of catch-ups between vehicles in different traffic streams [as manifested by the factor $\rho_n\rho_m/\rho^2$ in (24)]; hence, there is less improvement on the IPS by the large relative vehicular speed, as compared with a VANET with evenly distributed vehicular densities between traffic streams. This situation can be observed in the real world on freeways connecting the business and residential districts. Densities of the traffic streams in opposite directions vary a great deal, depending on the time of day when people go to work or come back home.

On the other hand, it is shown in Fig. 10 that a small amount (e.g., $\rho/10$) of vehicular traffic in an opposite direction can still significantly increase the IPS. Therefore, it is meaningful to exploit the traffic streams with different vehicular moving speed. This phenomenon can be also observed in Fig. 11, owing to the fact that large relative speed between vehicles reduces the catch-up delay, as discussed earlier. It is worth noting that the IPS can still be large in a VANET where most, or even all, vehicles are in the same traffic stream, which is different from previous studies (e.g., [7] and [8]), considering that all vehicles travel at the same time-invariant speed where the catch-up events can only occur between vehicles in different traffic streams.

Fig. 12 compares the results on IPS given in Section IV with the previous results of related studies. In Fig. 12(a), the radio range is 250 m, and the vehicular density is varied from 0 to 0.015, whereas in Fig. 12(b), the vehicular density is 0.036,[2] and the radio range is varied from 0 to 90 m. It can be seen that the results presented in this paper, which considers the effects of pseudo catch-up events in detail, have better accuracy compared with previous results such as [8] and [13]. Moreover, it can be also seen that the impact of pseudo catch-up events can be significant.

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[2]Consider a common case that the mean distance between two consecutive vehicles on the same lane is around 110 m [26], leading to a mean vehicle density of around 9 vehs/km/lane. Consider two lanes in each direction, leading to an overall mean vehicle density of around 36 vehs/km [26].
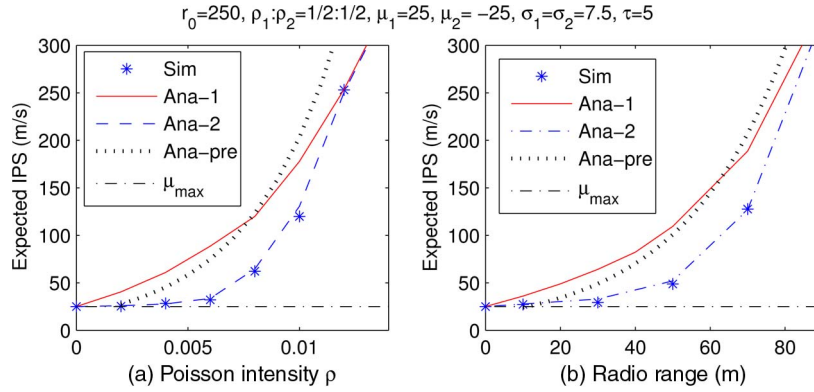
Fig. 12.   Expected IPS in a VANET with two traffic streams, where *Ana-1* is the previous result given in [13], *Ana-2* is the result given in this paper, and *Ana-pre* is the result given by [8].
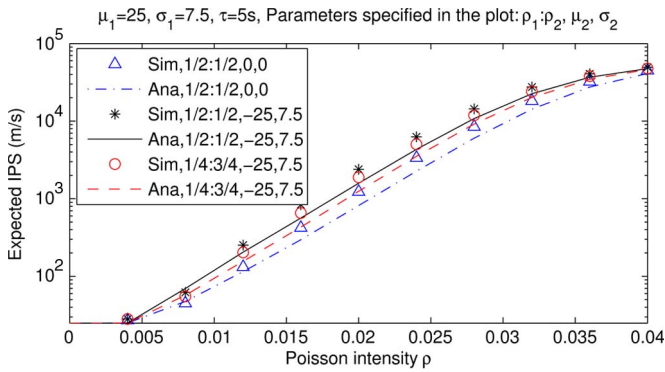

Fig. 13.   Simplified results of the expected IPS, where *Ana* is given by (44).

Fig. 13 shows the simplified results of the IPS, derived in Section VII. Without the detailed analysis of the catch-up process, as shown in Section V, the simplified results are only valid for VANETs with more than one traffic stream, i.e., vehicles have different mean speeds. Nevertheless, it can be seen that the simplified results, although do not match exactly with simulation results, capture the impact of fundamental parameters (e.g., vehicular density and speed) on the IPS.

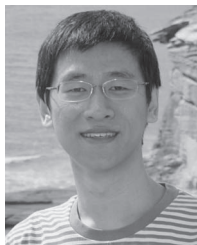## IX. CONCLUSION AND FUTURE WORK

In this paper, analytical studies have been presented for the information propagation process in a mobile wireless network formed by vehicles. Vehicles can be categorized into a number of traffic streams, where the mean vehicular speeds in different traffic streams are different. We found that the IPS can be boosted significantly by exploiting the existence of even a small number of vehicles traveling with a different mean speed, such as vehicles traveling in the opposite direction or heavy trucks moving in the same direction but with a slower speed. Through detailed analysis, comprehensive results that accurately calculate the IPS have been presented. Furthermore, simplified results have been also provided to reveal the inter-action among performance-determining parameters. By taking real-world measurements such as λ, μ_n, σ_n, and τ, our results can provide both an accurate and quick estimate of the IPS, which provides useful guidelines on the design of wireless communication networks for intelligent transportation systems.

The analysis conducted in this paper has been based on the unit disk communication model. In the future, the impact of channel randomness on the IPS can be investigated using the technique introduced in [27]. Furthermore, the analysis in this paper can be extended to 2-D VANETs, e.g., to consider the Manhattan topology [28]. Moreover, the distribution of vehicles in a VANET can be different from the homogeneous Poisson process considered in this paper, particularly in urban areas. In view of this, it is also necessary to study the information propagation process in a VANET with inhomogeneous vehicu-lar densities in the future.

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