Analysis of k-Hop Connectivity Probability in 2-D Wireless Networks with Infrastructure Support

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***Abstract*—Wireless multi-hop networks with infrastructure support have been actively studied to solve the scalability problem in large scale vehicular and sensor networks that the end-to-end throughput and other performance metrics decrease sharply with the increase in the number of nodes in the network. In the infrastructure-based networks, wireless nodes are allowed to access the base stations either directly or via a multi-hop path. In order to provide meaningful services, it is often desirable to limit the number of hops in the wireless multi-hop path. In this paper, we study a 2-D wireless network where users are Poissonly distributed in a square area and base stations are placed at the four corners of the square area as a typical component of a larger network where users are randomly distributed and base stations are regularly deployed. We obtain analytically the exact and approximate k-hop connectivity probability for $k = 2$, i.e. the probability that all users can access to at least one base station in at most two hops, under a generic channel model. The results are verified by simulations and can be used in network planning, design and resource management.**

***Index Terms*—wireless network, infrastructure-based, generic channel model, connectivity probability, two-hop-relay.**

I. Introduction

Wireless ad hoc networks have been actively studied in the recent decades. However, the studies have shown that ad hoc networks are not scalable. For example, consider an ad hoc network formed by having n nodes uniformly distributed on a unit disk area and each node is capable of transmitting at W bits per second, Gupta and Kumar [1] reported that the throughput obtainable by each node for a randomly chosen destination is $\Theta(\frac{W}{\sqrt{n \log n}})$ bits per second under a protocol or physical model. The result suggests that the per-node throughput approaches zero as the number of nodes increases. This shortcoming of the ad hoc networks could seriously impact its usefulness in real world deployment.

The key factor which causes the large degradation in per-node throughput of a large scale ad hoc network is found to be the average hop count for each source node to reach its destination node. In particular, Li *et al.* [2] have shown that for the per-node throughput to scale with the network size the average hop count between source and destination nodes *must* remain small as the network grows. Therefore, it is meaningful to limit the communication path length between nodes in the network to keep the average hop count in the network small. A possible solution not to affect the selection of source/destination pairs, namely the traffic pattern, in the network is to introduce some long-range links in the network [3] by using base stations which are interconnected by a wired backbone network. This type of network is well known as the wireless network with infrastructure support. By going through the base stations and the backbone network, the source nodes can then leapfrog a number of hops and reach their destination nodes in a much reduced hop count, thus guaranteeing QoS. Further we can restrict the maximum allowable hop count from any node to their respective nearest base stations. This has led to the proposal of the so-called the *k-hop connectivity problem*, i.e. all nodes must be either directly or indirectly connected to at least one base station in at most k hops.

Solving the k-hop connectivity problem is important, which can be seen from the following. First, it limits the communication path length between any source/destination pair in the network. By keeping k small as the number of network nodes grows, the scalability of the network can be guaranteed. Second, limiting the maximum hop count between a user and a base station may provide additional benefits, e.g. in localization. Particularly, it was shown in [4] that the location estimation error of a sensor increases with its hop count to the anchors (e.g. base stations with known location). By limiting the maximum distance between a user and a base station, it also helps to improve the localization accuracy.

To solve the k-hop connectivity problem, we can start by assigning k a small positive integer. For $k = 1$, the system is essentially the same as a Cellular system, whose properties have been extensively explored. For $k = 2$, the system is practical because it has comparatively simpler system and protocol design than those with $k > 2$. The 2-hop connectivity network does not require complicated ad hoc routing while enjoying a better service coverage than the 1-hop connectivity network [5]. In this paper, we develop an analytical model to investigate the performance of the 2-hop connectivity network. We obtain the closed-form formula for calculating the exact connectivity probability. To the best of our knowledge, the

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exact 2-hop connectivity probability has not been presented so far for 2-D networks. Unlike other work which focuses only on the unit disk communication model, a generic channel model is used in our analysis where the results can also be applied to the unit disk model. In addition to the exact result, an approximate connectivity probability is also derived. Although the approximate connectivity probability is less accurate but gives better intuitive understanding of the interactions among the performance impacting factors. At the current stage, we assume the nodes in the network are static. Nevertheless, we expect our analysis in this paper to hold with a good degree of approximation also for the mobile networks with the random waypoint mobility model.

