On the Information Propagation Speed in Mobile Vehicular Ad Hoc Networks

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Abstract—In this paper, we study the information propagation speed in a 1D mobile ad hoc network formed by vehicles Poissonly distributed on a highway and travelling in the same direction but with random Gaussianly-distributed speeds, independent between vehicles. Assume that time is divided into time slots of equal length and that each vehicle changes its speed at the beginning of each time slot, independent of its speed in other time slots. We derive analytical formulas for the IPS in the above network under the unit disk model. Using the formula, we can straightforwardly study the impact on the information propagation speed of various parameters such as vehicle density, speed and radio range. The accuracy of the formula is validated using simulations. The research provides useful guidelines on the design of vehicular ad hoc networks.

I. INTRODUCTION

Vehicular ad hoc networks (VANETs) have attracted significant interest in recent years due to a large number of potential applications. In this paper, we study the expected propagation speed for a piece of information to be broadcasted along the road, referred to as the information propagation speed (IPS). Due to the mobility of vehicles, the topology of a VANET is changing over time. Further, traffic density on a road can vary significantly depending on time-of-day or day-of-week. Therefore the properties of the information propagation in a VANET can be quite different from those in a static network. Our analysis considers the impact on the IPS of various parameters, e.g. radio range, vehicular traffic density and the time variation of vehicular speed.

It has been shown that a VANET is usually partitioned into a number of clusters [1], [2], where a *cluster* is a maximal set of vehicles in which every pair of vehicles are connected by at least one path. Due to the mobility of vehicles, the clusters are splitting and merging over time. Therefore the information propagation in a VANET is typically based on a store-and-forward scheme as that in a delay tolerant network [1]. Consider the example illustrated in Fig. 1, a piece of information starts to propagate from the origin toward the

This research is partially supported by ARC Discovery project: DP0877562. This research is partially sponsored by the Air Force Research Laboratory, under agreement number FA2386-10-1-4102. The U.S. Government is authorized to reproduce and distribute reprints for Governmental purposes notwithstanding any copyright notation thereon. The views and conclusions contained herein are those of the authors and should not be interpreted as necessarily representing the official policies or endorsements, either expressed or implied, of the Air Force Research Laboratory or the U.S. Government. positive direction of the axis at time t_0 . The vehicles that have received this piece of information are referred to as *informed vehicles*, where other vehicles are *uninformed*. As indicated by the left most ellipse, the first informed vehicle is inside a cluster of four vehicles at time t_0 . At time t_1 , the message is forwarded, in a multihop manner, to the frontmost vehicle in its cluster. The propagation of the message within a cluster, which begins at t_0 and ends at t_1 , is called a *forwarding process*. The propagation speed in a forwarding process is determined by the *per-hop delay* β , which is the time required for a vehicle to receive and process a message before it is available for further retransmission [3]. The value of β depends on the practical implementation, and a common assumption for the value of β is 4ms [3]. We show that the per-hop delay has a significant impact on the IPS, especially when the vehicle density is high.



Fig. 1. Illustration of the topology of a VANET at different time instance. The positive direction of the axis is the direction of information propagation.

Define the *head* (resp. *tail*) at time t to be the informed (resp. uninformed) vehicle with the largest (resp. smallest) coordinate at time t. We say two vehicles can directly communicate with each other iff their Euclidean distance is not greater than the radio range r_0 (We are adopting the *unit disk model*. Other channel models are expected to have a similar IPS, which is left as future work.). As shown in Fig. 1, at time t_1 the tail is outside the radio range of the head. Then a *catch-up process* begins, during which the informed vehicles hold the information until the head catches up the tail. We investigate the IPS by studying both the forwarding process and the catch-up process.

