On the Hop Count Statistics in Wireless Multi-hop Networks Subject to Fading

Zijie Zhang, *Student Member, IEEE,* Guoqiang Mao, *Senior Member, IEEE,* and Brian D. O. Anderson, *Life Fellow, IEEE*

Abstract—Consider a wireless multi-hop network where nodes are randomly distributed in a given area following a homogeneous Poisson process. The hop count statistics, viz the probabilities related to the number of hops between two nodes, are important for performance analysis of the multi-hop networks. In this paper, we provide analytical results on the probability that two nodes separated by a known Euclidean distance are k hops apart in networks subject to both shadowing and small-scale fading. Some interesting results are derived which have generic significance. For example, it is shown that the locations of nodes three or more hops away provide little information in determining the relationship of a node with other nodes in the network. This observation is useful for the design of distributed routing, localization and network security algorithms. As an illustration of the application of our results, we derive the effective energy consumption per successfully transmitted packet in end-to-end packet transmissions. We show that there exists an optimum transmission range which minimizes the effective energy consumption. The results provide useful guidelines on the design of a randomly deployed network in a more realistic radio environment.

Index Terms—hop count, log-normal shadowing, Nakagami-m fading, energy consumption, wireless multi-hop networks.

1 Introduction

A wireless multi-hop network consists of a group of nodes that communicate with each other over wireless channels. The nodes in such a network operate in a decentralized and self-organized manner and each node can act as a relay to forward information toward the destination. Wireless multi-hop networks have large potential in military and civilian applications [1], [2].

There are three related probabilities characterizing the connectivity properties of such a multi-hop network. These are the probability that an arbitrary node is k hops apart from another arbitrary node, denoted by $\Pr(k)$; the probability that a node at an Euclidean distance x apart from another node is connected to that node in exactly k hops, denoted by $\Pr(k|x)$; and the spatial distribution of the nodes k hops apart from a designated node, denoted by $\Pr(x|k)$. These three probabilities are related through Bayes' formula and if one is computable, the other two will be computable using similar techniques. Therefore we call these problems collectively the *hop count statistics* problems. **We refer readers to Section 9 in the supplementary material for an extensive discussion on the use of the three probabilities in various applications.**

A major technical obstacle in the analysis of hop counts statistics is the so-called *spatial dependence problem* [3], [4]. The spatial dependence problem arises because in a wireless multi-hop network the event that a randomly chosen node is k hops apart from a particular node is not independent of the event that another randomly chosen node is i hops apart from the same node for $i \leq k$. It follows that an accurate analysis on the conditional probability $\Pr(k|x)$ needs to consider all previous hops, which makes the analysis complicated. This technical hurdle caused by the spatial dependence problem is recognized in the literature but remains unsolved [4]. In this paper, a significant improvement on the accuracy of computing $\Pr(k|x)$ is achieved by considering the positions of previous two hop nodes, compared to the results considering the positions of previous one hop nodes only. Further, we show that considering the positions of previous two hops nodes is enough to provide an accurate estimate of $\Pr(k|x)$. A detailed explanation of the spatial dependence problem is given in Section 3.1.

Further, much previous research (e.g. [5]–[8]) establishes results based on the assumption that the network is *connected*, i.e. there is at least one path between every pair of nodes. However in many wireless multi-hop network applications, it is not only impractical (due to the randomness of node deployment, a complex radio environment, or the high node density required for a large scale network to be connected [9]) but also unnecessary to require every node to be connected to every other node. For example, in applications which are not life-critical, e.g. habitat monitoring or environmental mon-

- *Z. Zhang and G. Mao are with the School of Electrical and Information Engineering, University of Sydney. G. Mao is also with National ICT Australia, Sydney. E-mail: {zijie.zhang, guoqiang.mao}@sydney.edu.au.*
- *B. D. O. Anderson is with the Research School of Information Sciences and Engineering, Australian National University and National ICT Australia, Canberra. E-mail: brian.anderson@anu.edu.au.*

This work is partially supported by Australian Research Council (ARC) Discovery project DP110100538, and by the Air Force Research Laboratory, under agreement number FA2386-10-1-4102. The U.S. Government is authorized to reproduce and distribute reprints for Governmental purposes notwithstanding any copyright notation thereon. The views and conclusions contained herein are those of the authors and should not be interpreted as necessarily representing the official policies or endorsements, either expressed or implied, of the Air Force Research Laboratory or the U.S. Government.

itoring, having a few disconnected source-destination pairs will not cause statistically significant change in the monitored parameters [10]. In addition, there is a downside on the capacity to have a high node density required for a connected network. Gupta and Kumar [11] showed that in a connected network, as the number of nodes per unit area n increases, the throughput per source-destination pair decreases approximately as $1/\sqrt{n}$. It was further pointed out in [12] that significant energy savings can be achieved by requiring most nodes but not all nodes to be connected. Therefore we consider the more realistic and practical scenario that the network is not necessarily connected. Previous results established on the basis of a connected network actually form special cases of the problem examined in this paper.

The main contributions of this paper are:

- Firstly, in a network with nodes distributed in a finite area following a homogeneous Poisson process, we derive the probability that two nodes separated by a known Euclidean distance x are k hops apart, i.e. $\Pr(k|x)$, considering both shadowing and small-scale fading and using a distributed routing algorithm, i.e. greedy forwarding;
- Secondly, we analyze the impact of the spatial dependence problem on the accuracy of $\Pr(k|x)$;
- Thirdly, considering a sparse network in which there is not necessarily a path between any pair of nodes, we derive the probability distribution of the number of hops traversed by packets before being dropped if the transmission is unsuccessful. (An end-to-end transmission is *unsuccessful* if the packet sent from a source to a destination has to be dropped at an intermediate node because it is unable to find a next-hop node.)
- As an application of the results, we derive the effective energy consumption per successfully transmitted packet in end-to-end packet transmissions. We show that there exists an optimum transmission range which minimizes the effective energy consumption. Further, the impacts of unreliable link, node density and path loss exponent on the energy consumption is included in the analysis.

The results in this paper provide a more complete understanding on the properties of wireless multi-hop networks in a more realistic and practical setting.

The rest of this paper is organized as follows: Section 8, in the supplementary material, reviews the related work. Section 2 introduces the network models and some definitions. The analysis of the hop count statistics and the end-to-end energy consumption under the unit disk communication model is given in Section 3, followed by the analysis on the impact of the spatial dependence problem on the accuracy of the hop count statistics in Section 4. In Section 5, we further include the log-normal shadowing and small-scale fading in the analysis. The simulation results and discussions are given in Section 6. Finally Section 7 concludes this paper and proposes possible future work.

2 System Model

2.1 Network model

In this paper, we consider a wireless multi-hop network where nodes are identically and independently distributed (i.i.d.) in a square according to a homogeneous Poisson point process with a known intensity ρ.

We consider that every node has the same transmission power. The simplest radio propagation model is the unit disk communication model. Under the unit disk model, the power attenuates with the Euclidean distance x from a transmitter like $x^{-\eta}$, where η is the path loss exponent. The path loss exponent can vary from 2 in free space to 6 in urban areas [18]. The received signal strength (RSS) at a receiver separated by Euclidean distance x from the transmitter is $P_u(x) = CP_t x^{-\eta}$, where C is a constant, P_t is the transmission power. A transmission is successful iff the RSS exceeds a given threshold $P_{\min}$. Therefore the required transmission power P_t allowing a transmission range r_0 is $P_t = C_1 r_0^{\eta}$, where $C_1 = P_{\min}/C$.

The unit disk model is simple but unrealistic. In reality the RSS may have significant variations around the mean value, because of both large scale variation (i.e. shadowing) and small-scale fading. Considering a typical type of shadowing, i.e. the *log-normal shadowing model* [18] the RSS attenuation (in dB) follows a normal distribution with respect to the distance x between transmitter and receiver: $10\log_{10}(P_l(x)/CP_t x^{-\eta}) \sim Z$, where $P_l(x)$ is the RSS in the log-normal shadowing model and Z is a zero-mean Gaussian distributed random variable with standard deviation σ. When $\sigma = 0$ the model reduces to the unit disk model. In practice the value of σ is often computed from measured data and can be as large as 12 [18]. Denote by $q(z)$ the pdf (probability density function) of the shadowing fades; then: $q(z) = \frac{1}{\sigma\sqrt{2\pi}}\exp(-\frac{z^2}{2\sigma^2})$. As widely used in the literature [18], [20], [25], we assume that the shadowing fades Z between all pairs of transmitting node and receiving node are i.i.d. and the link is symmetric. **The limitation of the assumption of the independence between links is discussed in Section 10 in the supplementary material.**

Shadowing makes the RSS vary around its mean value over space, while the small-scale fading makes the RSS vary around its mean value over time. In this paper, we consider a generic model of small-scale fading, i.e. the *Nakagami-m fading* [26]. By choosing different values for the parameter m in the Nakagami-m fading model, the results easily include several widely used fading distributions as special cases, e.g. Rayleigh distribution (by setting $m = 1$) and one-sided Gaussian distribution (by setting $m = 1/2$) [26]. Subject to Nakagami-m fading, the RSS per symbol, ω, is distributed according to a Gamma distribution given by the following pdf [26]:

$$\zeta(\omega) = \frac{m^m \omega^{m-1}}{\bar{\omega}\Gamma(m)}\exp(-\frac{m\omega}{\bar{\omega}}), \quad \omega \geq 0 \tag{1}$$

where $\Gamma(.)$ is the standard Gamma function and $\bar{\omega} = P_l(x)$ is the mean RSS (over time), which is determined by path loss and shadowing.

We firstly conduct the analysis in the unit disk model, then we introduce the analysis in the more realistic *Log-normal-Nakagami* model, which takes into account statistical variations of RSS around the mean value due to both log-normal shadowing and Nakagami-m fading.

2.2 Per hop energy consumption

Assume that the time spent on transmitting a packet of unit size over a single hop is a constant T_t, and all nodes transmit at the same power P_t which results in a transmission range (without shadowing) of r_0 in the unit disk model. Therefore the energy consumed in transmitting a packet over a single hop is:

$$Eng(r_0) = \frac{T_t P_t + Eng_c}{1 - \alpha(r_0)} = \frac{C_2 r_0^{\eta} + Eng_c}{1 - \alpha(r_0)} \tag{2}$$

where $C_2 = C_1 T_t$ is a constant, Eng_c is another constant which includes the processing power consumption and receiving power consumption in each node and $\alpha(r_0) = \frac{2WN_b}{(W+1)^2+2WN_b}$ is the packet error rate [27], W_{min} is the minimum contention window size and N_b is the average node degree: $N_b = \rho\pi r_0^2$. Packet collision can increase the energy consumption, due to the consequent re-transmission of a packet, especially when the transmission range is large. To illustrate this effect, we implemented simulations (in Section 6) using the parameters shown in [27], i.e. $W_{min} = 64$. The values of C_2 and Eng_c are dependent on hardware specifications, where some typical values can be found in [28].