The rest of this paper is organized as follows. In Section II we introduce related work on infrastructure-based wireless networks. In Section III we define the system model. In Section IV we present the analysis of the connectivity probability under a generic radio channel model. In Section V we focus on two widely used radio channel models, i.e. the unit disk communication model and the log-normal shadowing model, and their analytical and simulation results, followed by conclusions in Section VI.

II. Related Work

Connectivity probability has been studied in the literature for 1-D [6], [7], [8] and 2-D [9], [10], [7] wireless networks with infrastructure support.

In 1-D networks, Miorandi and Altman [6] investigated the probability that a wireless node separated from the base station by a given Euclidean distance is connected (either directly or via multi-hop paths) to the base station under the assumption that other nodes are randomly distributed along the line. The unit disk communication model with a deterministic and random transmission range were considered. Dousse and Thiran [7] considered a network where nodes are Poissonly distributed on a line segment of length L and two base stations are placed at both ends of the line segment. Assume the unit disk model, they obtained analytically the probability that a node at distance x from the left base station is connected to at least one base station. For a network with a similar setup as in [7] but generalized to m base stations arbitrarily distributed in the line segment, Ng *et al.* [8] obtained analytically the probability that *all* nodes in the network are connected to at least one base station under the unit disk model.

For a 2-D network where a total of n nodes are uniformly distributed in a circular area of unit radius, Ojha *et al.* [9] obtained a lower bound on the transmission range required for all nodes in the network to be asymptotically connected to the base station at the center of the circular area as $n \to \infty$ under the unit disk model. Assume that both base stations and users are Poissonly distributed in $\Re^2$ and consider log-normal channel model, Mukherjee *et al.* [10] obtained a lower bound on the probability that an arbitrary user *cannot* reach any base station in less than or equal to t hops using the assumption that the event that one user can reach any base station in k hops is independent of the event that another user being able to reach any base station in k hops. In [7], Dousse and Thiran considered a square area where base stations are placed at the four corners of the square area, and other nodes are Poissonly distributed in the square area with known density λ. They obtained the probability that an arbitrary node is connected to at least one base station as $\lambda \to \infty$ under the unit disk model.

In the literature, most results on connectivity probability in 2-D networks are only applicable when the number of nodes (or the node density) goes to infinity (e.g. [9], [7]). Whether the results are still valid when the number of nodes *does not* go to infinity is yet to be justified. In this paper we assume users are Poissonly distributed with finite user density. Many studies on the networks with finite number of nodes only provide loose bounds to the connectivity probability (e.g. [10]). We obtain analytically the exact connectivity probability considering the maximum allowable hop count from each user to the base stations is two. A generic radio channel model is considered in the study. In addition, we also include the analytical approximate connectivity probability which provides reasonably close results to the true values. Our work on 1-D networks under the unit disk model can be found in [11].

III. System Model

We consider an infrastructure-based 2-D wireless network, as shown in Fig. 1, wherein base stations are deployed regularly at grid points, while users are randomly distributed in the same area following a Poisson distribution. We analyze the k-hop connectivity probability for $k = 2$, i.e. the probability that all users can access at least one base station (BS) within two hops, of the network by investigating a subnetwork in a square area bounded by four adjacent BSs. Let L be the side length (in kilometers) of the square area and ρ be the user density (in users per square kilometer). The probability that k users are found in the area A (with size $|A|$) is given as $f(k, A) = \frac{(\rho|A|)^k e^{-\rho|A|}}{k!}, k \geq 0$.

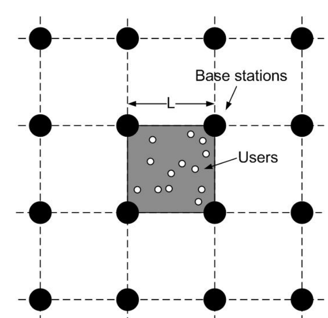

Fig. 1. An Infrastructure-based Two-Dimensional Wireless Network.

Assuming a generic channel model $\mathcal{C}$, let $g_u^{\mathcal{C}}(x)$ be the probability that two users separated by an Euclidean distance x are directly connected. Similarly, denote by $g_b^{\mathcal{C}}(x)$ the probability that a user and a BS separated by an Euclidean distance x are directly connected. We assume that the event that two users (or a user and a BS) are directly connected is independent of the event that another two users (or a user and a BS) are directly connected. We also assume that

$g_b^{\mathcal{C}}(x) \geq g_u^{\mathcal{C}}(x)$. This assumption is justified because it is often the case that a BS can not only transmit at a larger transmission power than a user/wireless node, it can also be equipped with more sophisticated antennas, which make it more sensitive to the transmitted signal from a user/wireless node.