The main contributions of this paper are: firstly, we provide analytical results on the distribution of the time required for a catch-up process. The impact of vehicle density, vehicle speed distribution and vehicle speed variation over time are considered, where previous research (e.g. [2], [4]) has been based on the assumption that vehicle speed does not change over time. Secondly, a first passage phenomenon, to be introduced later, is considered for the accurate analysis of the catch-up process. Thirdly, we report a closed form equation for the distribution of the length of a cluster, where in previous research only a numerical solution [5] or an approximate result [2] was provided. Based on the above results we derive the analytical results for the IPS, which provides useful guidelines on the design of a mobile VANET.

The rest of this paper is organized as follows: Section II reviews the related work. Section III introduces the network model. The analysis on the catch-up process is given in Section IV, followed by the analysis on the forwarding process in Section V. Based on the above results, we derive the IPS in Section VI. Finally Section VII concludes this paper.

II. RELATED WORK

The IPS is an important metric for the performance of VANETs. Through simulations, Wu et al. [3] showed that the IPS varies significantly with different vehicle density on a highway. In [5] Yousefi et al. provided analytical results on the distribution of the inter-vehicle distance in a 1D VANET under the Poisson arrival model: in the Poisson arrival model, the number of vehicles passing an observation point on the road during any time interval follows a homogeneous Poisson process with intensity λ . They further analyzed the connectivity distance, a quantity which is similar to the cluster length introduced later in this paper. However, they did not provide a closed form formula for the distribution of the cluster length. In [4] Agarwal et al. studied the IPS in a 1D VANET where vehicles are Poissonly distributed in the network, the vehicles are assumed to move at the same speed but either in the positive or negative direction of the axis. They derived upper and lower bounds for the IPS, which gave a hint on the impact of vehicle density on the IPS but the bounds are not tight because many factors were ignored in their analysis. In [2] Wu et al. considered a 1D VANET where vehicles are Poissonly distributed and the vehicle speeds are uniformly distributed in a designated range. They provided a numerical method to calculate the IPS when the vehicle density is either very low or very high. Same as [4], their analysis only considered a constant speed over time. In this paper, the time variation of vehicle speed is considered and is shown to have a significant impact on the IPS.

III. NETWORK MODEL

This paper considers a synchronized random walk model. Specifically, time is divided into time slots with equal length τ . Each vehicle randomly chooses its new speed at the beginning of each time slot, independent of other vehicles and its speed in other time slots, according to a Gaussian distribution with a mean value E[v]. The Gaussian speed distribution is a commonly used assumption for the VANETs on the highway [5]–[7]. A positive (resp. negative) value of the speed means that the vehicle is travelling in the same (resp. opposite) direction as the direction of information propagation. The speed of a vehicle can be considered as having a constant component E[v] and a variable component with a zero mean.

Accordingly the vehicular network can be decomposed into two components: a network in which vehicles travel at a constant speed and a network in which vehicles travel at speeds following a zero mean Gaussian distribution. Our analysis focuses on the IPS in the second network component. The first network component is considered separately and is combined into the result at the end of the analysis. Define $f_v(v)$ to be the pdf (probability density function) of the speed distribution. As a consequence of the Gaussian speed distribution: $f_v(v) = \frac{1}{\sigma\sqrt{2\pi}} \exp(\frac{-v^2}{2\sigma^2})$. The value of the time interval τ depends on practical

The value of the time interval τ depends on practical conditions, e.g. a heavy truck may change its speed much less frequently than a sports car. Reasonable values for the time interval can be from 1s to 25s [8]. The vehicle mobility parameters, i.e. E[v], σ and τ , are taken from practical measurements. Typical values for E[v] and σ are given in [7], where the usual record time intervals for a vehicle speed monitor are $\tau = 1s, 5s$ [9]. We conduct our analysis in the discrete time domain ($t = i\tau$) to obtain a closed-form analytical equations which give better insight into the impact of different parameters on the IPS. Extension to the continuous time domain is straightforward.