2.3 Routing algorithm

In addition to the impact of fading on the wireless channel between two nodes, fading also affects the performance of higher layer protocols. The impact of fading on higher layer protocols remains to be fully investigated [22]. In this paper, we consider the cross-layer issues by analyzing the performance of a wireless multi-hop network using the greedy forwarding routing algorithm, as a typical example of distributed routing algorithms. The GF routing belongs to the category of geographic routing algorithms and is a widely used routing algorithm for wireless multi-hop networks. Using GF, each node makes routing decisions independently of other nodes by using its own location information, the location information of its neighboring nodes and the locations of the source and the destination. GF has shown great potential in wireless multi-hop networks because of its distributed nature, low control overhead and capability of adapting to dynamic network topologies [14]. The area has attracted significant research interest, e.g. [5]–[8].

We consider a basic GF algorithm that operates following two rules [7]: 1) Every node tries to forward the packet to the node within its transmission range which is closest to the destination. 2) A packet will be dropped if a node cannot find a next-hop neighbor that is closer to the destination than itself, and hence the transmission becomes unsuccessful. **Moreover in the case of ties, viz. more than one node have the same Euclidean distance to the destination, an arbitrary one of those nodes can be chosen as the next hop node without affecting the results of our analysis. This is because the way to settle ties does not affect the probability distribution of the remaining distance to the destination at each hop, which is the quantity used to derive our results as shown in Section 3.2**.

Note that a number of complicated recovery algorithms have been proposed to route a packet around the routing void [29]. The quality of the path established by a greedy forwarding algorithm can be measured by the stretch factor, which quantifies the difference between a particular path and the shortest path [29]. By studying the stretch factor, it is shown in [29] that a basic GF algorithm can successfully find short routing paths in sensing-covered networks, without complex recovery algorithms. For analytical tractability and generality of the results, we consider the basic greedy forwarding algorithm without any recovery algorithm, as in [5]–[8].

2.4 Definitions of some terms

Denote by $E_s[k_s|x_0]$ the expected number of hops for a packet to reach the destination, conditioned on the Euclidean distance between source and destination being x_0 *and* the transmission being successful. For convenience, throughout this paper we use *conditioned on* x_0 for *conditioned on the Euclidean distance between source and destination being* x_0. Denote by $E_u[k_u|x_0]$ the expected number of hops traversed by a packet before it is dropped due to the nonexistence of a next hop node closer to the destination, conditioned on x_0 *and* the transmission being unsuccessful (in this case x_0 is the distance between the source and the *intended* destination).

It is worth noting that with the assumption that the network is connected, as used in say [7], the number of hops between two nodes increases as the transmission range (hence average node degree) decreases. In terms of energy consumption, the assumption of a connected network results in a misleading conclusion that a smaller transmission range is always better. This conclusion is misleading because the probability of having a multi-hop path between two nodes decreases as the transmission range decreases and this important fact was not considered. Consequently, this conclusion is in sharp contrast with the result obtained in this paper considering the possibility of disconnected networks that there exists an optimum transmission range that minimizes the energy consumption, as shown in Fig. 7. Our analysis does not rely on the assumption that the network is connected.

Let's consider a network with a total of N distinct source and destination pairs, where each source is separated from the associated destination by Euclidean distance x_0. Each source transmits a packet of unit size to the associated destination. Therefore there are a total of N packets transmitted. Assume M ($M \leq N$) packets can reach their respective destinations successfully.

Define $Eng_{eff}(r_0|x_0)$ to be the effective energy consumption per successfully transmitted packet for any pair of nodes separated by Euclidean distance x_0, viz $Eng_{eff}(r_0|x_0)$ is the total energy spent on transmitting all packets divided by the number of successfully received packets:

$$\begin{aligned} & Eng_{eff}(r_0|x_0) \\ = & \frac{MEng(r_0)E_s[k_s|x_0] + (N-M)Eng(r_0)E_u[k_u|x_0]}{M} \\ = & Eng(r_0)\frac{\phi_s(x_0)E_s[k_s|x_0] + (1-\phi_s(x_0))E_u[k_u|x_0]}{\phi_s(x_0)} \end{aligned} \quad (3)$$

where $\phi_s(x_0) := M/N$ is the probability of successful transmission between any pair of nodes separated by x_0.

In a network where the transmission range without shadowing and fading is r_0, given the distribution of the Euclidean distance between any pair of nodes $f(x_0)$, examples of which are given in [30], the average effective energy consumption is:

$$Eng_{eff}(r_0) = \int Eng_{eff}(r_0|x_0)f(x_0)dx_0 \quad (4)$$

The effective energy consumption is a measure of the energy spent on each successfully transmitted packet. A lower Eng_{eff} means a higher energy efficiency. We use Eng_{eff} as the metric to investigate the energy efficiency in end-to-end packet transmissions.

3 Analysis in the unit disk model

In this section, we analyze the hop count statistics (in particular the probability $\Pr(k|x)$) and the effective energy consumption under the unit disk model. We start with the calculations of the probability that two arbitrary nodes are k hops apart for $k \geq 3$ using GF. The analysis for $k = 1, 2$ is straightforward.

3.1 Spatial dependence problem

Before going into the analysis, we introduce the spatial dependence problem in the analysis of hop count statistics using the unit disk model as an example. The same problem also exists in other models. Generally, there are two types of spatial dependence problems.

First, it can be shown that the event that a randomly chosen node is a k^{th} hop node (S_k) from a randomly chosen source node (S) is not independent of the event that another randomly chosen node is a i^{th} hop node for $1 \leq i < k$. Denote by $C(S_k, r)$ the disk centered at S_k with radius r. As shown in the example in Fig. 1(a), the fact that S_{k-1} is a $k-1^{th}$ hop node from a source node S (not shown in the figure) implies that there is at least one node in the area $C(S_{k-3}, r_0) \cap C(S_{k-1}, r_0)$. On the other hand S_k is a k^{th} hop node from S implies that there is no node in the area $C(S_{k-3}, r_0) \cap C(S_k, r_0)$,

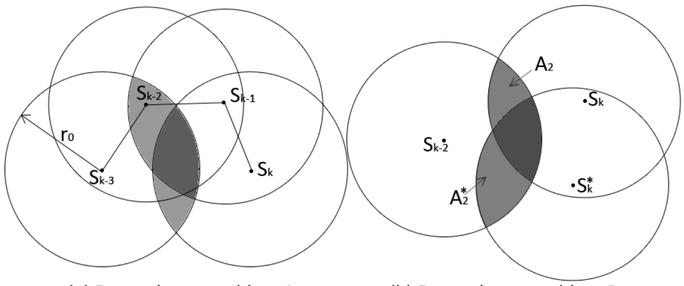


Fig. 1: Illustration of the spatial dependence problems in the hop count statistics using a unit disk model. S_k is the k^{th} hop node, where r_0 is the transmission range.

otherwise S_k will become a $k-2^{th}$ hop node. These two areas overlap which means that the event that S_k is a k^{th} hop node and the event that S_{k-1} is a $k-1^{th}$ hop node are not independent.

Second, it can be shown that the event that a randomly chosen node is a k^{th} hop node from S is not independent of the event that another randomly chosen node is a k^{th} hop node from S. As shown in the example in Fig. 1(b), S_k is a k^{th} hop node from S implies that there is at least one node (the $k-1^{th}$ hop node) in the area $A_2 = C(S_{k-2}, r_0) \cap C(S_k, r_0)$. Another node S_k^* is a k^{th} hop node from the same S implies that there is at least one node (the $k-1^{th}$ hop node) in the area $A_2^* = C(S_{k-2}, r_0) \cap C(S_k^*, r_0)$. These two areas overlap which means that the event that S_k is a k^{th} hop node and the event that S_k^* is a k^{th} hop node are not independent. In this paper, a significant improvement on the accuracy of $\Pr(k|x)$ is shown by reducing the inaccuracy associated with the first type of spatial dependence problem. **The second type of spatial dependence problem can be handled by a similar technique used in this paper. Specifically, when considering the area covered by the transmission range of a k^{th} hop node, we need to consider the overlapping of the area covered by the transmission range of the k^{th} hop node and the area covered by the transmission range of other k^{th} hop nodes. However, it can be seen in Section 7 that the result is fairly accurate after a proper handling of the first type of spatial dependence problem, so that specific handling of this second spatial dependence problem is effectively not warranted.**

3.2 Distribution of the remaining distance

Denote by $A(x, r_1, r_2)$ the intersectional area of two disks with distance x between centers and radii r_1 and r_2 respectively. The size of the area is [31]:

$$A(x, r_1, r_2) = \begin{cases} \min(\pi r_1^2, \pi r_2^2), & \text{for } x \leq |r_1 - r_2| \\ r_1^2 \arccos(\frac{x^2+r_1^2-r_2^2}{2xr_1}) + r_2^2 \arccos(\frac{x^2+r_2^2-r_1^2}{2xr_2}) \\ -\frac{1}{2}\sqrt{[(r_1+r_2)^2 - x^2][x^2 - (r_1-r_2)^2]}, \\ \quad \text{for } |r_1 - r_2| < x < r_1 + r_2 \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

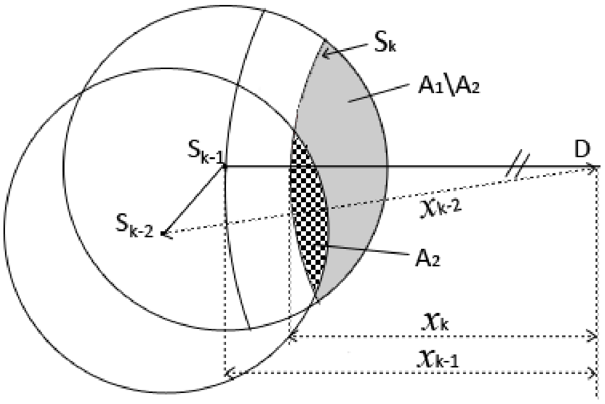


Fig. 2: Possible positions for the node at the k^{th} hop, denoted by S_k, are located on the arc, considering the positions of S_{k-1} and S_{k-2}, which are the nodes at the $k-1^{th}$ hop and $k-2^{th}$ hop from the source respectively. A_1, A_2, x_k, x_{k-1} and x_{k-2} are described in the following text.