IV. Analysis of The Connectivity Probability

Without loss of generality, assume that the BS located at the bottom left corner of the considered square area is labeled as BS_1 and has coordinate $(0,0)$. Similarly, the BSs at the bottom right, top left and top right corners are labeled as BS_2, BS_3 and BS_4, and have coordinates $(L,0)$, $(0,L)$ and (L,L) respectively. With a slight abuse of the notations let BS_1 to BS_4 also denote the coordinates of the BSs. Denote by $G(L,\rho,\mathcal{C})$ the subnetwork in a square area with side length L, user density ρ and channel model $\mathcal{C}$. Denote by S the area of the subnetwork. We investigate the probability p_c that all users in $G(L,\rho,\mathcal{C})$ are connected to at least one of the BSs at the four corners of the square area *in at most two hops*. The probability that a user which is located at $\boldsymbol{x} \in S$ is directly connected to either BS is equal to one minus the probability that the user is directly connected to none of the four BSs. Note that the probability that a user is directly connected to a BS is independent of the probability that the user is directly connected to another BS. Therefore,

$$p_1(\boldsymbol{x}) = 1 - \prod_{i=1}^{4}(1 - g_b^{\mathcal{C}}(\|\boldsymbol{x} - BS_i\|)) \qquad (1)$$

where $\|.\|$ denotes the Euclidean norm. In order to derive p_c we need the following lemmas.

Lemma 1. *Let K_1 be the set of users in the subnetwork $G(L,\rho,\mathcal{C})$ which are directly connected to either BS, then K_1 has an inhomogeneous Poisson distribution with density $\rho p_1(\boldsymbol{x})$ where $p_1(\boldsymbol{x})$ is given by Eq. (1).*

Proof: Let K denotes the set of users in $G(L,\rho,\mathcal{C})$. Then K has a homogeneous Poisson distribution with density ρ over the square area $S=[0,L]^2$. Consider a realization of K and remove a user located at $\boldsymbol{x}$ from this realization with probability $1-p_1(\boldsymbol{x})$, independent of the removal probability of other users. The remaining set of users forms a realization of K_1. Note that the above procedure which removes/retains users independently with some probabilities is called *thinning* and it generates an inhomogeneous Poisson point process of density function $\rho p_1(\boldsymbol{x})$ [12, pp. 9-10]. That is, let $\mathcal{N}(K_1)$ be the number of users in K_1, then

$$\Pr(\mathcal{N}(K_1)=j) = \frac{(\int_S \rho p_1(\boldsymbol{x}) d\boldsymbol{x})^j}{j!} e^{-\int_S \rho p_1(\boldsymbol{x}) d\boldsymbol{x}}. \qquad (2)$$

Refer to [12] for the detailed proof. ■

Lemma 2. *Let $p_c(Y)$ be the 2-hop connectivity probability of $G(L,\rho,\mathcal{C})$ conditioned on that the number of users directly connected to either BS is n and they are located at $Y = \{\boldsymbol{y}_1, \boldsymbol{y}_2, \cdots, \boldsymbol{y}_n : \boldsymbol{y}_i \in S = [0,L]^2, 1 \leq i \leq n\}$; let $p_Y(Y)$ be the probability density function (pdf) of Y conditioned on that there are n users directly connected to either BS. The following properties hold.*

$$\text{(i)} \qquad p_Y(Y) = \prod_{i=1}^{n} \frac{p_1(\boldsymbol{y}_i)}{\int_S p_1(\boldsymbol{x}) d\boldsymbol{x}} \qquad (3)$$

$$\text{(ii)} \qquad p_c(Y) = e^{-\int_S \rho(1-p_1(\boldsymbol{x}))\prod_{i=1}^{n}(1-g_u^{\mathcal{C}}(\|\boldsymbol{x}-\boldsymbol{y}_i\|))d\boldsymbol{x}} \qquad (4)$$

Proof: For $n=1$, $p_Y(\boldsymbol{y}_1) = \frac{p_1(\boldsymbol{y}_1)}{\int_S p_1(\boldsymbol{x}) d\boldsymbol{x}}$ is the probability that a user in K_1 is located at $\boldsymbol{y}_1$. As a consequence of Lemma 1, $p_1(\boldsymbol{y}_i)$ and $p_1(\boldsymbol{y}_j)$ are mutually independent for $i \neq j$, the result follows for Eq. (3).