We adopt a commonly used traffic model in vehicular traffic theory [6] in which vehicles travel independently in the same direction on a 1D infinite line and follow the Poisson arrival model with intensity λ veh/s. It is shown in [5] that if the vehicle speeds do not change over time, then at any time instant the spatial distribution of vehicles follows a homogeneous Poisson process with intensity $\rho = \lambda \int_{-\infty}^{\infty} \frac{f_v(v)}{v} dv$. Using the above result and mathematical induction, it can be shown that under the model considered in this paper, i.e. allowing time variation of vehicles still has the above property.

IV. ANALYSIS ON THE CATCH-UP PROCESS

Without loss of generality, we assume the catch-up process starts at time 0. Define l_c to be the initial Euclidean distance between the head and the tail at time 0. For convenience, a catch-up process where the distance between the head and tail is l_c at the beginning of the catch-up process is referred to as a *catch-up process with gap* l_c . Denote by H_i (resp. P_i) the i^{th} vehicle to the left of the head H_0 (resp. to the right of the tail P_0) at time 0, as shown in Fig. 2. Define w_i to be the Euclidean distance between H_i and H_0 at time 0.

Denote by p(y,t) the probability that the displacement of the head is y at time t, given that the head is positioned at 0 at time 0. Note that the head vehicle at time 0 is not necessarily the head vehicle at time t because the original head may be overtaken by another informed vehicle during (0, t).

A. The movement of the head and the tail

In this subsection, we ignore temporarily the possibility of overtaking, i.e. we consider *a basic catch-up process* involving only the vehicle, which is the head at time 0, catching up with the vehicle, which is the tail at time 0. Therefore the movement of the head is the same as the movement of a single vehicle.



Fig. 2. Illustration of the VANET at the beginning of a catch-up process.



Fig. 3. Illustration of the displacements of the head and tail at time t in a basic catch-up process.

It is obvious that $p(y, \tau)$ is determined by the speed distribution because the speed does not change during a time slot:

$$p(y,\tau) = \frac{1}{\sigma\tau\sqrt{2\pi}}\exp(\frac{-y^2}{2(\sigma\tau)^2})$$
(1)

Due to the independence of the speeds of the vehicle in different time slots, we have:

$$p(y,i\tau) = \overbrace{(p*p*\ldots*p)}^{1-101d} (y,\tau) \tag{2}$$

For $t = i\tau$, we can obtain:

$$p(y,t) = p(y,i\tau) = \frac{1}{\sigma_t \sqrt{2\pi}} \exp(\frac{-y^2}{2\sigma_t^2})$$
(3)

where $\sigma_t^2 = i(\sigma\tau)^2 = \frac{t}{\tau}(\sigma\tau)^2 = t\sigma^2\tau$.

Denote by $\tilde{p}(\tilde{y}, t)$ the probability that the displacement of the tail is \tilde{y} at time t, with regards to its location at time 0. Due to symmetry of the Gaussian distribution: $\tilde{p}(\tilde{y}, t) = p(\tilde{y}, t) = p(-\tilde{y}, t)$.

B. Catch-up delay in a basic catch-up process

Define the *catch-up delay* to be the time taken from the start of the catch-up process till the time when the head and tail move into the radio range of each other for the first time, i.e. $t_2 - t_1$ in Fig. 1. A catch-up delay is t iff the distance between the head and tail reduces to r_0 at t for the first time since time 0. This first passage phenomenon has a significant impact on the analysis of the catch-up delay.

Denote by $p_H(z,t)$ the probability that the reduction of the distance between the head and the tail is z at time t, with regards to their original distance at time 0. As illustrated in Fig. 3, it can be shown that:

$$p_H(z,t) = \int_{-\infty}^{\infty} p(y,t)\tilde{p}(z-y,t)dy \qquad (4)$$

$$= (p*p)(z,t) = \frac{1}{\tilde{\sigma}_t \sqrt{2\pi}} \exp(\frac{-z^2}{2\tilde{\sigma}_t^2}) \quad (5)$$

where $\tilde{\sigma}_t^2 = 2t\sigma^2\tau$.