Define x_k to be the remaining Euclidean distance between the k^{th} hop node (S_k) and the destination (D). Define $A_1 = A(x_{k-1}, r_0, x_k)$ to be the intersectional area of the disks $C(S_{k-1}, r_0)$ and $C(D, x_k)$. Similarly we have $A_2 = A(x_{k-2}, r_0, x_k)$. Next we record the form of $\frac{\partial A(x,r_1,r_2)}{\partial r_2}$, which will be used later. For $|r_1 - r_2| < x < r_1 + r_2$:

$$\begin{aligned} \frac{\partial A(x, r_1, r_2)}{\partial r_2} &= \frac{-r_1^2}{\sqrt{1-S^2}}(\frac{\partial S}{\partial r_2}) + 2r_2 \arccos(T) \\ &\quad + r_2^2(\frac{-1}{\sqrt{1-T^2}})(\frac{\partial T}{\partial r_2}) - \frac{1}{4\sqrt{W}}(\frac{\partial W}{\partial r_2}) \\ \text{where} \quad S &= \frac{x^2 + r_1^2 - r_2^2}{2xr_1}, \quad T = \frac{x^2 + r_2^2 - r_1^2}{2xr_2}, \\ W &= ((r_1 + r_2)^2 - x^2)(x^2 - (r_1 - r_2)^2) \end{aligned} \tag{6}$$

Define $f(x_k, k|x_0)$ to be the joint pdf of the remaining Euclidean distance to the destination from S_k being x_k *and* the packet having been successfully forwarded k hops, conditioned on x_0. Due to the spatial dependence problem, $f(x_k, k|x_0)$ depends on the remaining distances of all previous hop nodes, i.e. x_{k-1}, x_{k-2}, ..., x_0. We consider no more than two previous hops and the justification is given in Section 4.

Define $g(x_k, k|x_{k-1}, x_{k-2}, k-1)$ to be the joint pdf of the remaining Euclidean distance to the destination at S_k being x_k *and* the packet having been successfully forwarded k hops, conditioned on $\mathcal{B}$, where $\mathcal{B}$ is the event that the remaining distances at S_{k-1} and S_{k-2} are x_{k-1} and x_{k-2} respectively *and* the packet has been successfully forwarded $k-1$ hops. (Note that a packet has been successfully forwarded $k-1$ hops necessarily means that it has been successfully forwarded i hops for $i \leq k-1$.) Accordingly define the cdf (cumulative distribution function) of the remaining distance at the k^{th} hop node to be $\Pr(X_k \leq x_k, k|x_{k-1}, x_{k-2}, k-1)$. Ignoring the boundary effect, whose impact will be discussed in detail later, the cdf is equal to the probability that there is at least one node in area $A_1 \setminus A_2$ as indicated by the uniform-shaded area in Fig. 2. The area A_2 needs to be excluded because if there is a node in this area, that node will be closer to the destination than S_{k-1}, which violates the condition that S_{k-1} is the $k-1^{th}$ hop node using GF. We approximate the size of $A_1 \setminus A_2$ by $A_1 - A_2$. This approximation will greatly simplify the calculation while giving a sufficiently accurate result, as validated in Section 6. Due to space limitation, we omitted analytical studies on the accuracy of the approximation. Then:

$$\begin{aligned} &\Pr(X_k \leq x_k, k|x_{k-1}, x_{k-2}, k-1) \\ &= 1 - \exp(-\rho(A(x_{k-1}, r_0, x_k) - A(x_{k-2}, r_0, x_k))) \end{aligned} \tag{7}$$

For any two nodes close to the border, the intersectional area of the transmission ranges of the two nodes may be partially located outside the network area, which causes an error in computing the size of the area $A_1 \setminus A_2$ in Eq. 7. This effect is due to the boundary effect. Ignoring the boundary effect may generally cause an overestimation on the size of $A_1 \setminus A_2$, hence an overestimation on the probability of finding the next hop node. However, simulation results in Section 6 show that the boundary effect has very limited impact on the accuracy of the analytical results.

Taking the derivative of the cdf with respect to x_k, we have:

$$\begin{aligned} &g(x_k, k|x_{k-1}, x_{k-2}, k-1) \\ &= \frac{\partial \Pr(X_k \leq x_k, k|x_{k-1}, x_{k-2}, k-1)}{\partial x_k} \\ &= \rho \left(\frac{\partial A(x_{k-1}, r_0, x_k)}{\partial x_k} - \frac{\partial A(x_{k-2}, r_0, x_k)}{\partial x_k} \right) \\ &\times \exp\left(-\rho(A(x_{k-1}, r_0, x_k) - A(x_{k-2}, r_0, x_k))\right) \end{aligned} \tag{8}$$

where the partial differentiations are given by Eq. 6.

Define $h(x_k, x_{k-1}, k|x_0)$ to be the joint pdf of the remaining Euclidean distances at the k^{th} hop node and $k-1^{th}$ hop node being x_k and x_{k-1} respectively *and* the packet having been successfully forwarded k hops, conditioned on x_0.

For $k = 1$, it is straightforward that:

$$f(x_1, k = 1|x_0) = \rho \frac{\partial A(x_0, r_0, x_1)}{\partial x_1} e^{-\rho A(x_0, r_0, x_1)} \tag{9}$$

For convenience, $f(x_1, k = i|x_0)$ is denoted by $f(x_1, i|x_0)$ hereafter. Based on the above result, for $k = 2$ we have:

$$h(x_2, x_1, 2|x_0) = g(x_2, 2|x_1, x_0, 1) f(x_1, 1|x_0) \tag{10}$$

For $k > 2$, $h(x_k, x_{k-1}, k|x_0)$ can be calculated recursively:

$$\begin{aligned} h(x_k, x_{k-1}, k|x_0) &= \int_{r_0}^{x_0} g(x_k, k|x_{k-1}, x_{k-2}, k-1) \\ &\quad h(x_{k-1}, x_{k-2}, k-1|x_0) dx_{k-2} \end{aligned} \tag{11}$$

Finally for $k > 1$ we have:

$$f(x_k, k|x_0) = \int_{r_0}^{x_0} h(x_k, x_{k-1}, k|x_0) dx_{k-1} \tag{12}$$

3.3 Hop count statistics

Define $\Pr(k|x_0)$ to be the probability that the destination can be reached at the k^{th} hop conditioned on x_0. The destination can be reached at the k^{th} hop if the $k-1^{th}$ hop node is within the transmission range of the destination. Therefore:

$$\Pr(k|x_0) = \int_0^{r_0} f(x_{k-1}, k-1|x_0)dx_{k-1} \quad (13)$$

3.4 Results for successful transmissions

Denote by $\Pr_s(k_s|x_0)$ the conditional probability that a packet can reach its destination at the k_s^{th} hop, conditioned on x_0 *and* the transmission being successful. It follows that $\Pr(k_s|x_0) = \Pr_s(k_s|x_0)\phi_s(x_0)$, and:

$$\begin{aligned} \sum_{k_s=1}^{\infty} k_s \Pr(k_s|x_0) &= \phi_s(x_0) \sum_{k_s=1}^{\infty} k_s \Pr_s(k_s|x_0) \\ &= \phi_s(x_0) E_s[k_s|x_0] \quad (14) \end{aligned}$$

where $\phi_s(x_0)$ and $E_s[k_s|x_0]$ are defined in Section III.D. In reality, an upper bound on k_s can be found beyond which $\Pr(k_s|x_0)$ is 0. So Eq. 14 and other similar equations only need to be computed for a finite range of k_s.

An end-to-end packet transmission is successful if a packet can reach the destination at any number of hops. Therefore:

$$\phi_s(x_0) = \sum_{k_s=1}^{\infty} \Pr(k_s|x_0) \quad (15)$$

3.5 Results for unsuccessful transmissions

Define $\phi_u(k|x_0)$ to be the probability of a packet having been successfully forwarded k hops from the source toward the destination x_0 apart, but *not* reaching the destination in k hops, which distinguishes $\phi_u(k|x_0)$ from $\Pr(k|x_0)$. Therefore:

$$\phi_u(k|x_0) = \int_0^{x_0} f(x_k, k|x_0)dx_k \quad (16)$$

Based on the example introduced in Section III.D, we further assume that only M_k out of N packets reach the k^{th} hop nodes. Then $\phi_u(k|x_0) = M_k/N$. At the next hop, there are three possibilities for each of these M_k packets: 1) a packet reaches the destination at the next hop; 2) a packet makes another hop without reaching the destination; 3) the packet is dropped. Let W_{k+1} and M_{k+1} be the number of packets for which the first and second possibilities apply.

Define $\psi(k_u|x_0)$ to be the probability of the packets being dropped at the k_u^{th} hop. Then:

$$\begin{aligned} \psi(k_u|x_0) &= \frac{M_{k_u} - M_{k_u+1} - W_{k_u+1}}{N} \\ &= \phi_u(k_u|x_0) - \phi_u(k_u+1|x_0) - \Pr(k_u+1|x_0) \quad (17) \end{aligned}$$

The average number of hops for unsuccessful transmissions between a source and a destination separated x_0 apart is the expected value of k_u whose pdf is given by $\psi(k_u|x_0)$. Similar to the way to derive Eq. 14, we have: $\sum_{k_u=1}^{\infty} k_u \psi(k_u|x_0) = (1-\phi_s(x_0))E_u[k_u|x_0]$.

Given the above analysis, the effective energy consumption can be computed using Eq. 4, which is shown in Section 6. Further, the above results can also be useful in the analysis of delay, throughput or reliability of end-to-end packet transmissions [8], [20], as well as localization [8], [32], which is left as our future work.

4 Impact of Spatial Dependence Problem

In this paper, we considered that the remaining distance at the k^{th} hop node (S_k) depends on the remaining distance at previous two hops nodes (S_{k-1} and S_{k-2}). Due to the spatial dependence problem, it can be shown that correct analysis of the hop count statistics requires all previous hops to be considered, but the calculation is more complicated than if an independence assumption is made. Previous research, e.g. [7], usually considered the dependence on only previous one hop. In this section we study the impact of the spatial dependence problem on the accuracy of the $\Pr(k|x)$.

Define A_m to be the intersectional area of the disk centered at S_{k-m} with radius r_0 and the disk centered at D with radius x_k. Therefore the precise area that should be considered in the calculation of Eq. 7 is $A = A_1 \setminus (A_2 \cup A_3 \cup ... \cup A_k)$ instead of $A_1 \setminus A_2$.

Consider only previous one hop, then: $A \approx A_1$. Consider only previous two hops, then: $A \approx A_1 \setminus A_2 = A_1 \underline{- A_1 \cap A_2}$. Consider only previous three hops, then:

$$\begin{aligned} A &\approx A_1 \setminus (A_2 \cup A_3) = A_1 - A_1 \cap (A_2 \cup A_3) \\ &= A_1 - A_1 \cap A_2 \underline{- A_1 \cap A_3 + A_1 \cap A_2 \cap A_3} \quad (18) \end{aligned}$$

The underlined terms are the additional terms introduced when considering one more previous hop. In considering the previous m hops instead of previous $m-1$ hops, the improvement is bounded by a term determined by $A_1 \cap A_m$. Furthermore, it is evident that $x_{k-1} < x_{k-2} < ... < x_0$. Therefore $A_1 > A_2 > ... > A_k$ and the size of $A_1 \cap A_m$ is dominated by the size of A_m.