For Eq. (4), note that a user at $\boldsymbol{x}$ is not connected to any BS in at most two hops if it is not directly connected to any BS (the probability is $1-p_1(\boldsymbol{x})$) *and* it is not directly connected to any user located at $\boldsymbol{y}_i \in Y$ (the probability is $1-g_u^{\mathcal{C}}(\|\boldsymbol{x}-\boldsymbol{y}_i\|)$ for $1 \leq i \leq n$). Therefore, user at $\boldsymbol{x}$ cannot access any BS in at most two hops with probability

$$(1-p_1(\boldsymbol{x}))\prod_{i=1}^{n}(1-g_u^{\mathcal{C}}(\|\boldsymbol{x}-\boldsymbol{y}_i\|)). \qquad (5)$$

Eq. (5) is valid when $\boldsymbol{x} \notin Y$. When $\boldsymbol{x} = \boldsymbol{y}_j$ for arbitrary j, viz. the user being considered is in K_1, the probability that user at $\boldsymbol{x}$ cannot access any BS in at most two hops should be zero. If we assume that $g_u^{\mathcal{C}}(0) = 1$, then Eq. (5) is still valid when $\boldsymbol{x} \in Y$. Applying the thinning procedure and the technique used in Lemma 1, we have the number of users which are neither directly connected to any BS nor directly connected to any of the users at Y is an inhomogeneous Poisson random variable with density $\rho(1-p_1(\boldsymbol{x}))\prod_{i=1}^{n}(1-g_u^{\mathcal{C}}(\|\boldsymbol{x}-\boldsymbol{y}_i\|))$. The result follows immediately. ■

Theorem 1 (Exact result). *Denote by p_c the 2-hop connectivity probability of $G(L,\rho,\mathcal{C})$, i.e. the probability that all users in the subnetwork $G(L,\rho,\mathcal{C})$ are connected to either BS in at most two hops. Then*

$$p_c = \sum_{n=0}^{\infty} \Pr(\mathcal{N}(K_1)=n) \left[\int_{S^n} p_c(Y) p_Y(Y) dY\right] \qquad (6)$$

where $p_c(Y)$ and $p_Y(Y)$ are given by Lemma 2; $S=[0,L]^2$; $\Pr(\mathcal{N}(K_1)=n)$ is given by Eq. (2). When $n=0$, we declare

$$\int_{S^n} p_c(Y) p_Y(Y) dY \bigg|_{n=0} = p_c(Y) p_Y(Y) \bigg|_{n=0} = e^{-\int_S \rho(1-p_1(\boldsymbol{x}))d\boldsymbol{x}}.$$

Proof: Eq. (6) directly follows from the law of total probability, so the details are omitted here. ■

Eq. (6) gives an exact formula for the connectivity probability. However, the equation is very time consuming to compute numerically. In the following, we derive an approximate formula for the connectivity probability which consumes less computational power. The approximation relies on the assumption that the event that a user is connected to either BS in two hops and the event that another user is connected to either BS in two hops are *independent*. The following lemma proves, in a way, that such events are *dependent*.

Lemma 3. *Let $h(\boldsymbol{x})$ be the probability that a user at $\boldsymbol{x}$ is not directly connected to any user in K_1; let $h(\boldsymbol{x}_1, \boldsymbol{x}_2)$ be the probability that two users, at $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ respectively, are not directly connected to any user in K_1. Then, $h(\boldsymbol{x}_1, \boldsymbol{x}_2) \geq h(\boldsymbol{x}_1)h(\boldsymbol{x}_2)$.*