Define $\xi(z,t)$ to be the first passage probability [10] of $p_H(z,t)$, viz the probability that the reduction of distance between the head and tail is z for the first time at time t.

The relationship between $\xi(z,t)$ and $p_H(z,t)$ can be studied as the first passage time in a stochastic process [11].

We apply a standard procedure to determine the first passage probability [10], [11]. Consider a class of random walks start at time 0 and walk from point 0 to z' that must proceed by going through a point z. The transition from 0 to z' can be decomposed into two independent stages: in the first stage an agent walks from 0 to z for the first time in time t; in the second stage the agent walks from z to z' in the remaining time t' - t, not necessarily for the first time. Then we can obtain the following equation [10], [11]:

$$p_H(z',t') = \sum_{t=0}^{t'} \xi(z,t) p_H(z'-z,t'-t)$$
(6)

The convolution in Eq. 6 can be simplified by the Z-transform with regards to t, which is denoted by superscript *. Inspired by Eq. 6.4 in [11], $p_H(z,t)$ in Eq. 5 can be rewritten in the following form:

$$p_H(z,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp(-jz\alpha - \frac{\tilde{\sigma}_t^2}{2}\alpha^2) d\alpha \tag{7}$$

where j denotes $\sqrt{-1}$.

Then perform the Z-transform on Eq. 7 with regards to t:

$$p_{H}^{*}(z,s) = \sum_{t=0}^{\infty} e^{-st} p_{H}(z,t)$$
(8)

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp(-jz\alpha) \sum_{t=0}^{\infty} \exp(-st) \exp(-\frac{\tilde{\sigma}_t^2}{2}\alpha^2) d\alpha$$

With $\tilde{\sigma}_t^2 = 2t\sigma^2\tau$, there holds:

$$p_H^*(z,s) \tag{9}$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp(-jz\alpha) \sum_{t=0}^{\infty} \exp(-st) \exp(-t\sigma^2 \tau \alpha^2) d\alpha$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp(-jz\alpha)(s+\sigma^2\tau\alpha^2)^{-1}d\alpha$$
(10)

$$= \frac{1}{2\pi} \frac{\pi \exp(-z\sqrt{s/(\sigma^2 \tau)})}{\sqrt{s\sigma^2 \tau}}$$
(11)

Then according to the convolution theorem for the Z-transform, from Eq. 6 we have:

$$p_H^*(z',s) = \xi^*(z,s)p_H^*(z'-z,s)$$
(12)

$$\xi^*(z,s) = \frac{p_H^*(z',s)}{p_H^*(z'-z,s)} = \exp(-z\sqrt{s/(\sigma^2\tau)})$$
(13)

Then by the inverse Z-transform we can obtain:

$$\xi(z,t) = \frac{z}{2t\sqrt{\pi\sigma^2\tau t}}\exp(-\frac{z^2}{4\sigma^2\tau t}) \qquad (14)$$

C. Catch-up delay considering overtaking permitted

In this subsection, we include the possibility of overtaking in the catch-up process, in a way that balances the trade-off between the accuracy and complexity of the result. Denote by $F_{\xi n}(z,t)$ the probability that the reduction of the distance between the head and tail has reached z during time (0,t). The subscript n indicates that n vehicles to the left of the head (H_0) at time 0 are considered, i.e. we consider the possibility that one of these n vehicles $(H_1, H_2,...,H_n)$ may overtake H_0 during (0,t) and become the new head. It is referred to as a n-overtake catch-up process. For a basic catch-up process studied in the previous subsection, i.e. n = 0, $F_{\xi 0}(z, t_c)$ is the cdf of $\xi(z, t)$:

$$F_{\xi 0}(z, t_c) = \sum_{t=0}^{t_c} \xi(z, t) = \sum_{t=0}^{t_c} \frac{z}{2t\sqrt{\pi\sigma^2\tau t}} \exp(\frac{-z^2}{4\sigma^2\tau t}) \quad (15)$$