Define $h(x_k, x_{k-m}, k|x_0)$ to be the joint pdf of the remaining Euclidean distances at the k^{th} hop node and $k-m^{th}$ hop node being x_k and x_{k-m} respectively *and* the packet having been successfully forwarded k hops, conditioned on x_0. Then the expected size of A_m at the k^{th} hop can be calculated by:

$$\begin{aligned} E[A_m, k|x_0] &= \int_0^{x_0} \int_{x_k}^{x_k+r_0} A(x_{k-m}, r_0, x_k) \\ &\quad h(x_k, x_{k-m}, k|x_0)dx_{k-m}dx_k \quad (19) \end{aligned}$$

For $m=1$, $h(x_k, x_{k-1}, k|x_0)$ can be calculated using Eq. 11. For $m=2$, we have:

$$\begin{aligned} h(x_k, x_{k-2}, k|x_0) &= \int_{r_0}^{x_0} g(x_k, k|x_{k-1}, x_{k-2}, k-1) \\ &\quad h(x_{k-1}, x_{k-2}, k-1|x_0)dx_{k-1} \quad (20) \end{aligned}$$

For $m \geq 3$, the calculation becomes more complicated. But approximately $h(x_k, x_{k-m}, k|x_0) \approx$

$f(x_k, k|x_0)f(x_{k-m}, k-m|x_0)$, where $f(x_k, k|x_0)$ is given by Eq. 12. This approximation is valid because the distance between S_k and S_{k-m} generally increases as m increases, hence the size of the overlapping area decreases. Therefore the correlation between $f(x_k, k|x_0)$ and $f(x_{k-m}, k-m|x_0)$ reduces as m increases.

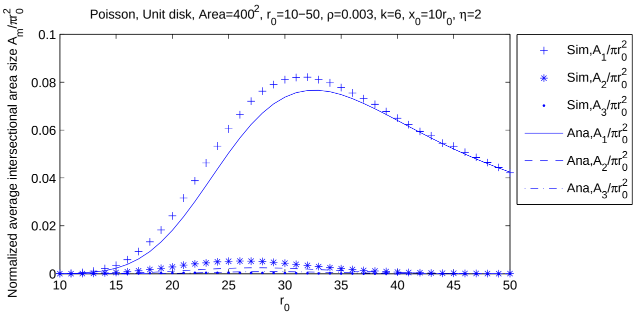


Fig. 3: Simulation (Sim) and analytical (Ana) results for the normalized average intersectional area size in the unit disk model. A_m is the intersectional area of the disk centered at S_{k-m} with radius r_0 and the disk centered at D with radius x_k.

Based on the approach introduced above, Fig. 3 shows the results for the average size of A_m when the source node and the destination node are separated by distance $x_0 = 10r_0$. The simulation parameters are introduced in Section 6. It is evident that the size of A_m, $m \geq 3$, are negligibly small (less than 1% of the size of the area covered by the transmission range) compared to the size of A_1 and A_2. It validates the claim that the improvement made by taking previous m hops into consideration will be marginal for $m \geq 3$, which explains our choice of considering two previous hops only.

Our results suggest that the accuracy of the analysis on $\Pr(k|x)$ can be significantly improved by considering previous two hops (compared to considering previous one hop only). However, moving beyond two hops results in marginal improvement in accuracy of the analysis. Therefore, the conclusion can be drawn that the locations of nodes three or more hops away provide little information for a node to determine its geometric relationship with other nodes. This conclusion provides analytical support for observations, to this point unsupported by analysis, in routing, localization and network security that taking into account the (location or link status) information of two-hops neighbors can significantly improve the routing [33] (respectively localization [34], network security [35]) performance compared with using one-hop neighborhood information only. However beyond two hops, taking into account more neighborhood information only has marginal impact. Therefore many distributed routing, localization and network security protocols use two-hop neighborhood information.

5 Analysis in the Log-normal-Nakagami Model

The technique to incorporate the impacts of both shadowing and small-scale fading is through the use of the random split property of a Poisson process.

5.1 Random split of a Poisson process

First, we introduce a random variable named the *Nakagami fades* Ω_0 which follows the Gamma distribution with mean 1. Therefore the pdf of Ω_0 is

$$\zeta_0(\omega_0) = \frac{m^m \omega_0^{m-1}}{\Gamma(m)} \exp(-m\omega_0), \quad \omega_0 \geq 0 \tag{21}$$

where m is introduced in Section 2.1.

It can be shown that the random variable $P_l(x)\Omega_0$ follows the Gamma distribution with mean $P_l(x)$, where $P_l(x) = CP_t x^{-\eta} 10^{Z/10}$ is the RSS given by the log-normal shadowing model introduced in Section 2.1. Then in the Log-normal-Nakagami model, the RSS at a receiver at distance x from the transmitter is $P_N(x) = P_l(x)\Omega_0 = CP_t x^{-\eta} 10^{Z/10}\Omega_0$, where Z is a zero-mean Gaussian distributed random variable and Ω_0 is a Gamma distributed random variable with mean 1.

According to the random split property of a Poisson process [36], the subset of nodes whose RSS from a particular transmitting node with shadowing fades $Z \in [z, z+dz]$ *and* Nakagami fades $\Omega_0 \in [\omega_0, \omega_0 + d\omega_0]$ are i.i.d. following a Poisson process with intensity $\rho q(z) dz \zeta_0(\omega_0) d\omega_0$. Via the splitting of the Poisson process, we can study the sub-process by the same technique used in the unit disk model.

Remark 1: The aforementioned technique can be extended to other communication models (e.g. the class of random connection models [37]). In a random connection model [37], two arbitrary nodes separated by Euclidean distance x are directly connected with probability $\gamma(x)$, where $\gamma(x)$ satisfies two conditions: 1) the probability is a non-increasing function mapping from the positive real numbers into $[0, 1]$; 2) the event that a pair of nodes are directly connected is independent of the event that another pair of nodes are directly connected.

Define $\Pr_l(k|x_0)$ to be the probability that two arbitrary nodes separated by Euclidean distance x_0 are k hops apart using GF in the Log-normal-Nakagami model. We start with k=1.

5.2 Probability of direct connection

Under the Log-normal-Nakagami model, two nodes separated by distance x are directly connected iff the RSS exceeds a given threshold $P_{\min}$. Without shadowing and small-scale fading, the model reduces to the unit disk model where $P_{\min} = CP_t r_0^{-\eta}$. With shadowing and fading, we have:

$$\begin{aligned}
\Pr(P_N(x) \geq P_{\min}) &= \Pr(CP_t x^{-\eta} 10^{Z/10}\Omega_0 \geq CP_t r_0^{-\eta}) \\
&= \Pr(Z \geq 10\eta \log_{10}(\frac{x}{r_0 \Omega_0^{1/\eta}})) \quad (22)\\
&= \Pr(x \leq r_0 \Omega_0^{\frac{1}{\eta}} \exp(\frac{Z \ln 10}{10\eta})) \quad (23)
\end{aligned}$$

Thus two nodes are directly connected if either of the following two conditions is satisfied: 1) Given the distance x and Nakagami fades value ω_0, two nodes are directly connected iff the (random) shadowing fades

$Z \geq 10\eta \log_{10}(\frac{x}{r_0 \omega_0^{1/\eta}})$. 2) Given that the shadowing fades z and Nakagami fades value ω_0, two nodes are directly connected iff their distance $x \leq r_0 \omega_0^{1/\eta} \exp(\frac{z \ln 10}{10\eta})$.

Based on the first condition, the probability of having a direct connection between two arbitrary nodes separated by x_0 is:

$$\Pr_l(k=1|x_0) = \int_0^{\infty} \int_{10\eta \log_{10}(\frac{x}{r_0 \omega_0^{1/\eta}})}^{\infty} q(z) dz \zeta_0(\omega_0) d\omega_0 \quad (24)$$

$$= \int_0^{\infty} \frac{1}{2} \left(1 - erf(\frac{10\eta \log_{10}(\frac{x}{r_0 \omega_0^{1/\eta}})}{\sqrt{2\sigma^2}}) \right) \zeta_0(\omega_0) d\omega_0 \quad (25)$$

where $erf(.)$ is the error function.

Remark 2: Without small-scale fading, viz. considering the log-normal shadowing model only, the probability of having a direct connection between two arbitrary nodes separated by x_0 is: $\frac{1}{2}(1 - erf(\frac{10\eta \log_{10}(\frac{x}{r_0})}{\sqrt{2\sigma^2}}))$. Similarly, the following analysis can be reduced to the analysis without small-scale fading by simply removing the integral with respect to ω_0.

In order to derive $\Pr_l(k|x_0)$ for $k > 1$, we use the second condition to study the probability of a direct connection. Define $r_N(z_S, \omega_S)$ to be the transmission range of a transmitter (S) conditioned on the shadowing fades and Nakagami fades being z_S and ω_S respectively. Then:

$$r_N(z_S, \omega_S) = r_0 \omega_S^{1/\eta} \exp(\frac{z_S \ln 10}{10\eta}) \quad (26)$$

Therefore any node, whose RSS from the transmitter (S) has shadowing fades $Z_S \in [z_S, z_S + dz_S]$ and Nakagami fades $\Omega_S \in [\omega_S, \omega_S + d\omega_S]$, is directly connected to S iff its Euclidean distance to the transmitter is smaller than or equal to $r_N(z_S, \omega_S)$. This allows us to apply the analysis used in the unit disk model.

5.3 Distribution of the remaining distance

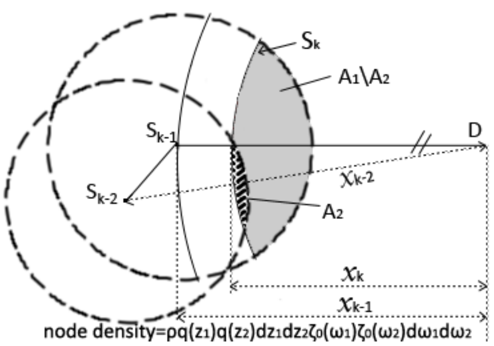


Fig. 4: Possible positions for the k^{th} hop node (S_k), are located on the arc. Consider the nodes whose RSS from S_{k-1} has fades $Z_1 \in [z_1, z_1 + dz_1]$ and $\Omega_1 \in [\omega_1, \omega_1 + d\omega_1]$; while its RSS from S_{k-2} has fades $Z_2 \in [z_2, z_2 + dz_2]$ and $\Omega_2 \in [\omega_2, \omega_2 + d\omega_2]$. The dashed-line circles represent the transmission range of S_{k-1} (resp. S_{k-2}) conditioned on the above values of shadowing and Nakagami fades. A_1 and A_2 are described in the following.