Proof: To begin the proof, imagine we partition $S = [0, L]^2$ into $(L/dl)^2$ non-overlapping square area of differential side length dl. Let $dS_{\boldsymbol{y}}$ be the differential area of side length dl and centered at $\boldsymbol{y}$. Since dl is a very small value, the probability that there exist more than one user within $dS_{\boldsymbol{y}}$ can be ignored and the probability that there exists exactly one user within $dS_{\boldsymbol{y}}$ is $\rho dS_{\boldsymbol{y}}$. The probability that there exists a user in $dS_{\boldsymbol{y}}$ which is also in K_1 is then given by $\rho p_1(\boldsymbol{y}) dS_{\boldsymbol{y}}$. Note that the complement of the previous probability, i.e. the probability that either there is no user in $dS_{\boldsymbol{y}}$ or the user in $dS_{\boldsymbol{y}}$ is not in K_1, is $1 - \rho p_1(\boldsymbol{y}) dS_{\boldsymbol{y}}$. In addition, note that the users at $\boldsymbol{x}$ and $\boldsymbol{y}$ are directly connected to each other with probability $g_u^{\mathcal{C}}(\|\boldsymbol{x} - \boldsymbol{y}\|)$. Therefore, the probability that a user at $\boldsymbol{x}$ is *not* directly connected to a user in K_1 and is located in $dS_{\boldsymbol{y}}$ is $(1 - g_u^{\mathcal{C}}(\|\boldsymbol{x} - \boldsymbol{y}\|))\rho p_1(\boldsymbol{y}) dS_{\boldsymbol{y}}$. So, the probability that the user at $\boldsymbol{x}$ is *not* directly connected to any of the users in K_1 is given by

$$h(\boldsymbol{x}) = \prod_{dS_{\boldsymbol{y}} \subset S} \big[(1 - g_u^{\mathcal{C}}(\|\boldsymbol{x} - \boldsymbol{y}\|))\rho p_1(\boldsymbol{y}) dS_{\boldsymbol{y}} + (1 - \rho p_1(\boldsymbol{y}) dS_{\boldsymbol{y}})\big] \tag{7}$$

$$= e^{-\int_S g_u^{\mathcal{C}}(\|\boldsymbol{x} - \boldsymbol{y}\|)\rho p_1(\boldsymbol{y}) d\boldsymbol{y}} \tag{8}$$

where from Eq. (7) to Eq. (8) we apply $e^{-z} = 1 - z$ for very small value of z. Using the similar approach and the probability that two users, at $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$, are *not* directly connected to a user in K_1 and is located in $dS_{\boldsymbol{y}}$ is $(1 - g_u^{\mathcal{C}}(\|\boldsymbol{x}_1 - \boldsymbol{y}\|))(1 - g_u^{\mathcal{C}}(\|\boldsymbol{x}_2 - \boldsymbol{y}\|))\rho p_1(\boldsymbol{y}) dS_{\boldsymbol{y}}$, we have

$$h(\boldsymbol{x}_1, \boldsymbol{x}_2) = \prod_{dS_{\boldsymbol{y}} \subset S} \big[(1 - g_u^{\mathcal{C}}(\|\boldsymbol{x}_1 - \boldsymbol{y}\|))(1 - g_u^{\mathcal{C}}(\|\boldsymbol{x}_2 - \boldsymbol{y}\|)) \rho p_1(\boldsymbol{y}) dS_{\boldsymbol{y}} + \ (1 - \rho p_1(\boldsymbol{y}) dS_{\boldsymbol{y}})\big] \tag{9}$$

$$= e^{-\int_S [g_u^{\mathcal{C}}(\|\boldsymbol{x}_1 - \boldsymbol{y}\|) + g_u^{\mathcal{C}}(\|\boldsymbol{x}_2 - \boldsymbol{y}\|)]\rho p_1(\boldsymbol{y}) d\boldsymbol{y}} \times e^{\int_S [g_u^{\mathcal{C}}(\|\boldsymbol{x}_1 - \boldsymbol{y}\|) g_u^{\mathcal{C}}(\|\boldsymbol{x}_2 - \boldsymbol{y}\|)]\rho p_1(\boldsymbol{y}) d\boldsymbol{y}} \tag{10}$$

$$\geq h(\boldsymbol{x}_1)h(\boldsymbol{x}_2) \qquad \text{(from Eq. (8))}$$

where the derivation from Eq. (9) to Eq. (10) is similar to the derivation from Eq. (7) to Eq. (8). ■

Before obtaining the approximate result of the 2-hop connectivity probability, we introduce the following lemma.