The distance between the head and tail does *not* reduce to z iff *none* of the distances between H_i and the tail for i = 0, 1, ..., n reduces to z. Therefore:

$$1 - F_{\xi n}(z, t_c) = (1 - F_{\xi 0}(z, t_c))$$
(16)
$$\prod_{i=1}^{n} (1 - \int_{0}^{\infty} F_{\xi 0}(z + w_i, t_c) f_{w_i}(w_i) dw_i)$$

where w_i is the distance between H_i and H_0 at time 0, $f_{w_i}(w_i)$ is the pdf of w_i . It can be shown that [12]:

$$f_{w_i}(w_i) = \frac{\rho e^{-\rho w_i} (\rho w_i)^{i-1}}{(i-1)!}$$
(17)

The choice of parameter n depends on the trade-off between computational complexity and accuracy, which will be discussed in Section IV-E. Finally the expected delay (t_c) for a catch-up process with gap l_c is:

$$E[t_c|l_c] \approx \sum_{t=0}^{\infty} (1 - F_{\xi n}(l_c - r_0, t))$$
(18)

D. Distribution of the gaps l_c

Denote by $f_l(l)$ the pdf of the Euclidean distance between any two adjacent vehicles. Then: $f_l(l) = \rho e^{-\rho l}$. Denote by $f_{l_c}(l_c)$ the pdf of the Euclidean distance between any two adjacent but disconnected vehicles. It is straightforward that:

$$f_{l_c}(l_c) = \frac{f_l(l_c)}{1 - \int_0^{r_0} f_l(l)dl} \quad \text{, where} \quad l_c > r_0 \qquad (19)$$

E. Simulation results

In this section, we report on simulations to validate the accuracy of analytical results. The simulations are conducted in a VANET simulator written in C++. Each point shown in the figures is the average value from 2000 simulations. The radio range is $r_0 = 250m$ [2]. The mobility parameters are $E[v] = 25m/s, \sigma = 7.5m/s$ [7]. Only the results for $\tau = 5s$ are shown in this section since the results for $\tau = 1s$ have a similar accuracy. The traffic density is $\lambda = 0.3$ veh/s, which results in the Poisson intensity $\rho = 0.012$ veh/m, which is a low density resulting in many catch-up processes.

Fig. 4(a) shows the catch-up delay for a catch-up process with gap l_c . It can be seen that the analytical result, which considers a 4-overtake catch-up process, provides a good approximation. Although the discrepancy increases as the length of the gap increases, it can be seen from Fig. 4(b) that the probability for a gap to be larger than 600m is lower



Fig. 4. (a) Expected catch-up delay, (b) pdf of the length of the gap (l_c) .

than 0.1%. Hence using n = 4 is accurate enough in this case. Further, Fig. 4(b) also verifies that the inter-vehicle distance, under our network model and the Gaussian speed distribution, still follows an exponential distribution with $\rho = 0.012$. This property is also expected to be the same in some other speed distributions, an issue which is left as future work.

V. ANALYSIS ON THE FORWARDING PROCESS

A. Cluster length

Define the cluster length x_0 to be the Euclidean distance between the vehicles at the two ends of a cluster. The pdf of cluster length can be studied as the length of busy period in queueing theory [13]. In the following we report the pdf of cluster length without derivation due to page limit.

$$f_{x_0}(x_0) = \frac{\rho}{e^{\rho r_0} - 1} \sum_{i=0}^{\lfloor \frac{x_0}{r_0} \rfloor} \left(\frac{(-\rho e^{-\rho r_0})^i}{i!} \left((x_0 - r_0 i)^i + \frac{i(x_0 - r_0 i)^{i-1}}{\rho} \right) \right)$$
(20)

B. hop count statistics in a cluster

Two vehicles are k hops apart if the length of the shortest path between them is k hops. Define $\phi_k(x_0)$ to be the probability that two vehicles separated by Euclidean distance x_0 are k hops apart. The probability $\phi_k(x_0)$ has been studied in [14]. Using $\phi_k(x_0)$ it is straightforward to calculate $E[k|x_0]$, which is the expected number of hops between two vehicles, in the same cluster and separated by distance x_0 .