Define area size $A_1 = A(x_{k-1}, r_N(z_1, \omega_1), x_k)$ and $A_2 = A(x_{k-2}, r_N(z_2, \omega_2), x_k)$, where $A(x, r_1, r_2)$ and x_k are defined in Section 3. Define $f_l(x_k, k|x_0)$, $g_l(x_k, k|x_{k-1}, x_{k-2}, k-1)$, the event $\mathcal{B}_l$, $\Pr_l(X_k \leq x_k, k|x_{k-1}, x_{k-2}, k-1)$ and $h_l(x_k, x_{k-1}, k|x_0)$ analogously as in Section 3 and use the subscript l to mark the corresponding probabilities in the Log-normal-Nakagami model. We will derive $\Pr_l(X_k \leq x_k, k|x_{k-1}, x_{k-2}, k-1)$ by studying the following two events. Denote by $\mathcal{C}$ the event that there is at least one node whose Euclidean distance to the destination is smaller than x_k *and* has a direct connection to S_{k-1} *and* has no direct connection to S_{k-m} for $m \in [2, k]$ where S_0 is the source node. Denote by $\mathcal{D}$ the event that the node S_{k-1} is not directly connected to the destination. Events $\mathcal{C}$ and $\mathcal{D}$ are independent because of the independence of the shadowing and Nakagami fades. It is evident that:

$$\Pr_l(X_k \leq x_k, k|x_{k-1}, x_{k-2}, k-1) = \Pr(\mathcal{C}|\mathcal{B}_l) \times \Pr(\mathcal{D}|\mathcal{B}_l) \quad (27)$$

We start with the analysis of event $\mathcal{C}$. In this paragraph we only consider the subset of nodes whose RSS from S_{k-1} has fades $Z_1 \in [z_1, z_1 + dz_1]$ and $\Omega_1 \in [\omega_1, \omega_1 + d\omega_1]$; while its RSS from S_{k-2} has fades $Z_2 \in [z_2, z_2 + dz_2]$ and $\Omega_2 \in [\omega_2, \omega_2 + d\omega_2]$. Due to the independence of the fades and the property of Poisson process, these nodes are distributed following a homogeneous Poisson process with intensity $\rho q(z_1) q(z_2) dz_1 dz_2 \zeta_0(\omega_1) \zeta_0(\omega_2) d\omega_1 d\omega_2$. Denote by $\mathcal{E}$ the event that $Z_1 \in [z_1, z_1 + dz_1]$ *and* $Z_2 \in [z_2, z_2 + dz_2]$ *and* $\Omega_1 \in [\omega_1, \omega_1 + d\omega_1]$ *and* $\Omega_2 \in [\omega_2, \omega_2 + d\omega_2]$. $\Pr(\mathcal{C}, \mathcal{E}|\mathcal{B}_l)$ is equal to the probability that there is at least one node in area $A_1 \setminus A_2$, as shown in Fig. 4. We approximate the size of area $A_1 \setminus A_2$ by $(A_1 - A_2)^+$, where $(A_1 - A_2)^+ = \max\{0, A_1 - A_2\}$. (Through this approximation we ignored some rare events that cause $A_1 - A_2 < 0$, which can possibly occur when $r_N(z_2, \omega_2)$ is much larger than $r_N(z_1, \omega_1)$. In contrast under the unit disk model it is always the case that $A_1 - A_2 \geq 0$.) Considering this subset of nodes only, $1 - \Pr(\mathcal{C}, \mathcal{E}|\mathcal{B}_l)$ is equal to $1 - \exp(-(A_1 - A_2)^+ \rho q(z_1) q(z_2) dz_1 dz_2 \zeta_0(\omega_1) \zeta_0(\omega_2) d\omega_1 d\omega_2)$, which is the probability that there is no node in area $A_1 \setminus A_2$. Note that A_1 depends on z_1 and ω_1; while A_2 depends on z_2 and ω_2.

Then considering all subset of nodes, we have:

$$\begin{aligned} & \Pr(\mathcal{C}|\mathcal{B}_l) \\ = \; & 1 - \prod_{z_1, z_2 \in (-\infty, +\infty), \omega_1, \omega_2 \in (0, +\infty)} \exp(-(A_1 - A_2)^+ \rho \\ & q(z_1) q(z_2) dz_1 dz_2 \zeta_0(\omega_1) \zeta_0(\omega_2) d\omega_1 d\omega_2 \quad (28) \\ = \; & 1 - \exp(-\int_0^{\infty} \int_0^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (A_1 - A_2)^+ \rho \\ & q(z_1) q(z_2) dz_1 dz_2 \zeta_0(\omega_1) \zeta_0(\omega_2) d\omega_1 d\omega_2) \quad (29) \end{aligned}$$

Since the event $\mathcal{D}$ only depends on x_{k-1}, we have:

$$\Pr(\mathcal{D}|\mathcal{B}_l) = 1 - \Pr_l(1|x_{k-1}) \quad (30)$$

Then substitute Eq. 29 and Eq. 30 into Eq. 27:

$$\begin{aligned}&\Pr_l(X_k \le x_k, k|x_{k-1}, x_{k-2}, k-1) \quad (31)\\ =\ & (1-\exp(-\int_0^\infty\int_0^\infty\int_{-\infty}^\infty\int_{-\infty}^\infty (A_1-A_2)^+\rho\\ & q(z_1)q(z_2)dz_1dz_2\zeta_0(\omega_1)\zeta_0(\omega_2)d\omega_1d\omega_2))\\ & \times(1-\Pr_l(1|x_{k-1}))\end{aligned}$$

By Leibniz integral rule:

$$\begin{aligned}& g_l(x_k, k|x_{k-1}, x_{k-2}, k-1) \quad (32)\\ &= \frac{\partial \Pr_l(X_k \le x_k, k|x_{k-1}, x_{k-2}, k-1)}{\partial x_k}\\ &= \int_0^\infty\int_0^\infty\int_{-\infty}^\infty\int_{-\infty}^\infty \frac{\partial(A_1-A_2)^+}{\partial x_k}\rho q(z_1)q(z_2)dz_1dz_2\\ &\zeta_0(\omega_1)\zeta_0(\omega_2)d\omega_1d\omega_2\\ &\times\exp(-\int_0^\infty\int_0^\infty\int_{-\infty}^\infty\int_{-\infty}^\infty (A_1-A_2)^+\rho q(z_1)q(z_2)\\ &\zeta_0(\omega_1)\zeta_0(\omega_2)dz_1dz_2d\omega_1d\omega_2)(1-\Pr_l(1|x_{k-1}))\end{aligned}$$

where $\partial A_1/\partial x_k$ and $\partial A_2/\partial x_k$ can be calculated by Eq. 6.

It is straightforward that for $k=1$, we have:

$$\begin{aligned}& f_l(x_1, 1|x_0) \quad (33)\\ &= \int_0^\infty\int_{-\infty}^\infty \frac{\partial A(x_0, r_N(z_1), x_1)}{\partial x_1}\rho q(z_1)dz_1\zeta_0(\omega_1)d\omega_1\\ &\times\exp(-\int_0^\infty\int_{-\infty}^\infty A(x_0, r_N(z_1), x_1)\rho q(z_1)dz_1\\ &\zeta_0(\omega_1)d\omega_1)(1-\Pr_l(1|x_0))\end{aligned}$$

For $k=2$, the pdf of the remaining distance of the previous hop is given by Eq. 33. Therefore:

$$h_l(x_2, x_1, 2|x_0) = g_l(x_2, 2|x_1, x_0, 1)f_l(x_1, 1|x_0) \quad (34)$$

For $k>2$, the joint pdf of x_k and x_{k-1} is calculated recursively:

$$\begin{aligned}h_l(x_k, x_{k-1}, k|x_0) &= \int_0^{x_0} g_l(x_k, k|x_{k-1}, x_{k-2}, k-1)\\ & h_l(x_{k-1}, x_{k-2}, k-1|x_0)dx_{k-2} \quad (35)\end{aligned}$$

Finally for $k \ge 2$, we have:

$$f_l(x_k, k|x_0) = \int_0^{x_0} h_l(x_k, x_{k-1}, k|x_0)dx_{k-1} \quad (36)$$

5.4 Hop count statistics

Because of shadowing and small-scale fading, the destination can be possibly reached in a single hop no matter how far the remaining distance from that hop is. Therefore for $k \ge 2$:

$$\Pr_l(k|x_0) = \int_0^{x_0} \Pr_l(1|x_{k-1})f_l(x_{k-1}, k-1|x_0)dx_{k-1} \quad (37)$$

Remark 3: Based on the above results, we can calculate the average number of hops for successful and unsuccessful transmissions, the probability of successful transmissions and the effective energy consumption by the same technique used in the unit disk model.

6 SIMULATION RESULTS

In this section, we report on simulations to validate the accuracy of the analytical results. The simulations are conducted in a wireless multi-hop network simulator written in C++. Nodes are deployed in a 400×400 square following a homogeneous Poisson process with intensity $\rho = 0.003$. The boundary effect is included in the simulation but it is shown to have a limited impact on the results. The route between two nodes is determined by the basic GF algorithm. The transmission range r_0 is varied from 10 to 50, which results in the average node degree varying from around 1 to 24. Note that r_0 is the transmission range without shadowing and small-scale fading. The value of r_0 can be specified by the network designer via adjusting the transmission power and receiver gain. The existence of a direct wireless link between an arbitrary pair of nodes will be further affected by shadowing and small-scale fading. Several values of the standard deviation in log-normal shadowing model have been used in our simulations, but only the results for $\sigma = 4$ are shown in this paper because other results show a similar trend. Further, we only include the results for $C_2 = 0.01$ and $Eng_c = 0.02$ (in Eq. 2) as an example and the value of Eng_c is found to have very limited impact on the results. In order to distinguish the impact on the network performance of different parameters, the packet error rate is not included (i.e. set $\alpha = 0$) except Fig. 8 and the small-scale fading is not included expect Fig. 6 and Fig. 7(b). Every point shown in the simulation result is the average value from 3000 simulations. As the number of instances of random networks used in the simulation is large, the confidence interval is too small to be distinguishable and hence is ignored in the following plots.

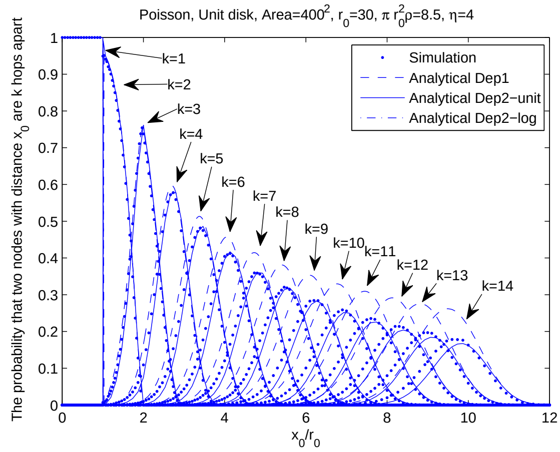
6.1 Hop count statistics

Fig. 5 shows the probability that two arbitrary nodes separated by distance x_0 are k hops apart using GF in the unit disk model and the log-normal shadowing model respectively. In the log-normal shadowing model [18], the received signal strength (RSS) attenuation (in dB) follows a normal distribution with standard deviation σ around the mean value. The mean value is given by the RSS under the path loss attenuation model, which is the model adopted in the unit disk model to determine the transmission range. Therefore when $\sigma = 0$, the log-normal shadowing model reduces to the unit disk model. As shown in Fig. 5, Dep2-unit and Dep2-log completely agree and the analytical results have a good match with the simulation results, which verifies the accuracy of our analysis in both the unit disk model and the log-normal shadowing model.