Lemma 4. *Denote by $p_a(\boldsymbol{x})$ the probability that the user at $\boldsymbol{x}$ is connected to either BS in at most two hops. Then*

$$p_a(\boldsymbol{x}) = 1 - (1 - p_1(\boldsymbol{x}))(1 - p_2(\boldsymbol{x})) \tag{11}$$

where $p_1(\boldsymbol{x})$ is given by Eq. (1); $p_2(\boldsymbol{x}) = 1 - h(\boldsymbol{x})$ is the probability that a user located at $\boldsymbol{x}$ is directly connected to at least one user in K_1 where $h(\boldsymbol{x})$ is given by Eq. (8).

Proof: The result follows immediately from the observation that the event that a user at $\boldsymbol{x}$ is directly connected to either BS is independent of the event that the same user is directly connected to at least one user in K_1. ■

Theorem 2 (Approximate result)**.** *Denote by p_c the 2-hop connectivity probability of $G(L, \rho, \mathcal{C})$, i.e. the probability that all users in the subnetwork $G(L, \rho, \mathcal{C})$ are connected to either BS in at most two hops. Assume that the event that a user is connected to either BS in at most two hops is independent of the event that another user is connected to either BS in at most two hops. Then*

$$p_c = e^{-\int_S \rho(1 - p_a(\boldsymbol{x})) d\boldsymbol{x}} \tag{12}$$

where $p_a(\boldsymbol{x})$ is given by Eq. (11) and $S = [0, L]^2$.

Proof: Let K_2 be the set of users in $G(L, \rho, \mathcal{C})$ which are connected to either BS in exactly two hops. Together with the definition of K_1 in Lemma 1, let $\overline{K_1 + K_2} = K \backslash (K_1 + K_2)$ be the set of users in $G(L, \rho, \mathcal{C})$ which are not connected to either BS in at most two hops. Apply the thinning procedure for K, i.e. consider a realization of K and remove each user located at $\boldsymbol{x}$ independently from this realization, with probability $p_a(\boldsymbol{x})$. The resulting set of users can be viewed as a realization of $\overline{K_1 + K_2}$ under our assumption that the event that one user is connected to either BS in at most two hops is independent of the event that another user is connected to either BS in at most two hops, and the probability that user at $\boldsymbol{x}$ is connected to either BS in at most two hops is $p_a(\boldsymbol{x})$. Using the same technique as that used in the proof of Lemma 1, it can be shown that $\overline{K_1 + K_2}$ has an inhomogeneous Poisson distribution with density $\rho(1 - p_a(x))$. Then all users in $G(L, \rho, \mathcal{C})$ are connected to either BS in at most two hops if and only if $\mathcal{N}(\overline{K_1 + K_2}) = 0$. The result follows. ■

V. Simulations

Based on the above analysis we discuss the 2-hop connectivity probability performance under two specific examples of the generic radio channel model, i.e. unit disk communication model and log-normal shadowing model.

A. Unit Disk Communication Model

In the unit disk model $\mathcal{U}$, assume that two users are directly connected if and only if their Euclidean distance is less than or equal to r; a user and a BS are directly connected if and only if their Euclidean distance is not more than R. That is,

$$g_u^{\mathcal{U}}(x) = \begin{cases} 1 & \text{if } x \leq r \\ 0 & \text{otherwise,} \end{cases} \qquad g_b^{\mathcal{U}}(x) = \begin{cases} 1 & \text{if } x \leq R \\ 0 & \text{otherwise.} \end{cases}$$

where r and R are predetermined values, commonly known as the transmission ranges. Typically we have $R > r$.