Define the *forwarding delay* to be the time required for a packet to be forwarded from the leftmost vehicle in a cluster to the rightmost vehicle in the cluster, which is $t_1 - t_0$ in Fig. 1. Assume that the cluster does not become disconnected during the forwarding process since the forwarding delay is relatively small. Then the expected forwarding delay in a cluster with length x_0 is: $E[t_f|x_0] = \beta E[k|x_0]$.

C. Simulation results

In addition to the simulation settings introduced earlier, the per-hop delay is $\beta = 4ms$ [2]. Fig. 5(a) shows the expected forwarding delay in a cluster with a given length. Fig. 5(b) shows the pdf of cluster length. It can be seen that the analytical results well match the simulation results. The



Fig. 5. Expected forwarding delay and the pdf of the cluster length.

results for other values of the parameters have similar accuracy and are omitted.

VI. INFORMATION PROPAGATION SPEED

A. Analytical results

The entire information propagation process can be considered as a renewal reward process [15, Chapter 7.4] where each cycle consists of a catch-up process followed by a forwarding process and the reward is the information propagation distance. As mentioned in Section III, E[v] is the constant component of the vehicle speed. It can be shown that [1], [2]:

$$E[v_{ip}] \approx \frac{\text{expected length of one cycle}}{\text{expected time duration of one cycle}} + E[v] \quad (21) \text{ }$$

$$= \frac{\int_{r_0}^{\infty} l_c f_{l_c}(l_c) dl_c + \int_0^{\infty} x_0 f_{x_0}(x_0) dx_0}{\int_{r_0}^{\infty} E[t_c|l_c] f_{l_c}(l_c) dl_c + \beta + \int_0^{\infty} E[t_f|x_0] f_{x_0}(x_0) dx_0} + E[v]$$

B. Simulation results

In addition to the simulation settings introduced earlier, the Poisson arrival rate λ is varied from 0 to 1.5. With E[v] = 25, the spatial distribution of the vehicles follows a homogeneous Poisson process with intensity ρ ranging from 0 to 0.06. For completeness of the plot, $\rho = 0$ is included which means there is only one vehicle on the road. Therefore the average number of neighbors (average node degree) varies from 0 to 30, which represents a large range of traffic densities.



Fig. 6. The expected information propagation speed for $\tau = 1s, 5s$. The confidence interval is too small to be distinguishable.

Fig. 6 shows the expected IPS for $\tau = 1s$ and 5s. It can be seen that when the vehicle density is low, the IPS is determined by vehicle speeds because there is little packet forwarding in the network. When the vehicle density increases,

small clusters are formed and the IPS is determined by the catch-up delay, which is further determined by the mobility of the vehicles. We can see that the more frequently the speed changes, the slower the information propagates. This is mainly because changing speed has the potential to interrupt the catch-up process, i.e. during a catch-up process, the tail may speed up and the head may slow down. As the vehicle density further increases, clusters become larger and the forwarding process starts to dominate. Therefore the IPS increases until it reaches the maximum value, which is determined by the per-hop delay in the forwarding process. The maximum IPS is equal to r_0/β .

VII. CONCLUSIONS AND FUTURE WORK

We provided analytical results on the IPS in mobile VANETs by studying the forwarding process and the catch-up process. We showed that various parameters such as vehicle density and speed variation can have significant impact on the IPS. By taking real world measurements such as λ , E[v], σ and τ , our results can provide a quick estimation of the IPS with a good accuracy. The results provide useful guidelines on the design of mobile VANETs. The analysis in this paper is conducted under the unit disk communication model and Gaussian speed distribution. In the future we will consider the impact of channel randomness as well as a generic speed distribution.

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