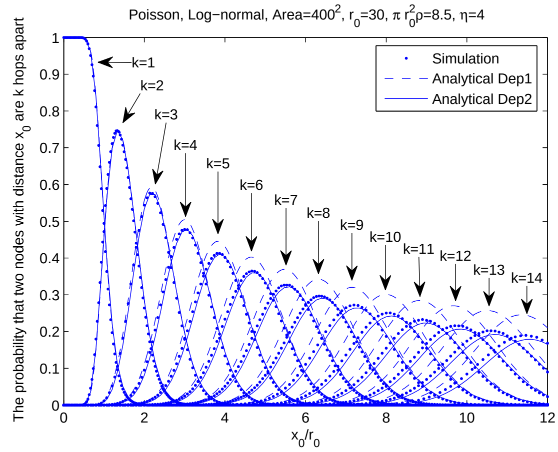
In addition, we can see that the accuracy is significantly improved by considering two previous hops (the result from this paper) compared with previous analysis considering only one previous hop (e.g. [23]). Further, it can be seen in Fig. 5 that the improvement of accuracy will be marginal if more than two previous hops are considered, which also confirms the analysis in Section 4. We expect this observation to be extended to many other areas (e.g. routing, localization, network security) and the approach used for shedding the independence assumption can be seen in a broader context. Specifically, our approach for shedding the independence assumption is to show that one can improve the accuracy of $\Pr(k|x)$ by taking into account the locations of previous m-hops nodes ($1 \le m \le k-1$). However the improvement becomes marginal as $m > 2$. This suggests that the locations of nodes three or more hops away provide little information in determining the geometric relationship of a node with other nodes in the network. This observation further confirms our assertion in Section 4.

Furthermore, it is interesting to see that packets can be transmitted to a larger distance under the log-normal shadowing model than under the unit disk model, at the same number of hops. This is because log-normal shadowing introduces a Gaussian variation of the transmission range around the mean value, and with a higher chance a node can find a next-hop neighbor closer to the destination. This phenomenon is also observed in the study of connectivity [19].

Fig. 6 shows the probability that two arbitrary nodes separated by distance x_0 are k hops apart in the Log-normal-Nakagami model when the Nakagami parameter $m = 1$. Therefore, the corresponding network subjects to log-normal shadowing and Rayleigh fading. The result shown in Fig. 6 verifies the accuracy of our analysis. Further, it can be seen that Rayleigh fading reduces the probability that two nodes are connected by a path with k hops. This can be explained by the exponentially distributed RSS over the mean value caused by the Rayleigh fading which reduces the probability of direct connection. Therefore, Rayleigh fading has a negative impact on the network connectivity. A similar result is also observed in the next subsection.



(a) In the unit disk model, Dep2-unit is the result in the unit disk model from this paper, while Dep2-log is the result in log-normal shadowing model by letting $\sigma = 0$. Dep2-log is indistinguishable in the plot because the curve fully agrees with Dep2-unit.



(b) In the log-normal shadowing model

Fig. 5: The probability that two arbitrary nodes separated by Euclidean distance x_0 are k hops apart. $Dep1$ stands for the result calculated by considering the dependency on previous one hop. $Dep2$ is the result from this paper.

6.2 Effective energy consumption

Fig. 7 shows the probability of successful transmissions and the Eng_{eff}. It can be seen that, unsurprisingly, the probability of successful transmissions increases from nearly 0 to nearly 1 as r_0 increased from 10 to 50. In contrast, the effective energy consumption could hardly have been predicted by heuristic reasoning, and needs more explanations.

Take the results under the unit disk model as example. When r_0 is small, the network is made up of a large number of small components. An increase in r_0 will cause an increase in the size (number of nodes) of the components and also a reduction in the number of components. Therefore the average number of hops for unsuccessful transmission increases, and the energy wasted on unsuccessful transmission also increases. Thus there is an initial increase in Eng_{eff} with the increase in r_0. As r_0 further increases, although the average number of hops for successful/unsuccessful transmission still increases, the energy wasted on unsuccessful transmission starts to decrease as more source-destination pairs become connected. The balance of the two effects causes Eng_{eff} to peak at $r_0 \approx 19$. Above this transmission range, the decrease in wasted energy starts to dominate, which causes a subsequent decrease in Eng_{eff}. As r_0 increases further, the average number of hops approaches its maximum and the energy wasted on unsuccessful transmission also reduces to a small amount. These cause Eng_{eff} to reach its minimum at $r_0 \approx 31$. Above this transmission range, most source-destination pairs are connected as shown in Fig. 7 (a.1). Another effect starts to dominate. That is, the increase in r_0 causes the increase in the per-hop energy consumption (like r_0^2) and the decrease in the number of hops (approximately like $1/r_0$). The net effect is an increase in Eng_{eff} with the increased r_0. Most previous studies have only considered this last stage of the relation between the energy consumption

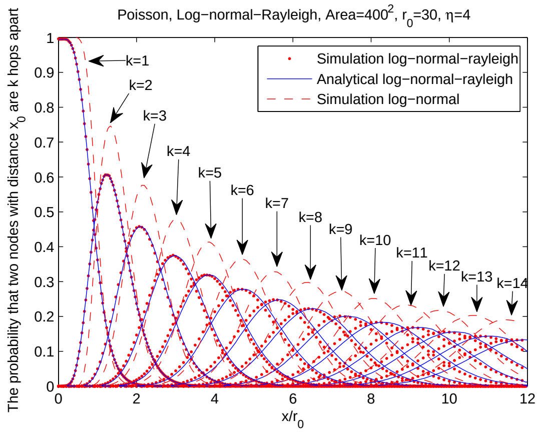
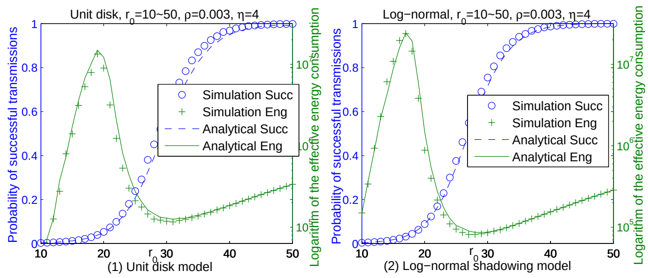
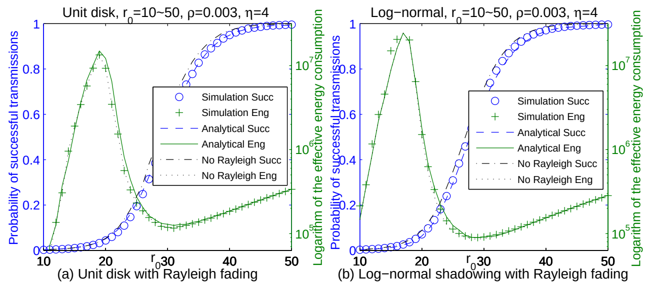


Fig. 6: The probability that two arbitrary nodes separated by distance x_0 are k hops apart in a network subject to log-normal shadowing and Rayleigh fading.



(a) Without small-scale fading



(b) With small-scale fading

Fig. 7: Probability of successful transmissions (Succ) and effective energy consumption (Eng) in the network. Subfigure (a) shows the results without small-scale fading and (b) shows the results with Rayleigh fading. Further, "No Rayleigh" is the result without small-scale fading shown in (b) for comparison.

and the transmission range and therefore cannot give a complete understanding of the energy efficiency in end-to-end packet transmissions.

It is interesting to note that the energy optimizing transmission range is around 31, which corresponds to a network with most (around 70%) source-destination pairs connected but not all of them. (Note that for $r_0 < 20$, Eng_{eff} may be smaller than the minimum Eng_{eff}. However at such value of r_0 most source-destination pairs are disconnected using GF and no meaningful service can be provided by the wireless multi-hop network.) In order for more than 99% source-destination pairs to be connected, r_0 has to be larger than 47 and Eng_{eff} will increase to more than 225% of its minimum value in the unit disk model. A similar result can also be found in the log-normal shadowing model and the models with Rayleigh fading. Therefore significant energy savings can be obtained by requiring most nodes, instead of all nodes, in the network to be connected. This observation also agrees with the analytical results in [12]. In addition, our result gives the amount of energy that can be saved. Purely from an energy-saving perspective and without consideration of other implications, this interesting result shows that the most energy-efficient topology control algorithms should be designed to let 70% (under this network setting) of the source-destination pairs be connected at the same time. The result sheds insight on the design of large wireless multi-hop networks where energy-efficiency is a important issue.

Further, Fig. 7 (b) shows that the probability of successful transmissions is slightly lower in a network with Rayleigh fading compared to a network without Rayleigh fading. This confirms our assertion in the previous subsection.

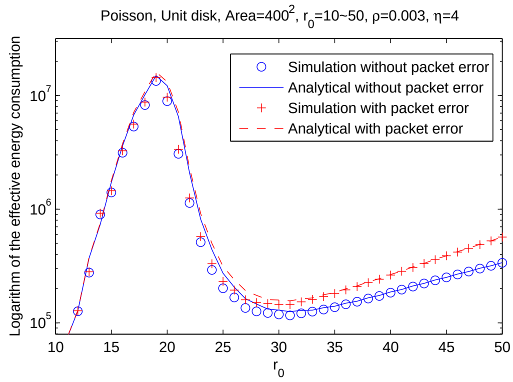


Fig. 8: The effective energy consumption subject to packet error.

Fig. 8 shows the effective energy consumption with a non-zero packet error rate as shown in Eq. 2. The packet error rate increases from 0.004 when $r_0 = 10$ to 0.40 when $r_0 = 50$. It can be seen that as the transmission range increases, the tail of the effective energy consumption increases faster than its error-free counterpart. This is because an increase in the transmission range causes an increase in the number of neighbors and also an increase in the distance between the transmitter and the receiver. This in turn increases the packet error rate and the energy consumption. Therefore when the packet error rate is non-zero, the energy optimizing transmission range becomes smaller as can be seen in Fig. 8.

Fig. 9 shows the effective energy consumption under the log-normal shadowing model with various values of standard deviations. It can be seen that a larger variance in the log-normal shadowing model leads to a lower energy consumption and a smaller optimum transmission range. This is because a larger variance provides a larger probability for a node to forward the packet to a further node that is closer to the intended destination, which is similar to the observation obtained in Section 6.1.