B. Log-normal Shadowing Model

The log-normal model $\mathcal{L}$ [13] is commonly used to model the real world signal propagation considering the shadowing effect caused by surrounding environment. In this model, we formulate the received power at a destination user as $p_{rx} = p_0 - 10\alpha \log_{10} \frac{l}{d_0} + N_\sigma$, where p_{rx} is the received power (in dBmW) at the destination user; p_0 is the power (in dBmW) at a reference distance d_0 from the source user; α is the path loss exponent; N_σ is a Gaussian random variable with zero mean and variance σ^2; l is the Euclidean distance between the two users (or a user and a BS depending on the context). A source user can establish a direct connection to a destination user if the received power at the destination user p_{rx} is greater than or equal to a certain threshold power p_{th}^u. Similarly, a source user can establish a two-way direct connection to a destination BS if the received power at the destination BS p_{rx} is greater than or equal to a certain threshold power p_{th}^b. In this paper, we assume that wireless connections between users, and between users and BSs, are symmetric. Note that when $\sigma = 0$, the log-normal model reduces to the unit disk model. Due to this fact, we assign $p_{th}^u = p_0 - 10\alpha \log_{10} \frac{r}{d_0}$, $p_{th}^b = p_0 - 10\alpha \log_{10} \frac{R}{d_0}$ so that the results under log-normal model can be compared with the results under the unit disk model later. It can be shown that under the log-normal model, $g_u^{\mathcal{L}}(x) = \Pr(p_{rx} \geq p_{th}^u) = Q(\frac{10\alpha}{\sigma} \log_{10} \frac{x}{r})$ where $Q(y) = \frac{1}{\sqrt{2\pi}} \int_y^\infty e^{-\frac{x^2}{2}} dx$ is the tail probability of the standard normal distribution. Similarly, $g_b^{\mathcal{L}}(x) = Q(\frac{10\alpha}{\sigma} \log_{10} \frac{x}{R})$.

C. Analytical and simulation results

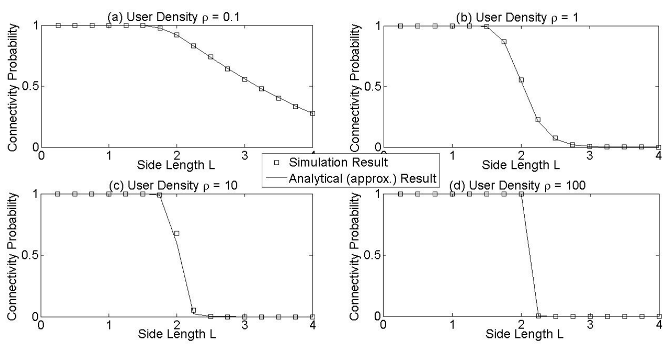

Fig. 2. Connectivity probability with L changing under the unit disk model, $R = 1$, $r = 0.5$, $\rho = 0.1, 1, 10, 100$ respectively.

Fig. 2 shows the connectivity probability given different values of L and ρ under the unit disk model. The analytical results are verified by the simulation results obtained from 40000 randomly generated network topologies. Due to tremendous amount of time required for computing the exact analytical results, only the approximate analytical results are plotted. However, the figure shows that the approximate analytical results match the true values in all circumstances.

Similarly, Fig. 3 shows the results under the log-normal model, specifically for $\alpha = 2$ and $\sigma = 2$. Again the approximate analytical results are verified by the simulation results obtained from 40000 samples. The approximate analytical results match the true values when the user density is low ($\rho = 0.1, 1$). The discrepancy between the approximate results and the true values occurs when the user density is high ($\rho = 10, 100$), but it is still within reasonable range.

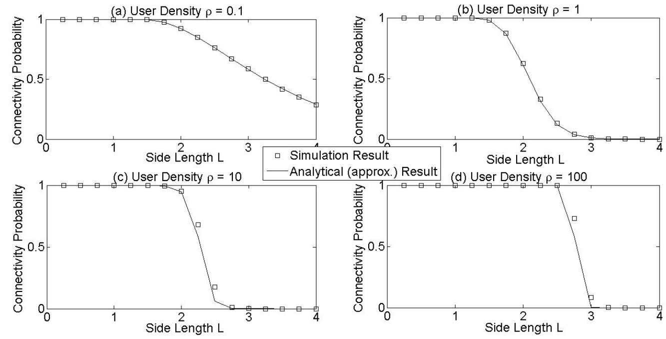

Fig. 3. Connectivity probability with L changing under the log-normal model, $R = 1$, $r = 0.5$, $\alpha = 2$, $\sigma = 2$, $\rho = 0.1, 1, 10, 100$ respectively.

VI. Conclusions

In this paper, we consider a square area where four base stations are placed at the corners of the square area, and users are Poissonly distributed in the same area with known density. Under a generic channel model, we derived the closed-form formulas for the (exact and approximate) probability that all users can access at least one base station in at most two hops. Taking the unit disk model and log-normal shadowing model as two specific examples of the generic channel model, the formulas are verified by simulations. The analysis can be useful in network planning, design and resource management.

In future, we plan to expand the current work on 2-hop connectivity to k-hop connectivity for $k > 2$.

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