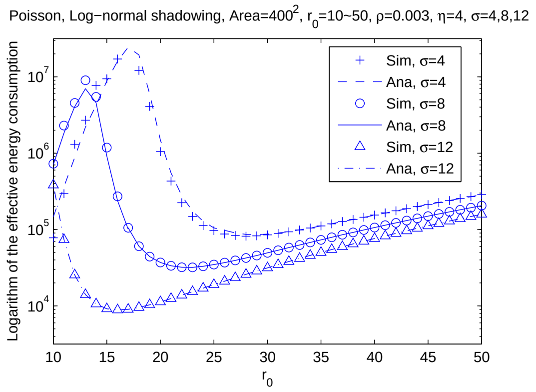


Fig. 9: The effective energy consumption under the log-normal shadowing model with various values of standard deviations.

6.3 Impact of node density and path loss exponent on the optimum transmission range

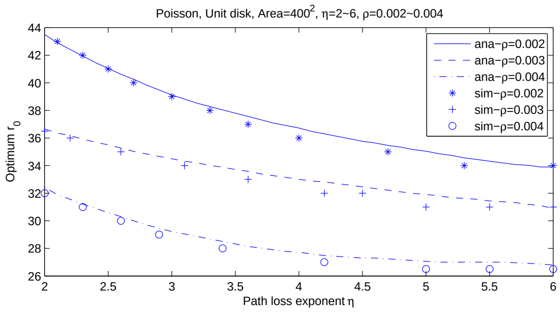


Fig. 10: Impact of node density and path loss exponent on the optimum transmission range.

Fig. 10 illustrates the impact of node density and path loss exponent on the optimum transmission range in the unit disk model. It can be seen that an increase in the node density will cause a decrease in the optimum transmission range. It is because an increase in node density without reduction in transmission range causes an increase in the average number of neighbors as well as the probability of successful transmissions between two nodes. Fig. 10 also shows that a higher path loss exponent will result in a smaller optimum transmission range. This is because an increase in the path loss exponent will cause an increase in the per-hop energy consumption, as given by Eq. 2. Therefore under a higher value of the path loss exponent it is more energy-efficient to have smaller components, hence a smaller optimum transmission range.

It has been shown that the probability $\Pr(k|x)$ and the energy consumption are affected by the node density and path loss exponent. Our analysis fully captures these effects and sheds insight on the design of a wireless multi-hop network.

7 Conclusions and Future Work

We investigated the hop count statistics and the energy consumed in the end-to-end packet transmissions in a wireless multi-hop network. Considering both shadowing and small-scale fading, we obtained analytical results on the probability distribution of the number of hops between two arbitrary nodes. Further, we analyzed the impact of the spatial dependence problem on the $\Pr(k|x)$. Considering the randomness of node deployment and a complex radio environment which may result in disconnected paths between nodes, we derived the distribution of the number of hops traversed by packets before being dropped if the transmission is unsuccessful. As an application of the above results, we derived the effective energy consumption per successfully transmitted packet in end-to-end packet transmission. We showed that there exists an optimum transmission range which minimizes the effective energy consumption. The research provides useful guidelines on the design of a multi-hop network in the presence of shadowing and fading.

The hop count statistics obtained in this paper will also be useful to determine other aspects of wireless multi-hop network performance, e.g. end-to-end throughput and delay. Allowing disconnected paths enables our results to be applicable to sparse network, which is essential to the study of partial connectivity [38]. Moreover, we plan to study the hop count statistics and energy consumption in a mobile ad-hoc network in the future.

References

[1] W. Hu, V. N. Tran, N. Bulusu, C. T. Chou, S. Jha, and A. Taylor, “The design and evaluation of a hybrid sensor network for cane-toad monitoring,” in *Fourth International Symposium on Information Processing in Sensor Networks*, 2005, pp. 503– 508.

[2] M. Conti and S. Giordano, “Multihop ad hoc networking: The reality,” *IEEE Communications Magazine*, vol. 45, pp. 88–95, 2007.

[3] X. Ta, G. Mao, and B. D. O. Anderson, “On the probability of k-hop connection in homogeneous wireless sensor networks,” *IEEE Communication Letters*, vol. 11, no. 8, pp. 662–664, 2007.

[4] ——, “Evaluation of the probability of k-hop connection in homogeneous wireless sensor networks,” in *IEEE Global Telecommunications Conference*, Washington DC, USA, 2007, pp. 1279–1284.

[5] J.-C. Kuo and W. Liao, “Hop count distribution of multihop paths in wireless networks with arbitrary node density: Modeling and its applications,” *IEEE Transactions on Vehicular Technology*, vol. 56, no. 4, 2007.

[6] S. Dulman, M. Rossi, P. Havinga, and M. Zorzi, “On the hop count statistics for randomly deployed wireless sensor networks,” *International Journal of Sensor Networks*, vol. 1, pp. 89–102, 2006.

[7] M. Zorzi and R. R. Rao, “Geographic random forwarding (geraf) for ad hoc and sensor networks: Multihop performance,” *IEEE Transactions on Mobile Computing*, vol. 2, no. 4, pp. 337–348, 2003.

[8] S. De, A. Caruso, T. Chaira, and S. Chessa, “Bounds on hop distance in greedy routing approach in wireless ad hoc networks,” *International Journal of Wireless and Mobile Computing*, vol. 1, no. 2, pp. 131 – 140, 2006.

[9] F. Xue and P. R. Kumar, “The number of neighbors needed for connectivity of wireless networks,” *Wireless Networks*, vol. 10, no. 2, pp. 169–181, 2004.

[10] P. Padhy, K. Martinez, A. Riddoch, H. L. . R. Ong, and J. K. Hart, “Glacial environment monitoring using sensor networks,” in *Real-World Wireless Sensor Networks Workshop*, 2005, pp. 10–14.

[11] P. Gupta and P. R. Kumar, “The capacity of wireless networks,” *IEEE Transactions on Information Theory*, vol. 46, no. 2, pp. 388–404, 2000.

[12] X. Ta, G. Mao, and B. D. O. Anderson, “On the properties of giant component in wireless multi-hop networks,” in *IEEE Infocom*, 2009.

[13] S. A. G. Chandler, “Calculation of number of hops required in randomly located radio network,” *Electronics Letters*, vol. 25, pp. 1669–1671, 1989.

[14] M. Mauve, J. Widmer, and H. Hartenstein, "Survey on position-based routing in mobile ad hoc networks," *IEEE Networks*, vol. 15, no. 6, pp. 30–39, 2001.

[15] K. Akkaya and M. Younis, "A survey on routing protocols for wireless sensor networks," *Ad Hoc Networks*, vol. 3, no. 3, pp. 325 – 349, 2005.

[16] P. Acevedo Contla and M. Stojmenovic, "Estimating hop counts in position based routing schemes for ad hoc networks," *Telecommunication Systems*, vol. 22, no. 1, pp. 109–118, 2003.

[17] Z. Zhang, G. Mao, and B. D. Anderson, "On the effective energy consumption in wireless sensor networks," in *Proceedings IEEE WCNC*, 2010, pp. 1 – 6.

[18] T. S. Rappaport, *Wireless Communications: Principles and Practice, 2nd Edition*. Prentice Hall, 2001.

[19] R. Hekmat and P. V. Mieghem, "Connectivity in wireless ad-hoc networks with a log-normal radio model," *Mobile Networks and Applications*, vol. 11, no. 3, pp. 351–360, 2006.

[20] S. Mukherjee and D. Avidor, "Connectivity and transmit-energy considerations between any pair of nodes in a wireless ad hoc network subject to fading," *IEEE Transactions on Vehicular Technology*, vol. 57, no. 2, pp. 1226–1242, 2008.

[21] D. Miorandi and E. Altman, "Coverage and connectivity of ad hoc networks in presence of channel randomness," in *INFOCOM 2005. 24th Annual Joint Conference of the IEEE Computer and Communications Societies. Proceedings IEEE*, vol. 1, 2005, pp. 491– 502.

[22] M. Haenggi, "On routing in random rayleigh fading networks," *IEEE Transactions on Wireless Communications*, vol. 4, no. 4, pp. 1553 – 1562, 2005.

[23] J. Deng, Y. S. Han, P.-N. Chen, and P. K. Varshney, "Optimal transmission range for wireless ad hoc networks based on energy efficiency," *IEEE Transactions on Communications*, vol. 55, no. 9, pp. 1772–1782, 2007.

[24] R. Zhang and J.-M. Gorce, "Optimal transmission range for minimum energy consumption in wireless sensor networks," in *IEEE Wireless Communications and Networking Conference*, 2008, pp. 757 – 762.

[25] C. Bettstetter and C. Hartmann, "Connectivity of wireless multihop networks in a shadow fading environment," *Wireless Networks*, vol. 11, no. 5, pp. 571 – 579, 2005.

[26] M. K. Simon and M.-S. Alouini, *Digital Communication Over Fading Channels*. John Wiley and Sons, Inc., 2000.

[27] M. M. Carvalho and J. J. Garcia-Luna-Aceves, "Delay analysis of IEEE 802.11 in single-hop networks," in *IEEE International Conference on Network Protocols*, 2003, pp. 146–155.

[28] O. Kasten, "Energy consumption," 2001. [Online]. Available: http://www.inf.ethz.ch/personal/kasten/research/bathtub/

[29] G. Xing, C. Lu, R. Pless, and Q. Huang, "Impact of sensing coverage on greedy geographic routing algorithms," *IEEE Transactions on Parallel and Distributed Systems*, vol. 17, no. 4, pp. 348– 360, 2006.

[30] L. E. Miller, "Distribution of link distances in a wireless network," *Journal of Research of the National Institute of Standards and Technology*, vol. 106, pp. 401–412, 2001.

[31] E. W. Weisstein, "Circle-circle intersection," 2009. [Online]. Available: http://mathworld.wolfram.com/Circle-CircleIntersection.html

[32] S. Dulman and P. Havinga, "Statistically enhanced localization schemes for randomly deployed wireless sensor networks," in *Proceedings of the Intelligent Sensors, Sensor Networks and Information Processing Conference*, 2004, pp. 403–410.

[33] N. Lo and H.-Y. Kuo, "Two-hops neighbor-aware routing protocol in mobile ad hoc networks," in *Lecture Notes in Computer Science*. Springer Berlin / Heidelberg, 2008, vol. 4611/2007, pp. 340–349.

[34] C. J. Mallery, S. Medidi, and M. Medidi, "Relative localization with 2-hop neighborhood," in *International Symposium on a World of Wireless, Mobile and Multimedia Networks*, 2008, pp. 1 – 4.

[35] I. Khalil, S. Bagchi, and N. Shroff, "Liteworp: A lightweight countermeasure for the wormhole attack in multihop wireless networks," in *Proceedings of the 2005 International Conference on Dependable Systems and Networks*, 2005, pp. 612 – 621.

[36] R. Nelson, *Probability, stochastic processes, and queueing theory: the mathematics of computer performance modelling*. Springer, 1995.

[37] M. Franceschetti and R. Meester, *Random networks for communication: From Statistical Physics to Information Systems*. Cambridge University Press, 2007.

[38] Z. Zhang, S. C. Ng, G. Mao, and B. D. O. Anderson, "On the k-hop partial connectivity in finite wireless multi-hop networks," in *Proceedings IEEE ICC*, 2011, pp. 1–5.

[39] P. Gupta and P. R. Kumar, "Critical power for asymptotic connectivity in wireless networks," *Stochastic Analysis, Control, Optimization and Applications: A Volume in Honor of W. H. Fleming, Eds. W. M. McEneany, G. Yin, Q. Zhang.*, p. 547 566, 1998.

[40] G. Mao, "Research on wireless multi-hop networks: Current state and challenges," in *submitted as invited position paper to International Conference on Computing, Networking and Communications*, 2012.

[41] S. C. Ng, G. Mao, and B. D. Anderson, "Analytical bounds on the critical density for percolation in wireless multi-hop networks," in *accepted by GLOBECOM*, 2011.

[42] S. C. Ng, W. Zhang, Y. Zhang, Y. Yang, and G. Mao, "Analysis of access and connectivity probabilities in vehicular relay networks," *IEEE Journal on Selected Areas in Communications*, vol. 29, no. 1, p. 140, 2011.

[43] S. Rajabi, M. Shahabadi, and M. ArdebiliPoor, "Modeling of the correlation coefficients of a receive antenna array in a mimo multipath channel," in *IEEE/IFIP International Conference in Central Asia on Internet*, 2006, pp. 1–4.

[44] P. Agrawal and N. Patwari, "Correlated link shadow fading in multi-hop wireless networks," *IEEE Transactions on Wireless Communications*, vol. 8, no. 8, p. 40244036, 2009.

[45] C. X. Wang, M. Patzold, and Q. Yao, "Stochastic modeling and simulation of frequency-correlated wideband fading channels," *IEEE Transactions on Vehicular Technology*, vol. 56, no. 3, pp. 1050 – 1063, 2007.

PLACE PHOTO HERE

Zijie Zhang received his BEng degree in Electronic and Communications Engineering from the University of Hong Kong in 2008. He is currently a PhD candidate in the School of Electrical and Information Engineering at the University of Sydney. His research interests include wireless multi-hop networks, vehicular ad-hoc networks, mobile ad-hoc networks and graph theory.

PLACE PHOTO HERE

Guoqiang Mao received PhD in telecommunications engineering in 2002 from Edith Cowan University. He joined the School of Electrical and Information Engineering, the University of Sydney in December 2002 where he is a Senior Lecturer now. His research interests include wireless localization techniques, wireless multi-hop networks, graph theory and its application in networking, and network performance analysis. He is a Senior Member of IEEE.

PLACE PHOTO HERE

Brian D. O. Anderson (M'66-SM'74-F'75-LF'07) was born in Sydney, Australia, and educated at Sydney University in mathematics and electrical engineering, with PhD in electrical engineering from Stanford University in 1966. He is a Distinguished Professor at the Australian National University and Distinguished Researcher in National ICT Australia. His awards include the IEEE Control Systems Award of 1997, the 2001 IEEE James H Mulligan, Jr Education Medal, and the Bode Prize of the IEEE Control System Society in 1992, as well as several IEEE best paper prizes. He is a Fellow of the Australian Academy of Science, the Australian Academy of Technological Sciences and Engineering, the Royal Society, and a foreign associate of the National Academy of Engineering. His current research interests are in distributed control, sensor networks and econometric modelling.

Supplementary Material

8 Related work

The hop count statistics were first investigated by Chandler [13] in 1989. He analyzed the probability that two randomly chosen nodes separated by a known distance can communicate in k or less hops where nodes are uniformly distributed over a plane. However the analysis was incomplete as the aforementioned spatial dependence problem was incorrectly ignored. Ta et al. [3] investigated the probability $\Pr(k|x)$ for nodes Poissonly distributed in a square. They pointed out the spatial dependence problem in the analysis of the probability $\Pr(k|x)$. Later in [4] the same authors empirically improved their earlier result in [3] by considering the impact of boundary effect and the spatial dependence problem.

In real applications, the packets are forwarded from a source to a destination according to certain routing algorithms. Many routing algorithms (e.g. LEACH, AODV or geographic routing [14]) share the similar idea of greedy forwarding (GF) [14], that is, to forward the message to the node that is closest to the destination [15]. Much research on the hop count statistics is based on the distributed routing algorithm GF, however the spatial dependence problem were incorrectly ignored. Specifically, two nodes are k hops apart if the path between them, *using GF*, is k hops. Zorzi et al. [7] proposed a GF algorithm for a network where nodes are Poissonly distributed in the coverage area of a transmitting node. They studied an upper and a lower bound on the average number of hops between two nodes separated by a known Euclidean distance, where the focus of this paper is on a complete characterization of the probability distribution of the number of hops between two arbitrary nodes in the network. **In [16], Contla and Stojmenovic considered position based routing schemes for a wireless multi-hop network where nodes are uniformly distributed in a square. They studied the average number of hops between an arbitrary pair of source-destination nodes.** As pointed out in Section 1, many applications need the knowledge of the probability distribution of the number of hops, instead of just the mean value. Dulman et al. [6] investigated the probability $\Pr(k|x)$ by estimating the expected progress per hop using GF. They considered the impact of the Euclidean distance between neighboring nodes in the previous hop on the progress in the current hop. Both [7] and [6] were established on the assumption that a packet can always reach the destination using GF. Further, the aforementioned research only considered the impact of one previous hop in their studies. Recently, the accuracy of the probability $\Pr(k|x)$ was significantly improved in [17] by considering the spatial dependence of two-hop neighbors.

The aforementioned results are all based on the *unit disk communication model*, in which two nodes are directly connected if and only if (iff) the Euclidean distance between them is smaller than or equal to the transmission range. The unit disk model is simple but unrealistic [18]. Considering the log-normal shadowing model, Hekmat and Mieghem [19] showed, through simulations, that the probability of a network being connected increases with increasing value of the shadowing parameter, which is the ratio between the standard deviation of shadowing and the path loss exponent [18]. Mukherjee and Avidor [20] considered the impact of the log-normal shadowing on the probability $\Pr(k|x)$ in a wireless ad hoc network where nodes are Poissonly distributed in a disk, which ignored the spatial dependence problem. In addition to shadowing, the communication between two nodes can be affected by the small-scale fading (e.g. Rayleigh fading). From the connectivity point of view, Miorandi and Altman [21] studied the node isolation probability in a network subject to both log-normal shadowing and Rayleigh fading. They showed that Rayleigh fading reduces the connectivity probability of the network. Moreover, Haenggi [22] studied the routing performance for large multi-hop networks, considering the impact of Rayleigh fading on the end-to-end delivery probability. It is shown that routing over many short hops is not as beneficial in a network subject to Rayleigh fading as that for a network without Rayleigh fading. In this paper, we considered the impact of both log-normal shadowing and small-scale fading on the hop count statistics. Further, our analysis takes into account the impact of the spatial dependence problem, which is a major technical hurdle in the accurate analysis of the probability $\Pr(k|x)$.

The analysis on the hop count statistics can be used in a number of areas in wireless multi-hop networks. This paper focuses on its use in energy-efficient operations of wireless multi-hop networks as an example. Minimizing energy consumption is one of the major considerations in the design of battery powered wireless multi-hop networks. In many applications it is difficult to change or re-charge a battery for the wireless nodes. From a designer point of view, a popular approach of reducing energy consumption is optimally choosing the transmission power. In [23], Deng et al. considered a network where nodes are Poissonly distributed in a circular area. They assumed that there is always a path between any pair of nodes using GF. By analyzing the average progress per hop that a packet is transmitted towards the destination, they obtained analytical results on the distance-energy efficiency, which is the ratio of the average progress to the energy consumed in a single transmission, and the optimum transmission range that maximizes the distance-energy efficiency for high-density networks. Zhang and Gorce [24] considered the impact on energy consumption of unreliable links. They postulated that with a certain probability a transmission between two directly connected nodes is unsuccessful, re-transmissions may then be required and energy consumption may be consequently higher. The extra energy consumed due to unreliable links is also considered in this paper.

9 Usefulness of the hop count statistics

The results on the hop count statistics hold the key to solving a large number of problems in wireless multi-hop networks:

- The summation of $\Pr(k)$ from $k = 1$ to $k = \infty$ provides the probability that two randomly chosen nodes are connected (via a multi-hop path), which in turn can lead to result on the probability of a connected network. The result will be valid for not only large-scale networks [39] but also for small-scale networks which are often encountered in real applications. So far, there are few analytical results on the connectivity of small-scale networks.
- The hop count statistics are also useful in network capacity analysis. In [40], it is demonstrated that the capacity scaling law of multi-hop networks observed in [11] can be easily explained by the increase in the average number of hops (hence the increase in the portion of bandwidth spent on relaying traffic) as the network becomes larger.
- The probability $\Pr(k)$ is also useful in estimating the energy consumption, the network lifetime and the reliability of end-to-end packet transmission [7]. As shown in this paper, $\Pr(k)$ can be used to estimate the effective energy consumption and help the network designers to choose the optimum transmission range/power to minimize the energy consumption. It can also be readily used to help a network designer to set the transmission range/power to provide a guaranteed performance on the end-to-end packet transmissions.
- The probability $\Pr(k|x)$ has been used in [41] to form a novel approach to obtain bounds on the critical density for percolation in wireless multi-hop networks, a well-known open problem in the area. In [38] results on $\Pr(k|x)$ are used as a main tool to study the partial connectivity of a wireless multi-hop network with infrastructure support. Besides its use in performance analysis, a protocol designer can use results on $\Pr(k|x)$ to help choose the optimum protocol parameters, e.g. the timeout parameter TTL used in many routing protocols, to balance bandwidth (or energy) consumption and probability of successful delivery [5].
- The probability density $\Pr(x|k)$ is useful in estimating the distance between two nodes from their neighborhood information and obtaining variance of such an estimate, which has in turn been used in forming a localization algorithm with improved performance [8], [32].
- Further, the technique used to derive $\Pr(k|x)$ can be simplified to study 1D networks or grid networks so that the analysis can be applied to study vehicular networks, see [42] for an example where $\Pr(k|x)$ is used to derive the access and connectivity probabilities of 1D vehicular networks.

10 Reliability of the assumption of the independence between links

In some environments, the assumption of independence of connections may not be accurate while in other environments (e.g. open space) it is a reasonable assumption. For example, it is generally accepted that if a pair of transmitters are separated by more than $\lambda/4$, where λ is the wavelength, their signals at a common receiver can be regarded as statistically independent. Further it was shown [43] that if a pair of receivers are separated by more than λ, their received signals from a common transmitter are only weakly correlated (with a correlation coefficient less than 0.15). At a typical frequency of 5GHz, $\lambda = 0.06$m. Thus the requirement on the separation of vehicles can be easily met. We also note that although field measurements in real applications seem to indicate that the connectivity between different pairs of geographically/frequency proximate wireless nodes are correlated [44], [45], the independence assumption is generally considered appropriate for far-field transmission and has been widely used in the literature under many channel models including the log-normal shadowing model [18], [20], [25].